Saint-Petersburg State University

Nikita G. Ivanov

Modeling and analysis of non-stationary stochastic processes in production enterprise management systems

Scientific specialisation 2.3.1.

System analysis, control and information processing, statistics

Dissertation is submitted for the degree of

candidate of physical and mathematical sciences

Translation from Russian

Scientific supervisor:

Doctor of Physical and Mathematical Sciences, Professor

Alexander V. Prasolov

Saint-Petersburg – 2024

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INTRODUCTION

Relevance of the topic. The dissertation considers the analysis of management systems for a full cycle production enterprise: raw material extraction, transportation, supply, production, storage, financial system, commercial system, and personnel management (Fig. 1). Auxiliary systems, such as business process organization, quality control, and lean production system (aimed at eliminating various types of losses), are not indicated in the figure. Similar systems, for example, are identified by Shiryayev E.V., Shiryayev V.I., Baev I.A. (Fig. 2). For each of these systems, a number of tasks can be identified for which there is a mathematical formulation, and for some of them, a mathematical solution is provided in the dissertation. Thus, if companies manage their work through standard systems, then these problem formulations and possibly problem solutions, using the example of a full-cycle production enterprise of peat soils and organo-mineral fertilizers, may be relevant to other researchers and entrepreneurs.

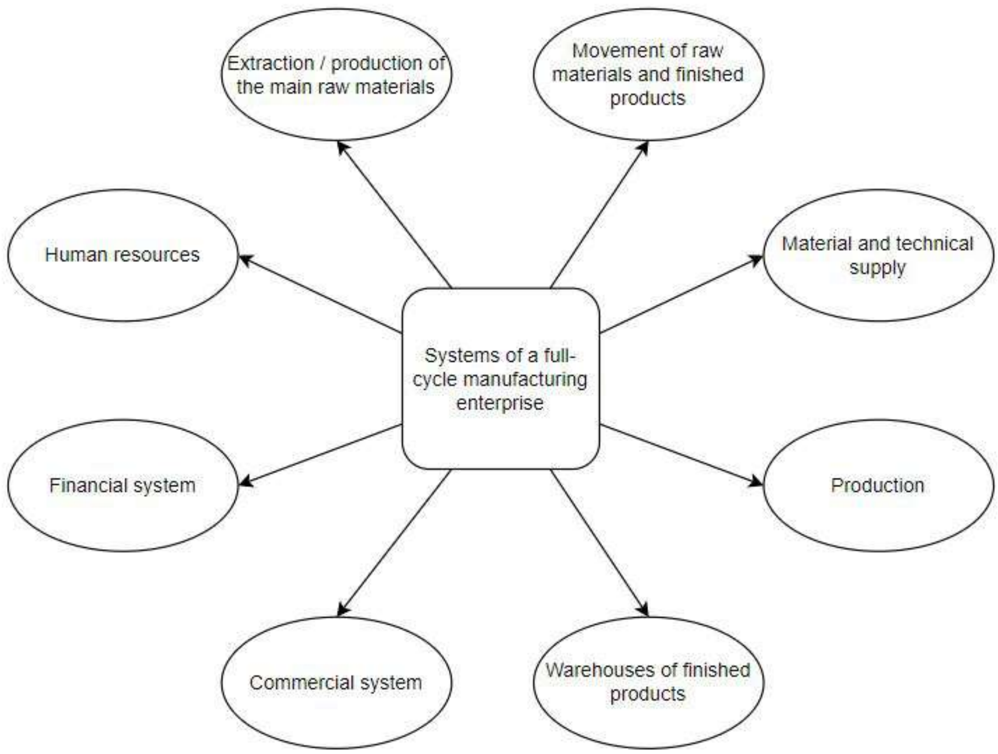

Figure 1 – Systems of a full-cycle production enterprise

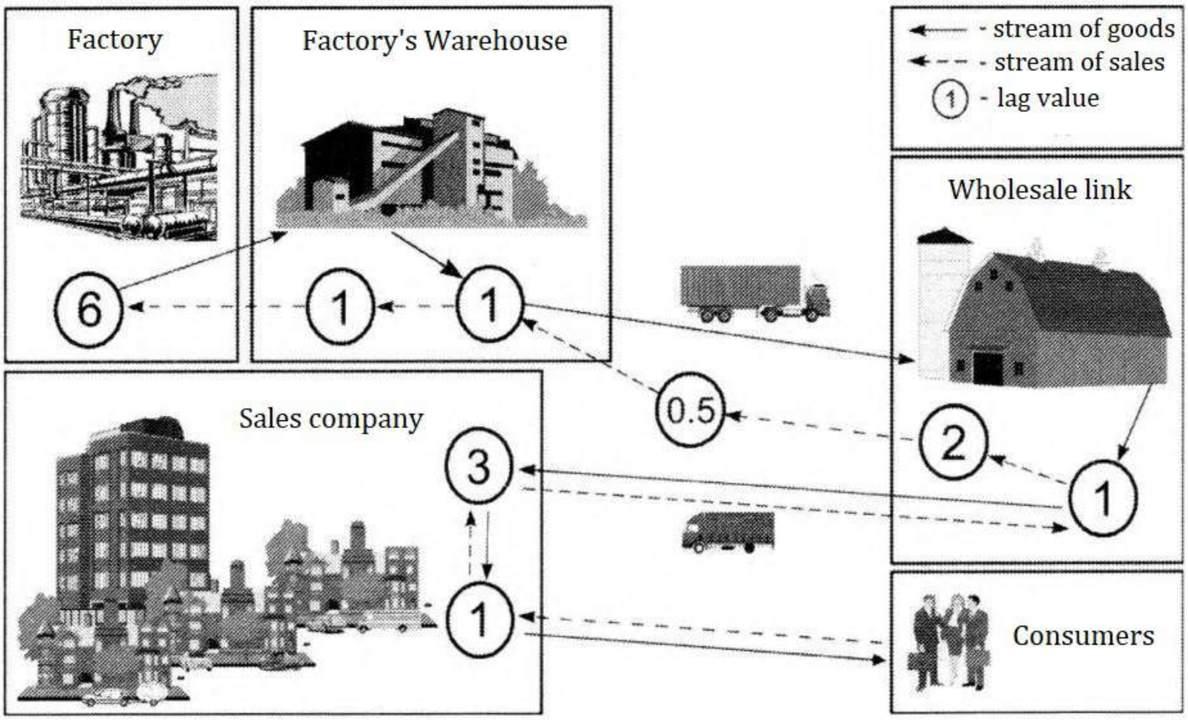

Figure 2 – Structure of a production company with delays in days

Modern companies face growing demands for speed and accuracy in their management decision-making. One of the key tools that enable this is forecasting. It allows for more accurate predictions of future changes, analysis of possible scenarios, and decision-making based on data. However, for the tasks of the considered systems, there is a problem with forecasting: either obtaining a forecast is a solution to a finite task, or subsequent chains of actions depend on the result of forecasting. As the subsequent management decisions depend on the forecast result, it is necessary to mathematically study these problems deeply enough. Standard forecasting methods often do not take into account complex dependencies between different factors and cannot predict non-stationary processes. Therefore, the development of new mathematical models and forecasting methods is a relevant task for scientific and practical areas. The main part of the work is devoted to mathematical modeling of non-stationary stochastic processes, including such aspects as trend determination, calculation of optimal forecasting horizon, dependence of depth on forecasting horizon, and extraction of piecewise-stationary processes from non-stationary ones.

The results of mathematical modeling, due to the study within non-stationary processes, are applicable to real processes. Therefore, the research results are relevant not only for enterprise tasks but also for financial time series, such as stock prices or currency exchange rates, where processes can change over time. Modeling can also be applied to meteorological time series, where parameter values such as temperature, atmospheric pressure, humidity, etc., can vary over time and external factors. In addition, non-stationary processes can be observed in biological and medical time series, such as pulse, blood pressure, hormone levels, etc. In general, modeling non-stationary processes can be applied to various types of time series where processes change over time and depend on various external factors. For example, in this dissertation, the author considered examples of temperature data, currency exchange rates, wheat and «Brent» oil prices, and sales of peat soil.

Aim and Objectives. The aim of this research is to conduct a comprehensive analysis of enterprise management systems, formulate mathematical problems, and develop algorithms for their solution in mathematical form. In particular, it is necessary to study non-stationary stochastic processes in order to develop algorithms for trend determination and forecasting horizon, and to test these algorithms on time series of various nature, including in enterprise management tasks.

To achieve this aim, the following tasks need to be solved:

1. Study relevant scientific research related to this topic and identify promising directions for research.
2. Systematize enterprise management problems into separate blocks and study each problem with different levels of mathematical elaboration.
3. Study the application of Fourier series and Chebyshev polynomials for modeling the trend of a time series.
4. Develop an algorithm for forecasting the horizon of time series.
5. Study the time series model as a piecewise stationary process.

6. Create an algorithm for determining the weight coefficients for the weighted least squares method using piecewise stationary modeling.
7. Test the algorithms on real data and illustrate the results.

Methodology and methods of the research were based on the application of mathematical algorithms and models for solving problems of analyzing enterprise management systems and analyzing time series in various formulations. The main idea was to create mathematical algorithms and derive mathematical models for finding solutions, followed by their testing on test examples. To achieve the goals of the research, methods of linear algebra, mathematical analysis, multicriteria optimization, and numerical methods were used.

Theoretical and practical significance of the work. The work has significant scientific and practical value due to the development of new mathematical methods and algorithms for analyzing time series data and enterprise management systems. The original management problems can be further investigated using more in-depth mathematical analysis. Time series analysis also has potential for further development, taking into account possible relationships between forecasting horizon and depth. Additionally, the forecast itself can be examined using the developed mathematical models.

