Report on the thesis prepared by Vitalii Alexeyevich
Entitled "Equilibrium behaviour in dynamic models of competition
with network interactions"
for the Degree of Candidate of Physical and Mathematical Sciences
Scientific specialty: 1.2.3. Theoretical informatics, cybernetics

The PhD thesis is devoted to the study of dynamic competition with network interactions and the resultant equilibrium behavior. Network games, especially dynamic network games, have become a fast-developing branch of mathematical game theory in recent years. A distinctive feature of network games is the dependence of the players' payoffs on a network of connections and interactions. The possibility of communication between connected players also becomes available. Therefore, the specific interactions of players within a connected network offer a new opportunity to analyze the relations between players with different, similar, and unrelated interests mathematically. Theoretically and practically, networks of conflict, coalition, collaboration, and integration are more prevalent and interesting than games without network structures. The thesis constructs a dynamic investment and network modification in the Cournot oligopoly with exogenously formed network interaction that yields unique open-loop and feedback Nash equilibria. Using this convenient model, the effect of network parameters, the firms' mutual influence, the equilibrium behavior of firms, profits and the dynamics of competitiveness are derived. The thesis also demonstrated that the conceptual approach to dynamic models with endogenous network interaction can be applied to problems in the economic sector, such as the conditions for selecting business partners. The derived new results on dynamic competition models with network interaction complement the existing models of game-theoretic analysis (which was without structural interaction of players). The thesis contributes to the development of the network game theory by introducing the game components as dynamics and multicomponent behavior of players through the players' network structural interactions.

In Chapter 1, a dynamic competition model with double-component (production and investment) behavior of firms with exogenous formation of their network interaction is constructed. Both the open-loop and feedback information structures are considered. The existence of unique Nash equilibria in both cases are obtained. A comparative analysis of the results demonstrates the impacts of network parameters and structures on the equilibria, market price, output, the firms' profits, and externalities with numerical simulation. Although the use a model with

specific functional forms may not be robust enough to prove the generality of the results, yet the results are important to start the analysis of the young field of dynamic games with network interactions.

Minor comments:

On page 20 -- $\rho \in (0 1]$ is the discount factor (and not factor rate).

On page 27 – "this matrix is negatively defined" should be "this matrix is negative-definite".

On page 28 bottom line – Networks g_1 and g_4 are star graphs. Check the graph of g_4.

On page 33 first line – g_4 there is a disconnected graph with an isolated vertex.

Actually g_2 is a disconnected graph with an isolated vertex. Check the graph.

Chapter 2 extends the study in Chapter 1 to the case in which the firms' behaviors are divided into network, production, and investment. Unilateral interactions represented by arcs and bilateral communications represented by edges in network structures described the formal component of the endogenous interaction variants and specified some components of the model under consideration. Open-loop Nash equilibrium under bilateral interaction of firms is searched. In particular, the thesis searches for the open-loop Nash equilibrium for models with constant network interaction between firms under unique-time and variable network communication costs, as well as the conditions of network interaction between firms. Using numerical simulations, benchmarks of the open-loop Nash equilibria for all models are obtained to evaluate the advantages and disadvantages of two network interaction durations — short-term (with rearrangement of the network structure at each decision point) and long-term (with the network established by firms at the initial time point). The chapter analyses three dynamic models with endogenous network formation (with both bilateral and unilateral connections), The similarities in the corresponding Nash equilibria are identified. An important result is that in models with endogenous formation of a permanent network structure, the Nash equilibria exhibit the stability of the network interaction structures in that no firm will have the incentive to change the existing network connections.

Chapter 3 attempts to use models of competition in real economic processes to demonstrate the application of the obtained results, both conceptual and methodological. For this purpose, the

thesis adopts a set of special assumptions that firms adhere to in real conditions of competitive production and conducts a benchmarking of short-term and long-term network interactions in the Nash equilibria under time-varying and constant investment behavior of firms. The thesis also investigates a model in which firms use one-time investments. In particular, it represents an attempt to respond to the fundamental questions of equilibrium behavior and the corresponding conditions for rational interaction of each pair of firms with a set of assumptions. The assumptions are considerably close to the practical situation. While the assumption of a one-time initial investment is not unrealistic, the assumption of a constant volume of production $u_i(t) = u_i$ needs more refinement in justifying it. (Especially u_i is an optimal response of the firms over time when things like costs are changing).

A point to note is that the Introduction deserves to be considered as the first Chapter (that is Chapter 1) of the thesis. It outlines the relevance of the research topic, provides a rather comprehensive literature review, states the research purpose and objectives, highlights the scientific novelty of the analysis, discusses the theoretical and practical significance of the research, gives the structure of the thesis and the main scientific results, specifies the research methodology, and lists the scientific presentations and publication of the results from the thesis. It provides the readers with a clear preview of the thesis and makes the organization of the thesis more formal with an Introductory Chapter. The thesis has completed all the tasks formulated within the framework and has achieved the objectives proposed.

Overall, the thesis is well-written with rigorous mathematical results and the contents are clearly presented. It presents a new way for the characterization and analysis of the equilibrium behavior of competing firms in dynamic models with network interaction. An innovative framework characterizes investment and network modification of the Cournot oligopoly in which firms competing in the common sales market are endowed with the ability to implement multi-component strategic behavior dynamically. The attempt for the adaptation of models with endogenous formation of network interaction to the characterization and analysis of the equilibrium behavior of firms in market conditions is successful. The main results of the thesis have generated three published scientific papers and ten presentations at noted scientific conferences. The thesis has scientific novelty and developed original knowledge in the Scientific specialty: 1.2.3. (Theoretical informatics, cybernetics). The thesis "Equilibrium behaviour in dynamic models of competition with network interactions" by Vitalii Alexeyevich

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deserves to be awarded the Degree of Candidate of Physical and Mathematical Sciences with very minor addresses/revisions and the defense of the main results as stated at the end of the Introduction (page 15) of the thesis.

Yours Sincerely,

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20.06.2024