Review of “Search for the Optimal Road Alignment On The Terrain In Terms of Construction Costs” by A.S. Sharlai

The aim of this thesis is to present several algorithms to identify a road design on a terrain, that minimises the cost of material used and transported. After a brief exposition of the formulation of the problem, the candidate presents and analyses four different approaches to solve it and presents results of numerical experiments to compare these approaches.

The first chapter of the thesis provides some background information, including the main notations and the mathematical framework under which the analysis will take place.

The second chapter introduces the topic. First, the problem of designing a road on uneven terrain is formulated mathematically, as an integro-differential problems. The basic assumptions that are used in the model are that the cost of building the road comprises the cost of construction itself, and the cost of delivering the building material. It is assumed that the construction technology is the same at all points along the trajectory, but that construction conditions vary from point to point. This enables the candidate to formulate the cost of constructing the road between two points. In the second part of this chapter, this function is analysed and necessary conditions for a trajectory to achieve minimum cost are obtained.

In the third chapter, several methods are proposed to find an approximate solution. The first method is based on interpolation by polynomials. Namely, a sequence of polynomial functions over a uniform grid covering the area is constructed to solve approximately the necessary conditions. It shown that the sequence of iterates converge towards the solution of the problem, which is confirmed using an example. The second

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approximation method is based on Ritz method. With this method, instead of using the necessary condition for optimality, the authors change the original problem, where the trajectory is replaced by a function from a specific family (say, polynomials, or trigonometric polynomials). The new problem can then be solved, and by increasing the size of the family of functions used (such as the degree of the polynomials) an increasingly accurate approximation to the actual trajectory is achieved. The third method proposed in the chapter is Galerkin's method. Here the solution to the equation governing the necessary condition for optimality is approximated using an generating function based on a specific basis (such as trignonometric functions), which satisfy some orthogonality conditions. Truncating the generating function yields a new equation that can be solved numerically.

The fourth and last chapter introduces another method for approximating the optimal trajectory, based on the shooting method. This method replaces the nonlinear differential equation with a linear approximation, which is then discretised. It is also shown in the chapter that the necessary condition for optimality actually has a unique solution within a set where the values of the trajectory and its derivative are bounded appropriately.

All methods discussed in this thesis are implemented and examples are provided throughout the thesis. The code for the implementations is provided in the appendix.

The results presented in this paper were published in peer reviewed journals. In my opinion the work submitted is interesting and worthwhile of a doctorate. The thesis provides a clear presentation of the results, which, to the best of my knowledge, are scientifically new and therefore acceptable for publication and for the fulfillment of the PhD degree completion of Mr Sharlai Artem Sergeevich, subject to some revisions and discussion outlined below and which we can discuss during the defence.

First, the English used in the thesis made it challenging to read. Especially, I found the fourth chapter very difficult to understand. It is difficult to write a document like that in a language that is not one's mother tongue, but nonetheless the candidate should make some efforts in using appropriate mathematical terms. A number of words in Russian are left in the translation. It is better to make sure those are properly translated: I suggest that a quick search for cyrillic letters through the document could have helped address this problem. There are also multiple issues with repeated words which could have been addressed from a careful re-reading.

I also have some comments on the mathematics, which I think would have helped improve the thesis:

1. p. 18. It would be good to explain the formulation of the functional $J(x)$. It is not obvious to see how the formulated assumptions lead to this particular formulation. It seems that $\sqrt{1 + y'(x)^2}$ represents the local cost at position x (which is then

intergrated appropriately), but it is not clear why. It would also be useful to explain that the first term corresponds to the material transportation cost and the second one corresponds to the construction cost.

2. p.44, proof of Theorem 4.2.3. I did not understand how the problem (4.1) is equivalent to the equation $Ay(x) = y(x)$. It would have been good to expand on this to provide some explanations.

3. p.50, equations (4.13) and (4.14). It would have been helpful to explain why these inequalities are true.

Also, there are minor comments below.

1. p. 12 - the notation $\mathbb{E}$ is used in Definition 1.1.1, but not explained before Definition 1.1.3. It should be explained the first time it is used.

2. p.14, top - "takes the *largest* value" is it really the largest? In the rest of the thesis, the candidate actually minimises this integral.

3. p.22 - the last formula for $\delta J(y*)$ is difficult to parse, because of the use of two integrals on the same variable (dx). At least I recommend adding parentheses around the inside integral to make it clearer.

4. p. 24: "It can be solved by considering the values of the function in the nodes". The notion of nodes is used before being defined.

5. p. 39: "Applying Galerkin's method we obtain"... The numbers obtained are exactly the same as at the top of the page, and then some other numbers are given in the next page. I think there is an issue that should be addressed here.

6. p.48, Definition 4.2.3: what is the meaning of "quite continuous"? This is not a common terminology, so it should be defined first.

7. p.48, Theorem 4.2.2: The sentence "A maps a bounded closed set S of Banach space onto its part" is not clear.

8. p.53, "Primer" 4.2.1. The example is a bit unfortunate, since the solution appears to be 0. It would be a good idea to test the method on a problem where the solution is *not* 0. Perhaps one of the problems from Chapter 3.

Julien Ugon December 05, 2024