The methods and algorithms proposed in the work can be used to solve various tasks in different fields, including forecasting economic indicators, meteorological data, planning production indicators, and analyzing financial markets. The developed methods and algorithms can be applied in practice to improve the efficiency of enterprise management and make more informed decisions.

The practical orientation of the work is its feature, supported by the successful application of methods and algorithms to real data of various nature. The implemented algorithms are also accompanied by software code, which can be used as a basis for creating software products to solve tasks related to forecasting horizon and monthly plan distribution on conveyor lines in a production enterprise.

The scientific validity and reliability are ensured by the correctness of the problem formulations obtained from scientific literature. The presented results were tested at multiple conferences, and the publications that formed the basis of the dissertation underwent review and were published in Russian and international journals. All proposed algorithms were programmed and tested on problems of different nature.

Personal contribution. The dissertation is the author's independent work. The main contributions presented in the defense represent the author's personal contribution. All results presented in the dissertation were achieved by the author, except where explicitly stated and referenced to the original source. A large part of the results is based on the author's publications in scientific journals and the author's final qualifying works at Saint Petersburg State University. The software code presented in the appendix was implemented by the author.

Approbation of the work. The results presented in this dissertation have been presented and discussed at conferences:

1. XLVI International Scientific Conference of Postgraduates and Students «Control Processes and Stability» (CPS'15), April 6-9, 2015, St. Petersburg, Russia.

2. III International Conference «Stability and Control Processes» (SCP 2015) dedicated to the 85th anniversary of Professor, Corresponding Member of the Russian Academy of Sciences V.I. Zubov, October 5-9, 2015, St. Petersburg, Russia

3. XLVII International Scientific Conference of Postgraduates and Students «Control Processes and Stability» (CPS'16), April 4-7, 2016, St. Petersburg, Russia.

4. XLVIII International Scientific Conference of Postgraduates and Students «Control Processes and Stability» (CPS'17), April 3-6, 2017, St. Petersburg, Russia.

5. All-Russian Forum «System of Distributed Situational Centers as the Basis for the Digital Transformation of Public Administration (SRSC-2017)» St. Petersburg, October 25-27, 2017, St. Petersburg, Russia.

6. 3rd International Conference on Applications in Information Technology (ICAIT–2018), November 1-3, 2018, Aizuwakamatsu, Japan.
7. IV International Conference «Stability and Control Processes» (SCP 2020) dedicated to the 90th anniversary of Professor, Corresponding Member of the Russian Academy of Sciences V.I. Zubov, October 5-9, 2020, St. Petersburg, Russia.
8. Management of Business in the Digital Economy. V International Conference, March 19, 2022, St. Petersburg, Russia.

Publications. The results on the topic of the dissertation are presented in eleven scientific publications [2–12], of which two articles are in the journal included in the list of journals recommended by the Higher Attestation Commission of the Russian Federation for publishing the results of dissertation research, and are indexed in WoS and Scopus [2, 3].

Support. The work was carried out with financial support from the Russian Foundation for Basic Research (project № 20-31-90063 «Development of mathematical modeling of non-stationary stochastic processes»).

Contents and structure of the work. The dissertation consists of an introduction, two chapters, a conclusion, and an appendix. The total volume of the dissertation is 129 pages, including 40 figures and 4 tables. The bibliography contains 73 references.

Brief summary of the work. The **introduction** justifies the relevance, sets out the objectives and tasks of the dissertation, presents the main positions to be defended, describes the scientific and practical value, and scientific novelty of the work. Information on the testing of the work, the main publications, and financial support is provided, followed by a brief summary of the work.

In the **first chapter**, the analysis of control systems for a full-cycle production enterprise manufacturing peat soils and organo-mineral fertilizers is considered in terms of enterprise management tasks.

A detailed model for calculating the monthly production plan, dependent on sales forecasts, actual and planned inventories at multiple warehouses in different cities, and inter-warehouse transfers is presented.

An algorithm for distributing the production plan among conveyor lines is developed. If the production is multi-assortment and packing machines can produce several product formats, and some conveyors are interchangeable with others in terms of formats, then the problem of distributing the monthly production plan by assortment and conveyor arises. The distribution criterion of minimizing the difference in conveyor operating times is considered, and a quasi-optimal gradient descent method is implemented to solve the problem.

A model for calculating the price based on a predetermined margin for network stores with retro bonuses proportional to the price is presented. Furthermore, a non-trivial real model for pricing on marketplaces is discussed, where the retro bonus and commission system is complex, and the problem of determining the price with given margin is solved by parameter estimation.

The mathematical problem of determining the recipe for a new mineral fertilizer with a specified N:P:K (Nitrogen (N): Phosphorus (P): Potassium (K)) basic mineral complex indicator by mixing several classic fertilizers is formulated.

A mathematical model for analyzing the assortment base and identifying a recommendation system for managers is applied. If customers order a certain product, the correlation matrix is used to determine what else they purchase with it, and the most popular choices are suggested to the customer by the manager.

An algorithm for selecting the optimal method of delivering products to customers, taking into account the possibility of direct delivery or through the company's own warehouses, is presented.

In addition to the aforementioned problems, for which mathematical formulations are created and solutions are found for some, not necessarily optimal, the chapter describes other problems from various enterprise management systems. At present, these problems do not have sufficient mathematical content, but they are necessary for a full description of enterprise management systems. Moreover, mentioning such problems in a mathematical dissertation can be useful for expanding knowledge in the field of enterprise management and attracting the attention of other researchers to these problems, possibly leading to new ideas and approaches for their solution.

The **second chapter** deals with a series of problems related to the development of non-stationary stochastic processes.

A model for approximating the trend of a non-stationary process using a Fourier series is considered. The inaccuracy of the approximation calculation of the Fourier series coefficients is taken into account by introducing and computing correction coefficients. The trend is approximated using a system of orthogonal Chebyshev polynomials, where the coefficients are determined using Bessel's formulas.

An algorithm is developed for computing the optimal forecasting horizon, while also determining the minimum number of harmonics needed to determine the trend of a time series. The algorithm is tested on real data.

To increase accuracy and account for the heterogeneity of non-stationary time series, the algorithm is modified to compute the mean value over several sections. A comparison is made between two methods of data averaging: the shift method and the stretch method.

A model of a time series as a piecewise stationary process, i.e., a set of sequential stationary intervals, is presented. An algorithm is developed for determining the region in which the trend is located for this model.

It is known that the pure least squares method for trend estimation is not commonly used in statistical analysis and econometric software packages. Often, a weighted least squares method is employed to ideally remove heteroscedasticity. The author proposes an algorithm for estimating the weighting coefficients for this method using piecewise stationary modeling.

In the **conclusion**, the main results of the dissertation research are listed..

In the **appendix**, a software implementation in the MATLAB package of the production distribution algorithm for conveyor lines is provided.

Main scientific results.

1. An algorithm for quasi-optimal distribution of the production plan along conveyor lines by the method of descent has been developed, which allows redistributing the production between interchangeable conveyors with the aim of equalizing the operating time [2].

2. A study of the model of trend approximation of a time series using Fourier series has been conducted, in which correction coefficients are introduced to improve the accuracy of the model approximation [6].
3. An algorithm has been proposed for determining the forecasting horizon of time series and determining the necessary number of harmonics to obtain a correct horizon [4, 8].
4. An algorithm has been developed for extracting a piecewise-stationary process from a non-stationary time series and determining the band around it, in which the realization of this process lies with a certain probability [5].
5. An algorithm for determining the weighting coefficients for the weighted least squares method using piecewise-stationary modeling has been proposed [3].

Thesis statements to be defended.

1. The systematization of enterprise management tasks was carried out with a description of all tasks and a proposal of ideas for their solution. Mathematical algorithms for solving some of the problems have been developed.
2. Developed a model for approximating the trend of a time series using the Fourier series for a given number of harmonics. The model has been improved by introducing correction coefficients. The use of Chebyshev polynomials for trend modeling has also been analyzed.
3. Developed an algorithm for determining the forecasting horizon of time series of any nature.
4. Developed an algorithm for extracting a piecewise-stationary process from a non-stationary time series with the determination of the band in which any realization of this process lies with a certain probability.
5. Created an algorithm for determining the weight coefficients for the weighted least squares method using piecewise-stationary modeling.
6. The above models and algorithms have been tested using an applied mathematical package on real data, confirming the correctness of the algorithms

taking into account the differences in the nature of the data.

CHAPTER 1. Analysis of Mathematical and Economic Problems of a Full-Cycle Enterprise

1.1. Introduction

The problems of the enterprise have been described by many authors, primarily in the field of economic research. V.E. Shiryaev [1] focused on systematizing information on firm management, while G.B. Burdo and N.A. Semyonov [13,14] studied the management system of a multi-assortment machine-building enterprise. Several other authors will be mentioned in the corresponding paragraphs of this chapter.

Let's consider an enterprise engaged in the production of peat soils and organo-mineral fertilizers. The company aims to increase profits while maintaining high product quality. The enterprise has its own plot for extracting the main raw material — peat, a mixing workshop for raw materials, a packaging workshop for products, a material warehouse, a finished product warehouse, as well as a branch sales network with its own or leased warehouses. The company sells its products both to households (mostly through intermediaries) and to agricultural farms.

The goal of the chapter is to review and analyze the tasks of the financial, economic, and analytical departments of the enterprise, as well as to describe solutions or ideas for solving these tasks in the form of mathematical models. The activities of a specific enterprise can be divided into 12 blocks (Figure 3), where PnL stands for budget of revenues and expenditures, Cashflow stands for budget of cash flow movement, and KPI stands for key performance indicators. Several tasks from different blocks will be detailed mathematically with solutions or at least with problem formulation. The remaining sections of the first chapter (mostly logistics block [15], margin calculation block [16], project management block [17–21], and business process description block) will have a more descriptive content for a comprehensive understanding of enterprise processes and interrelations.

