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Report on the thesis prepared by Gao Jingjing

Entitled
“Time of influence as a decision variable in game-theoretic models of opinion dynamics in social groups”
for the Degree of Candidate of Physical and Mathematical Sciences
Scientific specialty: 1.2.3. Theoretical information, cybernetics

This PhD thesis is devoted to studying strategic timing of influence controls and validations of opinion dynamics in social groups. Communication through social groups becomes a popular tool of interaction among people. The study of the thesis focuses on modeling opinion dynamics in small social networks under different constraints. It combines opinion dynamics, noncooperative games, and optimization to analyze how the opinions of social network members change over time. It first studies the effects of different modes of choosing significant periods to validate the agents’ opinions in a network to reach the target opinion. Then, it provides different modes to choose periods to control when the player has a limit on the number of such periods to influence agents. Finally, it derives the necessary conditions for the Nash equilibrium for two players in a competition on agents’ opinions when players minimize their costs in a linear-quadratic form. The thesis advances the development of the game theory involving social group dynamics and the practical application of time pattern strategies for optimization. The thesis also contributes to the foundation for the development of the network game theory where constraints on the patterns of the timing of actions are present.

In **Chapter 1**, models of opinion dynamics with different scenarios of restrictions on players’ or influencers’ behavior in small social networks are presented. Two restrictions are imposed in the analyses of this Chapter. First, the player assumes that only the agents’ opinion in the last time is significant. Her objective function would contain only the agents’ opinion in the terminal time. Second, the player can choose a set of periods in which the agents’ opinion is considered to be significant. This set of agents’ opinions will be incorporated in the objective function. The optimization problems are formulated for a single-player case and the necessary conditions for the optimal strategies are found. A series of numerical simulations is conducted to test the

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results and to make conclusions about parameter influence on optimal strategies.

The Chapter presents an innovative way for charactering the optimal outcomes in the presence of timing strategies. A comment of this Chapter is "On page 110, lines 3-7: the thesis stated that: "From our numerical experiments, we find that the increasing number of validation periods reduces the player's costs, and that if the validation period is optimal for the given number of such periods, then it will be also optimal if the number of periods is increased."

But according to Equation (1.12) on page 29

$$J(u)=\sum_{t=0}^{T-1}\delta^t c(u(t))^2+\sum_{j=1}^{k}\delta^{t_j}\left((x_1(t_j)-s)^2+(x_2(t_j)-s)^2\right)$$

$$+\delta^T\left((x_1(T)-s)^2+(x_2(T)-s)^2\right), \qquad (1.12)$$

an increase in the number of validation periods k would increase the player's cost $J(u)$.

I think this is likely a computer computation problem, and the general theory and direction of research still would hold. The problem is not a theoretical problem. The candidate's address/clarification is appreciated.

In **Chapter 2**, the models of opinion dynamics are subject to different scenarios of restrictions on players' or influencers' control strategy. These restrictions are as follows:

(i) the player can control the agents or validate their opinions, but these two actions cannot be done at the same time,

(ii) the player can choose the periods when she controls opinions and simultaneously validates the opinions, and the size of this set is limited, and

(iii) the player can choose the set of periods to control the opinions but they should be different from the time when she is validating opinions of agents.

In all these models, the optimization problems are formulated and necessary conditions for the optimal strategies are proved.

A very positive extension of the Chapter can be readily constructed to generate a

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general Theorem.

The analysis can be readily extended to a general theorem in which

(i) the player would make n_k validations, and

(ii) pursue m_p control actions.

(No need to satisfy the conditions $M \cup N = \{0,1,\cdots,T-1,T\}$ and $M \cap N = \phi$).
This will be a significant general theorem for solving the optimal time patterns of validation and control, and its applications can cover a wide range of examples.

Since the technical results have been developed, the candidate may address the issue without repeating all the technical results.

Chapter 3 considers game models of opinion dynamics in small networks with two players or influencers and examines their competition on social opinion under constraints on the number of validation periods or periods to control them.

Three main scenarios are (i) players validate the agents' opinions only in the terminal time, and (ii) players can choose the set of validation periods with the limitation on the size of this set, and this set is the same for both players, or (iii) this set may be different for the players. The necessary conditions for the Nash equilibria for all scenarios are obtained.

The Chapter develops an innovative way for the characterization and analysis of equilibria in dynamic games of social groups' opinion with timing strategies.

Finally, I would suggest a very minor modification of the title of the thesis to **"Strategic timing as a decision variable in game-theoretic models of opinion dynamics in social groups",**
which may reflect more closely the content of the thesis. (This is only a semantic suggestion).

Overall, the thesis is well-written with rigorous mathematical results and the contents are clearly presented. It presents an innovative way for the characterization and analysis of the equilibrium behavior of opinion dynamics in social groups. The thesis has successfully answered all specific research questions (stated on end of page

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12 and top of page13 of the thesis):

Question 1: What is the optimal set of significant periods which should be taken into account into the objective function of the influencer?

Question 2: When would it be an optimal set of periods for a player to control for agents' opinions?

Question 3: What is the effect of competition between influencers on opinion dynamics and what are their costs when the number of players is more than one? What are the equilibrium outcomes?

In sum, scientific novelty of thesis is created through new models and solution techniques.

The main results of the thesis have generated seven published scientific papers, including 3 indexed in Scopus and Zentralblatt MATH and 2 recommended by the Higher Attestation Commission of the Russian Federation. The main results of the thesis were presented at five conferences, 4 of them were international conferences. The thesis "Time of influence as a decision variable in game-theoretic models of opinion dynamics in social groups" by Gao Jingjing deserves to be awarded the Degree of Candidate of Physical and Mathematical Sciences with very minor addresses of the comments mentioned in this report.

Yours Sincerely,

31.03.2025

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