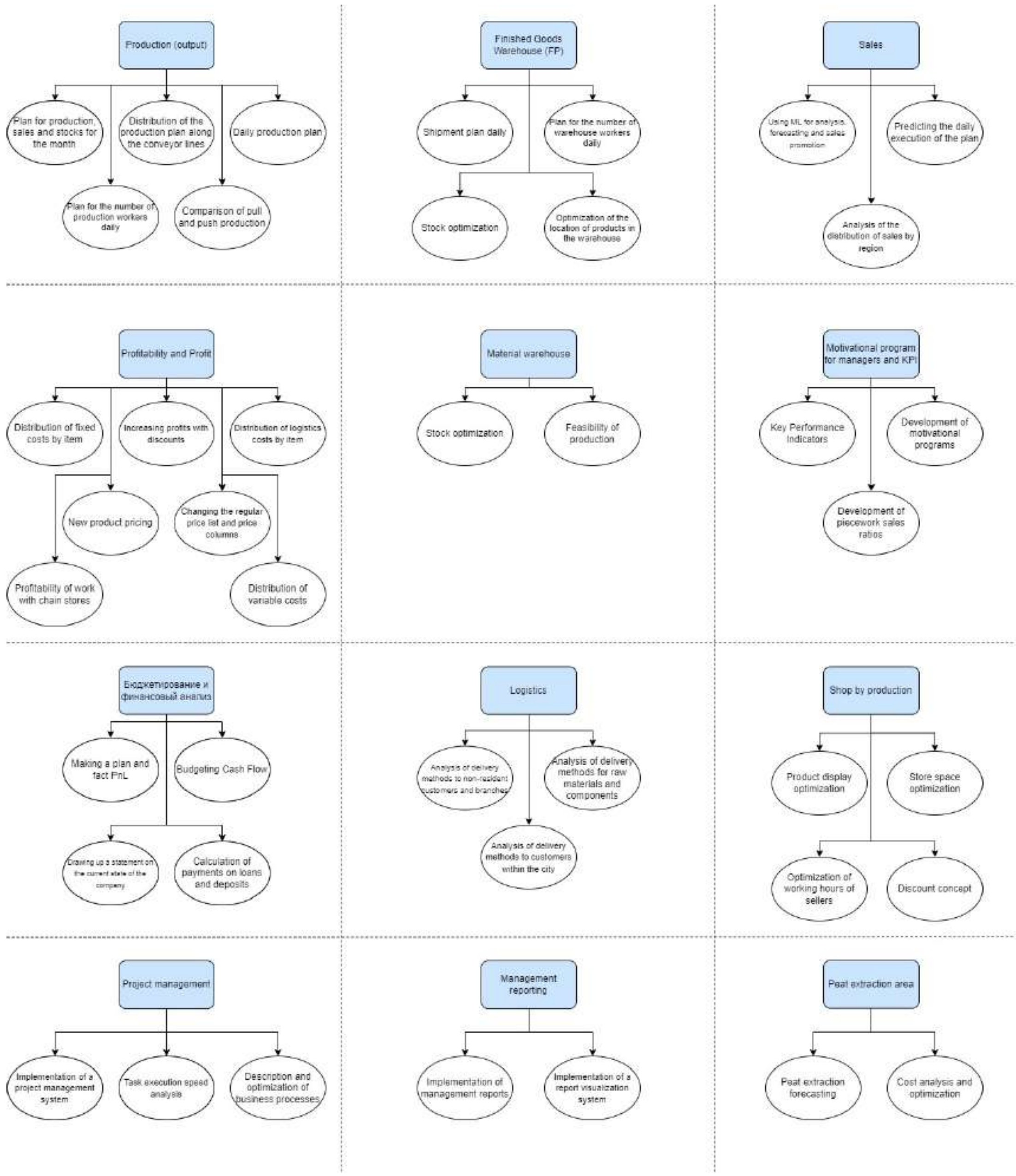

Figure 3 – Flowchart of analytical tasks

1.2. Production, sales, and inventory planning for one month

Profitability is influenced by the quality of production and sales plans forecasting, including understanding the optimal forecasting horizon [4, 5]. It is important to consider the accuracy of forecasting, taking into account demand constraints and shelf life [22].

There is a well-developed inventory management theory regarding the problem of optimal planning [23]. Let's consider an example of building an inventory management system for a specific production enterprise. The production enterprise includes a production complex with a warehouse and a sales branch, as well as other sales branches with their own warehouses. Every month, inventory levels are analyzed, sales statistics from past periods are reviewed, and a sales and inventory movement plan from the production warehouse to the branches is compiled. Then, a production plan is developed based on this calculation so that at the end of the month there are stocks of each item for a specified period ahead. After that, a constraint is introduced on the productivity of conveyor lines. The production plan is increased to the multiple of rolls of production film or pallets. This plan is then implemented in production.

It is important to make the sales plan forecast (and consequently the production plan) as accurate as possible because if the actual sales exceed the planned sales during production, there will not be enough stocks in the warehouse, and it will be necessary to urgently increase the production plan. This can negatively impact well-established processes. Conversely, if actual sales are lower than planned, warehouses become overcrowded, leading to both physical inconveniences in moving goods and financial issues such as renting additional space and purchasing additional shelves. Another significant aspect is that money remains tied up in inventory, which negatively affects the company's current financial position.

1.2.1. Problem statement

The company produces N types of products.

$P\,[N \times 1]$ – a monthly production plan showing the number of pieces to produce each of the N nomenclature items.

$T\,[N \times m]$ – a monthly plan for moving products for each of the m branches from the production warehouse to the branch warehouse.

$S\,[N \times m]$ – a monthly product sales plan for each of the m branches, displaying in each column the number of branch sales for each of the N nomenclature items.

$W\,[N \times m]$ – monthly sales plan from the production warehouse for each of the m branches.

$D\,[i \times m]$ – share of sales from the production warehouse of the total sales for each of the m branches.
$E\,[N \times m]$ – monthly sales plan from the branch warehouse for each of the m branches.
$F\,[N \times m]$ – the actual amount of sales for the 12 months of the branch for each of the m branches.
$C\,[N \times 1]$ – the seasonality of sales for each of the N nomenclature items in the month (in % of the year).
$O\,[N \times m]$ – actual product stocks for each of the m branches at the beginning of the month.
$R\,[N \times m]$ – the coefficient of change for each of the m branches relative to the same month of the previo

Then the mathematical problem can be formulated as follows. Required for the month of $k \in \{1, 2, ..., 12\}$ make a production plan $P\,[N \times 1]$, a sales plan $S\,[N \times m]$ (including $W\,[N \times m]$ and $E\,[N \times m]$) and a plan to move $T\,[N \times m]$ with the share of sales from the production warehouse set for each branch, fixed seasonality, set stocks at the beginning of the month, set limits on the next month's stocks, sales statistics for the last 12 months and under the condition of a given gain factor.

1.2.2. Model construction

First of all, let's clarify that

$$\begin{aligned} S_{i,j} &= W_{i,j} + E_{i,j}, \quad i = \overline{1,N}, j = \overline{1,m}; \\ W_{i,j} &= D_{i,j} S_{i,j}, \quad i = \overline{1,N}, j = \overline{1,m}; \\ E_{i,j} &= (1 - D_{i,j}) S_{i,j}, \quad i = \overline{1,N}, j = \overline{1,m}. \end{aligned}$$

Remark 1.2.1 A priori, we believe that calculations are carried out for the month k, however, if other months are also important for calculating some formula, then we will indicate the index from above.

Let's form the sales plan S based on the fact that the business is seasonal, and the change R is specified:

$$S_{i,j} = R_{i,j}\frac{F_{i,j}}{\sum_{x=1}^{12} C_{i,x}}C_{i,k}, \quad i = \overline{1,N}, j = \overline{1,m}.$$

Using $D_{i,j}$, we get $W_{i,j}$ and $E_{i,j}$, $i = \overline{1,N}$, $j = \overline{1,m}$. Let the warehouse of the branch № 1 be located during production, from which the products are moved to the warehouses of other branches and to the customers of all branches. Then $W_{i,1} = S_{i,1}, E_{i,1} = 0, D_{i,1} = 1, T_{i,1} = 0$, $i = \overline{1,N}$. Next, we will combine the leftovers, production and sales into a single model for the warehouse at the production and branch warehouses:

$$\begin{cases} O_{i,j}^k + P_{i,1}^k - \sum_{x=1}^m (T_{i,x}^k + W_{i,x}^k) = O_{i,j}^{k+1}, \quad i = \overline{1,N}, j = 1. \\ O_{i,j}^k + T_{i,j}^k - E_{i,j}^k = O_{i,j}^{k+1}, \quad i = \overline{1,N}, j = \overline{2,m}. \end{cases} \tag{1}$$

The company has enough production capacity to produce products without large stocks, moving to branches takes little time. Therefore, it was decided, at the end of the planned month, to have a stock of leftovers for half of the forecast sales of the next month from the warehouse in advance (although there may be another share of sales of the following months):

$$\begin{cases} O_{i,j}^{k+1} = \frac{1}{2}\sum_{x=1}^m W_{i,x}^{k+1}, \quad i = \overline{1,N}, j = 1; \\ O_{i,j}^{k+1} = \frac{1}{2}E_{i,j}^{k+1}, \quad i = \overline{1,N}, j = \overline{2,m}, \end{cases}$$

where

$$\begin{cases} S_{i,j}^{k+1} = R_{i,j}\frac{F_{i,j}}{\sum_{x=1}^{12} C_i^x}C_i^{k+1}, \quad i = \overline{1,N}, j = \overline{1,m}. \\ W_{i,j}^{k+1} = D_{i,j}S_{i,j}^{k+1}, \quad i = \overline{1,N}, j = \overline{1,m}. \\ E_{i,j}^{k+1} = (1 - D_{i,j})S_{i,j}^{k+1}, \quad i = \overline{1,N}, j = \overline{1,m}. \end{cases}$$

So, now we have sales for branches, stocks at the beginning of the month and stocks at the end of the month. Therefore, it is not difficult to deduce a plan for moving products to branches from the second equation of the system (1):

$$\begin{cases} T_{i,j}^k = 0, \quad i = \overline{1,N}, j = 1. \\ T_{i,j}^k = E_{i,j}^k + \frac{1}{2}E_{i,j}^{k+1} - O_{i,j}^k, \quad i = \overline{1,N}, j = \overline{2,m}. \end{cases} \tag{2}$$

Remark 1.2.2 If in the system (2) $T^k_{i,j} \leq 0, i = \overline{1, N}, j = \overline{2, m}$, then $T^k_{i,j} = 0$.

Let's return to the first equation of the system (1) and express P.

$$P^k_{i,1} = \sum_{x=1}^{m} (T^k_{i,x} + W^k_{i,x} + \frac{1}{2} W^{k+1}_{i,x}) - O^k_{i,1}, \quad i = \overline{1, N}. \tag{3}$$

Remark 1.2.3 If in the equation (3) $P^k_{i,1} \leq 0, i = \overline{1, N}$, then $P^k_{i,1} = 0$.

As a result, we determined a sales plan for S in the context of W and E, moving T for each branch, and a production plan for P.

1.3. Distribution of production plan across conveyor lines

With the monthly production plan in place, the production manager needs to understand how much and what type of products need to be produced on each conveyor. While the manager may be able to do this intuitively based on experience, it would be desirable to create a mathematical model for distributing the overall production plan into production plans for each conveyor line.

Remark 1.3.1 Variables will be re-introduced in this paragraph, they are not related to the previous paragraph.

1.3.1. Problem statement

With a large number of conveyor lines and the performance characteristics of each of them, it becomes necessary to solve the problem of optimal distribution of products from the production plan along the conveyors. Optimality criteria can be different: the minimum required number of conveyors to fulfill the plan, the minimum number of equipment changeovers (minimizing downtime), minimum production time, minimizing working hours (minimizing labor costs).

Such a task is not new in the scientific literature. There are mathematical models for the operation of production lines with many sequential and overlapping operations in the technological route [24]. However, such serious models are mainly used in large enterprises with a long assembly cycle and a large amount of work in progress. For small enterprises, the number of technological operations may even be one. The only difficulty lies in the fact that one technological operation can be performed in parallel on several interchangeable conveyors.

In one company producing peat soils in consumer packaging, there is a need to distribute the production plan so that it does not exceed the maximum capacity of the equipment, and the difference in the operating time of the conveyors would be minimal. Economically, such a criterion is related to the fact that all workers during each shift are engaged in the production of products, that is, maximizing labor productivity.

Let's consider the problem of distributing the production plan along the conveyors in a quasi-optimal way according to the criterion of minimizing the difference in the working time of the conveyors. Let the specifics of the products be that there are types of products (SKU — stock keeping unit), each of which has a parameter — volume (liter). Each of the conveyors can produce only certain volumes of products, and the volumes on different conveyors may overlap. The conveyors differ from each other both in the productivity of products for the same volumes, and products on the same conveyor for different volumes.

The company produces N types of products of p types of stained glass windows. Let's assume that $L_{N\times 2}$ is a production plan that displays in the first column the number of pieces for each of the N SKUs, in the second column is the number of SKUs (one of the p options). There are m conveyors in total, each of which can produce no more than p volumes. Let's introduce $C_{m\times p}$ — a matrix of conveyor performance (pieces per shift), $R_{m\times p}$— a matrix of distribution of the plan by pipelines and volumes in pieces, $S_{m\times 1}$ — a column vector, displaying the production time of each conveyor in shifts, r — the number of work shifts per month.

Let's formulate the problem strictly mathematically.

$$\begin{cases} \sum_{j=1}^{p}\sum_{k=1}^{m} R(k,j) = \sum_{i=1}^{N} L(i,1) \\ S_k = \sum_{j=1}^{p} \frac{R(k,j)}{C(k,j)}, \quad k=\overline{1,m},\ C(k,j)>0 \\ d(S) = [max(S) - min(S)] \to min \\ S_k \leqslant r, \quad k=\overline{1,m} \end{cases} \tag{4}$$

It is necessary to define the matrix R under the condition that the system is running (4), if the parameter r is known, the performance matrix C and the production plan L are specified. It can be seen from the system (4) that there are fewer equations than unknowns. In addition, the matrix R has an integer form, so the gradient descent method [25] is not applicable. Then we will solve the problem using a numerical method, which will actually be the descent method for a discrete function.

1.3.2. Solution algorithm

Let's assume that the production plan was set, the productivity of the equipment and the number of work shifts per month were set. Due to the fact that a numerical method is chosen for the solution, we will set the variable ε to stop the algorithm. In the performance matrix $C_{m\times p}$, if some conveyor cannot produce products of a certain volume, then the corresponding element of the matrix $C(k, j) = 0$ and the corresponding element of the matrix $R(k, j) = 0$.

1. Preliminary distribution of the plan along the pipelines.
   For each volume $j = \overline{1, p}$ we will distribute the products along the pipelines. To do this, we sum up i from 1 to N only those elements of the plan $L(i, 1)$ for which the condition $j = L(i, 2)$ is fulfilled. Next, we will distribute the resulting amount evenly across all pipelines with a capacity of j greater than zero. As a result, we get the filled matrix $R_{m\times p}$.

2. Pre-allocation of working time across pipelines.
   The sum of $j = \overline{1, p}$ of the ratios of the matrix R to the matrix C (if $C(k, j) = 0$, then this ratio is not included in the sum) shows the operating time of each pipeline $S(k, 1)$. After going through all the pipelines at $k = \overline{1, m}$, we get the vector S. Let's fix its maximum $\max(S)$ and minimum $\min(S)$ values.

3. Reduction of the maximum working time difference between pipelines.
   In order to reduce the operating time of the «maximum» conveyor, it is necessary to move the pre-distributed products of this conveyor to the «minimum» conveyor in the amount of predetermined q pieces, provided

that the conveyor is able to produce products of this volume. Thus, $\max(S)$ will decrease and $\min(S)$ will increase.

4. Checking the quasi-optimality of the solution
   4.a. as long as $d(S) \geq \varepsilon$, repeat step 3.4.;
   4.b. if $d(S)$ is greater than $d(S)$ in the previous iteration, then stop the algorithm and consider the previous result to be quasi-optimal, because otherwise the algorithm will start to loop.

Thus, the algorithm leads to a certain distribution of the production plan, it is not necessarily the only one, but it is important that the conditions (4) are met.

Remark 1.3.2 If some subsets of the set of pipelines do not intersect in volumes or in the composition of the contents of the package with other pipelines, then the above algorithm should be used for each such subset. It is also possible that only one type of conveyor produces products of the required volume or composition. For such pipelines, the preliminary distribution of the plan will coincide with the final one.

Remark 1.3.3 If, according to the results of executing the algorithm on some conveyor, the required number of production shifts to complete the plan is more than the number of shifts per month, then it is necessary to reduce the production plan by the corresponding volumes, and execute the algorithm again. The reduction of the plan can also be automated.

Remark 1.3.4 Depending on the number of q pieces being rearranged from one pipeline to another, the minimum value of ε is determined in one iteration.

Statement 1.3.1 Let all pipelines of the set intersect in volumes. If the stop condition is determined by an inequality

$$\varepsilon \geq \frac{q \times N \times 2}{min_{C(k,j)>0} C(k,j)}, \tag{5}$$

then the algorithm will not loop.

Proof. In order for the algorithm not to loop, it is necessary to define such a ε so that at each iteration of paragraph 4.a. of the algorithm, $d(S) =$

$[\max(S) - \min(S)]$ decreases slightly. In (5), the maximum possible numerator for $d(S)$ is set (all N positions are removed from one pipeline and added to another q pieces at a time) and the minimum denominator (the minimum non-zero element of the conveyor performance matrix). Thus, such a ε will be determined, in which $d(S)$ will not strictly decrease with each iteration.

The statement 1.3.1 has been proved.

Remark 1.3.5 With a more detailed analysis and in particular cases of the performance plan and matrix, most often ε can be done less than in the inequality (5), however, the algorithm may be forced to stop due to looping.

Remark 1.3.6 The parameter q in the primitive case is equal to 1 pc. To speed up the algorithm, sacrificing accuracy, q can be another constant natural value greater than one. Another option that speeds up the algorithm is the case when q varies from iteration to iteration depending on the intermediate R or S. To be even more precise, it should be borne in mind that q may have physical limitations, for example, in terms of the number of pieces on a pallet, the number of consumer packages from one roll of film. Then q can be a vector function of $q(i), i = \overline{1, N}$.

1.3.3. Example

A production plan is set for $L_{51\times2}$, with a maximum of 60 shifts per month (r=60). In total, there are 7 conveyor lines in production, each of which makes up to 5 types of stained glass windows. The hardware performance matrix $C_{7\times5}$ is known (Table 1).

Table 1. The C performance matrix for a 12-hour shift

Conveyor №	Volume, L				
	2,5	5	10	25	50
1	0	0	0	6187.5	5062.5
2	0	11 092	9787.5	0	0
3	0	11 092	9787.5	0	0
4	0	12 398	9787.5	0	0
5	12 398	12 398	9787.5	0	0
6	12 398	12 398	9787.5	0	0
7	12 398	12 398	9787.5	0	0

Let's form a preliminary matrix $R_{7\times5}$, distributing the plan $L_{51\times2}$ evenly over the volumes and conveyors, for which we calculate the initial vector of work shifts $S_{7\times1}$ for each conveyor (Table 2).

Table 2. Vector S and preliminary matrix R

Conveyor №	Num. of shifts, S	Volume, L				
		2,5	5	10	25	50
1	57.557	0	0	0	190 120	135 830
2	17.671	0	106 372	79 093	0	0
3	17.671	0	106 372	79 093	0	0
4	16.661	0	106 372	79 093	0	0
5	20.502	47 625	106 372	79 093	0	0
6	20.502	47 625	106 372	79 093	0	0
7	20.502	47 625	106 372	79 093	0	0

Since pipeline № 1 does not intersect with others in terms of volumes, the preliminary result for it will be equal to the final one. Conveyors № 2–7 are considered as a separate system.

We will use $q = 1$. Then $N = 51$, $min_{C(k,j)>0}C(k, j) = 9785.5$,

$$\varepsilon = \frac{1 \times 51 \times 2}{9785.5} = 0.0104.$$

According to the preliminary S, it can be seen that the number of shifts for all conveyors does not exceed r, so the production plan L does not need to be adjusted.

Having implemented the algorithm described above in the package MATLAB, we obtained in 3 032 iterations the final vector S (the span of the vector for pipelines No. 2–7 is 0.0095 (figure 4)) and a matrix R of the form (Table 3):

Table 3. Quasi-optimal vector S and quasi-optimal matrix R

Conveyor №	Num. of shifts, S	Volume, L				
		2,5	5	10	25	50
1	57.557	0	0	0	190 120	135 830
2	18.942	0	115 366	83 603	0	0
3	18.943	0	115 366	83 605	0	0
4	18.942	0	123 911	87 573	0	0
5	18.951	47 625	94 534	73 259	0	0
6	18.951	47 625	94 534	73 259	0	0
7	18.951	47 625	94 534	73 259	0	0

Thus, along conveyor № 1, it is necessary to bring workers out for 58 working shifts, along the rest of the conveyors - for 19 shifts (rounding up to a whole side).

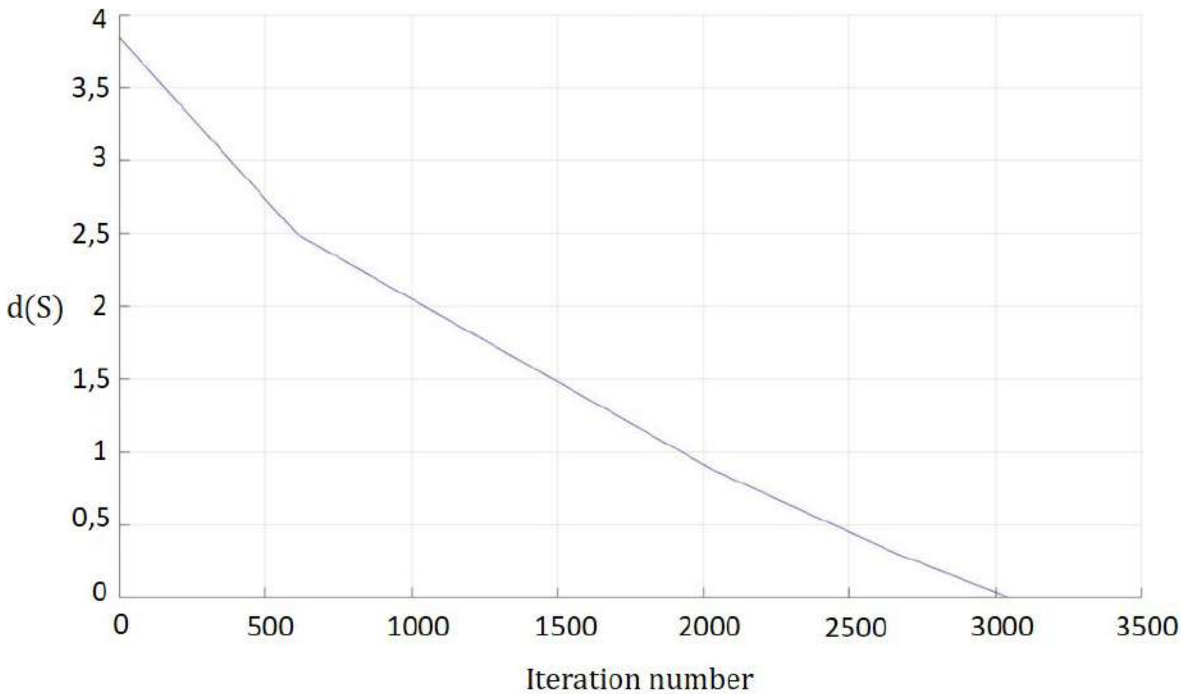

Figure 4 – Plot of d(S)

Remark 1.3.7 There are cases when, according to the performance matrix and the sales plan, it is unclear whether the set of pipelines is correctly divided into subsets. It means that the preliminary matrices R and C do not raise suspicions that $max(S)$ or $min(S)$ will not reach a value close to other elements of S. This happens, for example, when all pre-distributed products from $\max(S)$ were transferred to other conveyors according to some volumes, but at the same time the same conveyor remains the maximum, because other volumes, which can only be produced on this conveyor, require a large number of production shifts. Because of this, the algorithm stops decreasing the value of $d(S)$. Such cases require separate study and modernization of the algorithm, which is not included in the consideration of this scientific work.).

1.4. Creation of a recommender system

The sales unit uses both statistical methods and develops machine learning models. Regression, clustering, and classification tasks can be solved quickly and efficiently. With the help of such methods, you can analyze the consumer basket, and accordingly generate «tips» for managers what to offer and to whom (a recommendation system of related products).

1.4.1. Problem statement

Let's describe mathematically the problem and the construction of a solution to the recommendation system. Let there be $P_i, i = \overline{1,n}$ of nomenclature items. There is a binary matrix of order statistics $A_{m \times n}$, where the element $a_{i,j} = 1$ if the line of implementation i had products $P_j, j = \overline{1,n}$, otherwise $a_{i,j}$ =0.

The task is to create for each item $P_i, i = \overline{1,n}$ a sorted list of purchase recommendations if the customer has already bought the products P_i.).

1.4.2. Solution algorithm

Let's introduce the correlation matrix $B_{n \times n}$ with the elements $\{b_{i,j}\}$. Let's fill it with correlation coefficients:

$$b_{i,j} = \frac{\sum_{k=1}^{m} a_{k,i} a_{k,j}}{\sum_{k=1}^{m} a_{k,i}}, \quad i = \overline{1,n},\ j = \overline{1,n}$$

If the denominator of the fraction is 0, then $b_{i,j} = 0$.

For each row $k = \overline{1,n}$ of the matrix B, we introduce the matrix $C^k_{2\times n}$. Then $c^k_{1,j}$ is the ordinal number of the item item from the list $P_j, j = \overline{1,n}$, $c^k_{2,j} = b_{k,j}, j = \overline{1,n}$.

Let's sort the second row of the matrix C^k in descending order and sort the first row in the same order. Comparing the numbers of the first line of the names of the items according to their index and the compilation of the matrix B, we get a vector of purchase recommendations, where the purchase itself will be in the first place, and from the 2nd to the n position — recommendations in descending order of the correlation coefficient. Similarly, we define a recommendation vector for each $k = \overline{1,n}$.

1.5. Pricing of a new product

When a business develops, the market segments that the company does not cover are analyzed. The question arises about the production of new products. A simplified diagram of this process looks like this:

- Determines the type and volume (liters) of products in the new line;
- Recipes are made for them;
- Defines the pipelines and their performance on which the release will take place;
- All direct production costs are collected;
- An advertising component is added to promote new products
- The costs of discounts and product promotions are laid down;
- Fixed costs are collected by analogy with other products;
- A decision is made on the inclusion or non-inclusion of fixed costs to promote products at lower prices, the cost period is determined, which does not include fixed costs;

- Based on the solution, the price is determined and included in the price list;
- Delivery to branches is added to this price to determine the price for shipment from each warehouse;
- A detailed description of the product is brought to the managers, the possibility of motivating managers for the sale of new products is determined.

The list does not include a number of small but important points, such as technological tests, a tender for raw materials suppliers, obtaining state registration, drawing a layout, quality control, checking the properties and effectiveness of a new peat soil (in this case, this happens on plants for which the soil is intended).

The peculiarity of creating a formulation for mineral fertilizers is shown in the following task.

1.5.1. Problem Statement

Let's consider the task «compounding according to the specified characteristics». In addition to soils and liquid fertilizers, the company also produces mineral fertilizers. These products come in standard classic ones (they can be bought from a large supplier and packaged in consumer packages), and there are flour mixtures - a mixture of classic fertilizers that need to be mixed in their production. All mineral fertilizers have an indicator N:P:K (Nitrogen : Phosphorus : Potassium), usually recorded in the amount of grams per 100g. For example, Ammophos has an NPK ratio of 16:16:16, Diammonium phosphate 0:10:16, Nitrate 33:0:0.

So, we have a set of N classical fertilizers represented as vectors $x_{i_{3\times1}}, i = \overline{1,N}$. A client requests a compound fertilizer with NPK $= y_{3\times1}$ with an acceptable range $y \in [y-1; y+1]$. It is necessary to determine the weighting coefficients $p_i, i = \overline{1,N}$ to solve the following system, or prove that the system is unsolvable:

$$\begin{cases} y-1 \le \sum_{i=1}^{N} p_i x_i \le y+1; \\ \sum_{i=1}^{N} p_i = 1. \end{cases}$$

1.6. Profitability of working with network stores

In this section, we will consider tasks directly related to profit and profitability. Recall that if P is profit, S is sales, Z is costs, and R is profitability, then

$$P = S - Z; \quad R = \frac{P}{S}.$$

Delivery to chain stores is a special case of standard profitability. But when calculating the price for a particular network, it is important to take into account the nuances.

- In business expenses, you can identify specific managers who will be engaged in sales (network managers).
- It is important to take into account the network bonus in costs (usually it is a percentage of sales, sometimes a certain amount for the delivery of a pallet from a Distribution Center (DC) to retail outlets)
- Determine the method (if possible) of work - self-delivery to retail outlets, or delivery to one or more shopping centers in Russia, from where the network itself will deliver to all retail outlets. The second method usually comes out more expensive in terms of costs, because the DC is not located near the production, but the sales volume will increase greatly, because it is unprofitable to carry to distant points of Russia with a small number of branches and the absence of other customers at these points.
- To clarify the shares of product distribution by DC/outlets in order to get a more accurate profit/margin for each of them and eventually understand the overall profitability/marginality of working with the network.

Some chains hold a tender for Their Own Trademark (TOT), that is, in fact, products under the brand of the network, manufactured on the manufacturer's equipment and raw materials. Usually, the network is asked to provide recipes and other costs for the proposed trademark. In this case, there is no network bonus (advertising, marketing, promo, discounts, the network takes over). Otherwise, profitability is calculated according to the same list as when working with a network with its own products

1.6.1. Calculation of the price for a marketplace in the «Garden and kaleyard» category

Unlike traditional federal networks, marketplaces create a rather non-trivial bonus system. And in order to understand how much the supplier will earn if he sets the price c for the buyer, the marketplace creates a calculator where, when entering the price, weight, logistics direction, the output is the amount to be credited to the supplier minus all bonuses.

If the new supplier has thousands of positions, then the manual method becomes ineffective. The supplier wants to receive a certain amount of $\boldsymbol{m}$ from each product. Then he needs to solve the inverse problem, knowing the scheme of price formation $\boldsymbol{c}$, set $\boldsymbol{m}$ and get $\boldsymbol{c}$, then apply this method to all positions.

Let's look at a specific example of the marketplace [26]. Let's introduce the notation:

m — the amount to be charged (set in advance by the supplier) with VAT;

c — minimum recommended price for the end customer with VAT (calculated price);

S_1 — marketplace bonus: «last mile» - delivery from the pick-up point to the apartment;

S_2 — Marketplace bonus: logistics;

a — coefficient applied to the price (depending on the weight);

k_1 — the minimum possible value of the logistics bonus (depending on the weight);

k_2 — the maximum possible value of the logistics bonus (depending on the weight);

l — cluster multiplier.

Remark 1.6.1 The parameters k_1, k_2 and a are selected in threes from the table specified by the marketplace (part of which is given below (Table 4)), are always uniquely determined, and depend on the pre-known weight of the product.

Table 4. Dependence of k_1, k_2 and a on the mass of the product

Weight, kg	k_1	k_2	a
<0,1	38	50	0,05
[0,1-0,2)	39	50	0,05
[0,2-0,3)	40	60	0,05
...	...	...	...
[5;6)	130	250	0,06
...	...	...	...
[20;25)	500	650	0,07
[25;30)	650	1200	0,07
[30;35)	750	1200	0,07
>35	950	1200	0,07

The price of $\boldsymbol{c}$ is calculated using the equation

$$c = m + 0,135c + S_1 + S_2,$$

where

$$S_1 = \begin{cases} 20 & \text{if } 0,05c < 20 \Rightarrow c < 400 \\ 0,05c & \text{if } 0,05c \in [20; 250] \Rightarrow c \in [400; 5000] \\ 250 & \text{if } 0,05c > 250 \Rightarrow c > 5000 \end{cases},$$

$$S_2 = l \left(\begin{cases} k_1 & \text{if } ac < k_1 \Rightarrow c < \frac{k_1}{a} \\ ac & \text{if } ac \in [k_1; k_2] \Rightarrow c \in [\frac{k_1}{a}; \frac{k_2}{a}] \\ k_2 & \text{if } ac > k_2 \Rightarrow c > \frac{k_2}{a} \end{cases} \right).$$

If we combine the conditions S_1 and S_2, then on the c axis the values $\{400; 5000; \frac{k_1}{a}; \frac{k_2}{a}\}$ can be arranged as follows: 1. $400 < \frac{k_1}{a} < \frac{k_2}{a} < 5000$

2. $400 < \frac{k_1}{a} < 5000 < \frac{k_2}{a}$

3. $5000 < \frac{k_1}{a} < \frac{k_2}{a}$

4. $\frac{k_1}{a} < \frac{k_2}{a} < 400$

5. $\frac{k_1}{a} < 400 < \frac{k_2}{a} < 5000$

Let's consider even stricter conditions than shown in Table 4.

$$\begin{aligned}\min\frac{k_1}{a}&=\frac{\min k_1}{\max a}=\frac{38}{0.07}=542.86,\\ \max\frac{k_1}{a}&=\frac{\max k_1}{\min a}=\frac{950}{0.05}=19\,000,\\ \min\frac{k_2}{a}&=\frac{\min k_2}{\max a}=\frac{50}{0.07}=714.29,\\ \max\frac{k_2}{a}&=\frac{\max k_2}{\min a}=\frac{1200}{0.05}=24\,000.\end{aligned}$$

Thus, we are faced with a standard problem with the parameters [27]. Let's consider 3 options:

Option 1. At $400<\frac{k_1}{a}<\frac{k_2}{a}<5000$

$$c=\begin{cases}m+0.135c+20+k_1l, & c<400,\\ m+0.135c+0.05c+k_1l, & 400\le c<\frac{k_1}{a},\\ m+0.135c+0.05c+acl, & \frac{k_1}{a}\le c<\frac{k_2}{a},\\ m+0.135c+0.05c+k_2l, & \frac{k_2}{a}\le c<5000,\\ m+0.135c+250+k_2l, & c\ge 5000.\end{cases}$$

Expressing c, we get

$$c=\begin{cases}\frac{m+20+k_1l}{0.865}, & c<400\Rightarrow m<326-k_1l,\\ \frac{m+k_1l}{0.815}, & 400\le c<\frac{k_1}{a}\Rightarrow 326-k_1l\le m<0.815\frac{k_1}{a}-k_1l,\\ \frac{m}{0.815-al}, & \frac{k_1}{a}\le c<\frac{k_2}{a}\Rightarrow 0.815\frac{k_1}{a}-k_1l\le m<0.815\frac{k_2}{a}-k_2l,\\ \frac{m+k_2l}{0.815}, & \frac{k_2}{a}\le c<5000\Rightarrow 0.815\frac{k_2}{a}-k_2l\le m<4075-k_2l,\\ \frac{m+250+k_2l}{0.865}, & c\ge 5000\Rightarrow m\ge 4075-k_2l.\end{cases}$$

Option 2. At $400<\frac{k_1}{a}<5000<\frac{k_2}{a}$

$$c=\begin{cases}m+0.135c+20+k_1l, & c<400,\\ m+0.135c+0.05c+k_1l, & 400\le c<\frac{k_1}{a},\\ m+0.135c+0.05c+acl, & \frac{k_1}{a}\le c<5000,\\ m+0.135c+250+acl, & 5000\le c<\frac{k_2}{a},\\ m+0.135c+250+k_2l, & c\ge \frac{k_2}{a}.\end{cases}$$

Expressing c, we get

$$c=\begin{cases}\frac{m+20+k_1l}{0.865}, & c<400 \Rightarrow m<326-k_1l,\\ \frac{m+k_1l}{0.815}, & 400\le c<\frac{k_1}{a}\Rightarrow 326-k_1l\le m<0.815\frac{k_1}{a}-k_1l,\\ \frac{m}{0.815-al}, & \frac{k_1}{a}\le c<5000\Rightarrow 0.815\frac{k_1}{a}-k_1l\le m<4075-5000al,\\ \frac{m+250}{0.865-al}, & 5000\le c<\frac{k_2}{a}\Rightarrow 4075-5000al\le m<0.865\frac{k_2}{a}-k_2l-250,\\ \frac{m+250+k_2l}{0.865}, & c\ge\frac{k_2}{a}\Rightarrow m\ge 0.865\frac{k_2}{a}-k_2l-250.\end{cases}$$

Option 3. At $5000<\frac{k_1}{a}<\frac{k_2}{a}$

$$c=\begin{cases}m+0.135c+20+k_1l, & c<400,\\ m+0.135c+0.05c+k_1l, & 400\le c<5000,\\ m+0.135c+250+k_1l, & 5000\le c<\frac{k_1}{a},\\ m+0.135c+250+acl, & \frac{k_1}{a}\le c<\frac{k_2}{a},\\ m+0.135c+250+k_2l, & c\ge\frac{k_2}{a}.\end{cases}$$

Expressing c, we get

$$c=\begin{cases}\frac{m+20+k_1l}{0.865}, & c<400\Rightarrow m<326-k_1l,\\ \frac{m+k_1l}{0.815}, & 400\le c\le 5000\Rightarrow 326-k_1l\le m<4075-k_1l,\\ \frac{m+250+k_1l}{0.865}, & 5000\le c<\frac{k_1}{a}\Rightarrow 4075-k_1l\le m<0.865\frac{k_1}{a}-k_1l-250,\\ \frac{m+250}{0.865-al}, & \frac{k_1}{a}\le c<\frac{k_2}{a}\Rightarrow 0.865\frac{k_1}{a}-k_1l-250\le m<0.865\frac{k_2}{a}-k_2l-250,\\ \frac{m+250+k_2l}{0.865}, & c\ge\frac{k_2}{a}\Rightarrow m\ge 0.865\frac{k_2}{a}-k_2l-250.\end{cases}$$

Remark 1.6.2 In general, options 4 and 5 are possible: $\frac{k_1}{a}<\frac{k_2}{a}<400$ and $\frac{k_1}{a}<400<\frac{k_2}{a}<5000$, for them, the calculation systems $\boldsymbol{c}$ are similar to options 1–3 and will give 2 more options for 5 cases. However, in this example, $\frac{k_1}{a}>400$ and $\frac{k_2}{a}>400$, so we cut off these 2 options.

As a result, for this example, depending on the weight, a, k_1, k_2 and m, 1 out of 15 cases is uniquely determined (in general, 1 out of 25 cases), and using the corresponding formula, we calculate c.

1.7. Analysis and optimization of delivery methods to out-of-town clients and branches

There are quite a lot of logistical tasks. However, in the described company due to the fact that:

- the branch network is small, i.e. the logistician calculates the optimal routes manually, because they are standard and not very frequent;
- deliveries within the company go, with rare exceptions, from the production warehouse to the branch warehouse, that is, there is no movement between branches;
- transportation is hired, so there is no need to worry about the return load and the optimal route through the cities.
- the absence of an extensive production network, i.e. you do not need to choose where it is more profitable to carry cargo to the client,

the number of actual tasks for this particular company is decreasing.

Some of the price columns imply pickup, some include delivery within the city. If the client cannot or does not want to organize transportation and export products from the warehouse, then a contractor is hired and the products are delivered to the client with his help. To make it profitable for both the contractor and the supplier (in terms of 1 ton of goods), for such customers we limit the batch to the size of 1 truck / 1 railcar. It often turns out that a fairly profitable option is to rent warehouses. Then you can reach more small customers by reducing the order quantity (the customer will pick up by pickup), if you carefully monitor the stocks and turnover of warehouses.

We also arrange delivery to branches in multiples of a truck / wagon in order to spend the minimum logistical costs per piece. Sometimes there is an analysis of which branch's warehouse is more profitable to deliver products to the customer. This task is known as [28].

1.7.1. Problem statement

Each order has a certain weight. The products can be delivered to the warehouse of the branch by wagon, then through the warehouse of the branch by truck or by a machine of other load capacity, the alternative is to deliver by car from the production warehouse. We believe that moving to the warehouse always takes place by wagon (because at the moment the cost of transporting 1 kg of cargo from point A to point B is more profitable by wagon than by any car). Even if the client does not have the opportunity to accept the car himself (otherwise it is cheaper to deliver the car directly than to transship products at another warehouse, increasing the length of the route), that is, it can only be delivered to the client by car. Let's denote:

$G_{m\times 1}$ is the vector of all possible load capacities in kilograms in ascending order, and G_m is the load capacity of the wagon (the largest);
$Cps_{1\times p}$ - the cost of transporting one wagon to each of the company's warehouses;
$Csk_{m-1\times p}$ - the cost of transporting one machine of all load capacities from each warehouse p to the customer;
$Cpk_{m-1\times 1}$ - the cost of transporting one machine of all load capacities from production to the customer;
W is the weight of the order. Then the task is to determine the cheapest way to deliver 1kg of products.

1.7.2. Solution Algorithm

1. **Determination of the minimum cost of delivery from production to the customer by machine.**
   We choose the minimum cost of delivery of a kilogram, taking into account the physical limitation of the load capacity.
   $$C_1 = min\{\frac{Cpk_i}{W}, \quad i = \overline{1, m-1}, G_i > W\}.$$
2. **Determination of the minimum cost of delivery from production to the warehouse by wagon, then from the warehouse to the customer by car**.

We calculate the cost of delivery of 1 kilogram by a fully loaded railcar to each warehouse, then add the cost of delivery from this warehouse to the customer. We select the minimum cost of delivery of a kilogram, taking into account the physical limitation of the load capacity, and determine through which warehouse $j = \overline{1, p}$ to make transshipment.

$$C_2 = min\{\frac{Cps_j}{G_m} + \frac{Csk_{i,j}}{W}, \quad j = \overline{1, p},\, i = \overline{1, m-1},\, G_i > W\}.$$

3. **Determining the minimum from steps 1 and 2**.

We choose the most profitable delivery option from those calculated earlier.

$$C_{opt} = min\{C_1, C_2\}.$$

1.8. Some other tasks of enterprise management

There is no mathematical model for the tasks in this section, but for many of them the ideas of making such models are described. In addition, the combination of descriptions of all tasks helps to fully understand the standard enterprise management systems [29, 30] and their relationship.

1.8.1. Production plan per shift

The daily production plan (shift-daily task) is handled by the production director or shift supervisor with the assistance of a commercial analyst.

With a sufficient number of production capacities, this approach is applied to a shift assignment: a report «Deficit» has been created in the electronic accounting system of the enterprise. The date on which we want to generate the report is selected. All the nomenclature items are located in it along the lines. By columns: «production plan», «fact of production», «left to produce», «stock balance», «applications for transfer to branches», «applications for shipment to customers» and «deficit». In the last column, it is worth explaining that this is «stock balance» minus «applications for moving to branches» and «applications for shipment to customers». If you select today's date, then if there is a negative value in the column «deficit» for the nomenclature item, then it is urgently necessary to produce these products. If the absolute value of the deficit exceeds the value of the column «left to produce before the plan», the production sends a request to the commercial department to confirm the additional output. However, if there is no shortage, production is guided by a monthly production plan and a general recommendation - to have products for 2 weeks of sales at any given time.

1.8.2. Daily production workforce plan

The task is to optimize the number of man-hours required to complete the production plan. That is, it is necessary to try to make sure that as many people are involved in each working day as are minimally necessary to fulfill the production plan and at the same time that each of these people does their job throughout the shift. The «minimum» in the previous sentence refers to the minimum amount at the standard (normative) pace of work. First of all, we are talking about the workers of the production shop - adjusters, operators, loader drivers, auxiliary workers.

1.8.3. Implementation of lean manufacturing tools

There is a need to increase labor productivity without significant investment. Productivity in the simplest case is measured as the ratio of output in natural units of the site to the staff of this site. However, this item is not only about production, but also about the finished goods warehouse, the material warehouse.

There is a Japanese lean manufacturing approach «Gemba Kaizen» for enterprise management, especially for production workshops [31]. In 2018, the Government of the Russian Federation created a national project for Russian enterprises «Increase in labor productivity» [32]. This is an adapted version of the Japanese approach created in the 1950s at Toyota factories. The goal of the project is to increase labor productivity in Russia by 5% due to instructors who help diagnose and carry out improvements in enterprises in 6 months.

The author of this work is a certified internal trainer of a lean manufacturing company. The essence of this approach is to reduce losses and, as a result, increase safety, reduce cost, increase staff satisfaction, increase the level of customer service (price reduction, quality improvement, on-time delivery).

There are 7 types of losses:

1. Overproduction (the main loss that causes overstocking of warehouses and other losses)
2. Excess inventory (we are talking about raw materials and supplies)
3. Unnecessary movements (making movements of people who do not carry value to create a product)
4. Unnecessary transportation (moving using transport)
5. Waiting (downtime due to non-optimal business process)
6. Excessive processing (performing work that does not carry value for the customer)
7. Defective

There are a number of tools available to solve these problems. Most of them have no mathematical value, and talk about the organization of space («5S»), the construction of a value stream map, the search for problems in areas and operations that do not carry value (mapping), standardization, the search for root causes («5 why», «8D», «4W2H» and «Ishikawa diagram»), prioritization of problems (selection diagram «cost-efficiency» or «complexity-efficiency»), rapid equipment changeover (SMED).

However, although the other part of the tools is simple from the point of view of mathematics, it may be interesting as a basic tool for the reader in application to production practices.

One of these tools is the transition from push-out production to pull-out production. Push-out production implies the operation of production at full capacity, mainly without taking into account customer demand. Surplus products accumulate in warehouses, working capital decreases, because they «lie» in the remnants in warehouses. Also, in the absence of the possibility of placing products in a warehouse (multi-tiered racks in a closed warehouse), in the absence of own (non-leased) warehouses, additional costs for moving arise. An alternative is pull-out production: work «on order» with a small projected margin. The disadvantage of this approach may be in paid processing and lack of capacity. The development of a model of an optimal balance between pulling and pushing production from the point of view of profit is the task of this item.

The second tool is «Spaghetti diagram» [33]. It is necessary to mark up the map of the workshop (top view), in which to designate the equipment. Next, draw a solid line from each operation to each, without cutting corners, while timing the movement. This visual map shows the operations and the travel time between them. It becomes obvious how to move or combine equipment in order to minimize losses «unnecessary movements» and «excessive transportation»

The third tool «Planned production analysis». If many positions are made per day, then it is customary to do an hourly scheduled analysis. The analysis form must contain an hourly task, plan, fact, cumulative plan, cumulative fact, deviation, cumulative deviation, reasons for deviation. In order to build a planned hourly shift assignment, it is necessary to know the customer's need (in units per month), the number of working days per month, the number of shifts per day, working hours, breaks within working hours.

The algorithm for determining the hourly plan.

1. Define the general plan for the month of A products.
2. Calculate the number of working days in a month b_1 and the number of shifts in a day b_2: $b = b_1 \times b_2$ – the number of shifts in a month.
3. Calculate how much to produce per work shift $c = \frac{A}{b}$
4. Calculate the total break time $t_{free} = t_1 + ... + t_n$ (in minutes), where n is the number of hours per shift
5. From the total shift time T find the net time $T_{use} = T - t_{free}$ (in minutes)
6. Determine the standard output for each minute of time.

$$d = \frac{c}{T_{use}}$$

7. Schedule the net work time for each hour

$$T_{use_h} = [60min - t_1; 60min - t_2; .....; 60min - t_n]^T$$

8. Calculate production $P = T_{use_h} \times d$ for each hour.

Finally, the TEE tool (total Equipment efficiency). [34–40]

$$TEE = A \times P \times Q,$$

where A is availability, P is performance, and Q is quality.

To calculate the availability criterion A, it is necessary to calculate the operating time OT from the planned production time PPT (i.e., without scheduled stops) and the loss time due to unscheduled stops US:

$$OT = PPT - US;$$
$$A = \frac{OT}{PPT}.$$

To calculate the performance criterion P, we first calculate TP - output and IRR -theoretically the maximum number of products produced per unit of time.

$$P = \frac{\left(\frac{TP}{OT}\right)}{IRR}.$$

To calculate the quality criterion Q, it is necessary to calculate GP – the output of good Pieces – the actual number of units of good products produced during the operating time OT.

$$Q = \frac{GP}{OT}.$$

It can be noted that A,P and Q are fractions of the planned or theoretical value. The recommended target value within the framework of the national labor productivity project is $OEE = A * P * Q = 0.85$.

1.8.4. Optimization of stocks

The task of inventory management is present in one form or another for every company that sells tangible goods. The ideal situation is when a warehouse can continuously meet the demand for products in a timely manner, aiming at zero warehouse area, that is, selling products «from wheels».

The benefits of this are obvious - an increase in working capital, a reduction in the risks of product expiration date and presentation of packaging. Usually, «timely», «continuously» and «meeting demand» at the same time does not work without the presence of inventories due to limitations on equipment performance, travel time to the customer/branch and other factors. It may also be impractical due to equipment downtime most of the year, and full load only during the seasonal months.

There is a limitation in the enterprise in question: there must be products of each type in the warehouse of each branch for exactly 2 weeks of sales. This restriction allows you to keep the warehouse moderately filled, to have liquid stocks in case of unplanned large orders (the specifics of the business do not imply pre-orders in half a month, usually a shipment request arrives in 1-3 days).

1.8.5. Daily shipment plan

It is virtually impossible to schedule a shipment plan for each day of the month due to several factors: the inconstancy of the provision of rail and motor transport, the lack of pre-orders and other things. However, empirically, the company revealed that, on average, 95% of applications for shipment for tomorrow and 60% of applications for the day after tomorrow are known.

To address the challenge of emerging from such significant uncertainty, it is essential to first understand the demand from each customer as thoroughly as possible, so that you can anticipate orders even before receiving them. If you can achieve this with high accuracy, you can rely not on customer orders, but on your probabilistic data, allowing managers to individually tailor product offerings to their customers throughout the month.

Primarily, it is necessary to reduce the turnover of warehouses to the normative level, meaning understanding the demand for each of the branch warehouses.

1.8.6. Daily warehouse workforce plan

Having a shipment plan, you can adjust the number of necessary workers for these shipments. To do this, you need to output the average number of shipments for each employee (team) of the warehouse, then schedule these employees for the planned volume of shipments. At the same time, it is worthwhile to calculate in advance the optimal cost model, which takes into account the cost of overworking in the season (if you recruit fewer teams than necessary), and the employment of seasonal teams in the off-season (if there were enough teams in the season without overworking).

The obvious limitation will be that a staffing table is being drawn up for employees, so it is very important to have a shipment plan not for 1-2 days in advance, but for the whole month with a probability of at least 80%.

1.8.7. Optimization of product placement in warehouse

This task is mathematical. In the simplest case (and in the company in question) all products can be stored anywhere and have no special storage conditions. The task is to place pallets in the warehouse at the exit from production so that during further shipment the loading time of the transport is minimized. In other words, the necessary products for shipment should be as close as possible to the shipping area, provided that they were not specially additionally moved inside the warehouse.

The importance of this task is obvious: reducing loading time, increasing warehouse capacity, optimizing inventory, reducing fuel costs of the loader.

It is not always possible to know all the applications in advance, but it may not be necessary. It is important to identify popular products, and as their demand decreases, place them further from the shipping area, taking into account the average required volumes.

1.8.8. Forecasting daily plan execution

It is necessary to analyze the implementation of the sales plan daily for a month. This will help to identify the quality of the manager's work and also clarify the planning of inventory and production. It often turns out that by the end of the month, managers begin to sell more intensively in order to achieve the fulfillment of the plan set by them, with which they have a salary associated.

1.8.9. Analysis of sales distribution by regions

This paragraph does not address the basic analysis tasks that sales managers are engaged in: customer sales analysis, analysis of the item being sold, analysis of the average sales price, analysis of customer growth, and so on.

However, managers usually do not delve into the analysis by region, because this does not greatly affect their sales and is not tied to their key performance indicators. But it is quite interesting for the company to see the visualization on the geographical sales map, analyze high or low sales in the regions, and from here deduce sales per capita. The conclusion is to bring this information to managers and the marketing department that it is necessary to develop sales in «sagging» regions.

This analysis can also be considered in the context of one of the following paragraphs «Management reporting».

1.8.10. Increase profits by providing discounts

One of the basic rules of economics: the lower the price, the higher the demand [41]. Within a company/line of business, the demand curve is usually not defined as a clear function that does not change over time. Therefore, lowering the price with the expectation of an increase in demand and, as a result, profit is a risky undertaking. Firstly, there is no complete certainty what kind of increase will be achieved. But more importantly, the customer's behavior is completely unclear if the price is returned to its original position after an unsuccessful experiment: will customers agree to pay more again, or will they start looking for other suppliers with lower prices. That is, the psychology of the intermediary and the end customer, the marketing component, and the so-called "brand strength"are important here.

A safer way to increase profits is to provide temporary discounts. This approach suggests that the regular price remains in place, but there is a time-limited discount, thereby motivating demand. In any seasonal business, with the same discount in seasonal and non-seasonal months, you can expect different absolute increases.

The traditional approach to providing discounts. Analyzing the percentage of profitability of products, there is an understanding that you can give a discount of no more than a percentage of profitability (if it is positive, of course) in order not to work at a loss.

An alternative approach. Costs = conditionally constant (independent of the number of sales and output) + variables (sales and output costs). If the company did not give discounts on a regular basis before, then all previous sales were regular. At one point, the company realizes that given the current price and the strength of the brand in this market, sales cannot be increased, so it decided to sell more at the expense of discounts (additional sales). In this case, it can be assumed that fixed costs are allocated only to regular products, therefore, only variable costs are imposed on additional (discounted products). If fixed costs occupy a large share in the company, then the cost of discounted products will decrease significantly, which means that it will be possible to give a discount much more than with the traditional approach. At the same time, there must be regular sales at a regular price in a standard volume. In other words, additional sales either leave sales at the same level at regular prices, but give additional sales at discounted prices, or a new product with a lower price is created, for which fixed costs are not allocated. In this way, you can compete with sellers in a lower price segment.

In the absence of control over the transmission of a discount from an intermediary, the end customer needs restrictions on the volume of shipments of discounted products, otherwise there may be no regular sales at all in the period following the discount.

An approach that takes into account only variable costs is called unit economics [42], which is commonly used in startups and Internet products. The possibility of using this approach is advisable if fixed costs are small, or it is expected that very soon they will occupy a small part, provided the business scales and sales grow sharply (at the same time, the company is planned to be unprofitable, and fixed costs are covered by investments). This works especially well if several programmers have released a product (after spending some fixed time on development) and then sell a copy of it. That is, the time and resource costs are much lower than the production of physical goods. It is then that the costs of selling one product or attracting one customer come to the fore and, as a result, it becomes possible to have high margins both in the paradigm of the unit economy and in the traditional one. However, in a traditional company, constantly monitoring its profitability, the company can afford to produce and sell goods with low margins, for example, to increase representation in the low price segment, to increase the load of production lines, to reduce fixed costs per piece.

1.8.11. Distribution of logistics costs by assortment

When calculating the total cost of one item of a nomenclature item, one of the components is logistics. It is divided into delivery of products to branch warehouses and delivery to customers.

The price will ultimately depend on how the cost of delivery of one car/wagon will be distributed among the products. The specifics of the described business imply that the main products are polyethylene bags with peat of different liters and different weights.

Task: to distribute the costs of the products in the car. This approach is considered optimal: the car is limited in load capacity and body volume. The peat grunt is light, so there is a situation that depending on the moisture content of the peat, the number of pieces on the pallet, the volume of products may vary, the volume will fill faster or the weight will be exceeded. For example, for a truck, this is a limit on the carrying capacity of 20 tons and the volume of 32 Euro pallets.

The proposed solution is not completely accurate, but it is quite «honest». Reduce costs to 1 pc. That is, with a known weight and number of pieces per pallet, you can easily determine how many pieces will be 20 tons and 32 pallets, choose the minimum indicator (that is, choose an approach by weight or volume), then divide the cost of delivering the machine by this number of pieces. Thus, representing «monomachine»/«monorailcar> products, it is possible to calculate the standard amount of logistics costs per 1 piece in the corresponding transport.

1.8.12. Changing the regular price list and price columns

The previous paragraph described the appearance of the product price. However, due to changes in cost, and sometimes with the introduction of a new pricing policy, regular prices are revised.

The basis for price changes is the management report on actual costs for the period. An additional incentive is the rise in prices of suppliers due to high inflation (for example, in the spring of 2022) or an increase in tax (for example, at the beginning of 2019 there was an increase in VAT by 2

So, there is a sales document for each product and its costs. Therefore, profitability, marginality and profit from each item are calculated. A simplification is accepted - to put a single price on positions with slightly different composition and costs, usually based on a position with maximum costs. Their profitability varies by 2-4%. As a result of the price list adjustment, the price is pulled up to the level of planned profitability/marginality. At the same time, it is necessary to understand the realities of the market: in which price segment is the product, who is the target audience, what are the prices of competitors. In addition, the location of competitors' production is important (in order to compete on price, sometimes you have to sacrifice part of the logistics costs if the branch is far from production, unlike other manufacturers). Thus, we determine the average selling price, for which there will be a corresponding planned profitability/ marginality.

There are several price columns in the price list (the more the customer buys, the lower the price they receive). Accordingly, the same percentage is added to each price column by which the average selling price was increased so that the planned profitability converged.

The difference between the price columns can be considered as a subtask. To determine the base column, the cost plus a small margin is usually used. This column implies the largest possible volumes (key partners and regional distributors) and pickup (or full payment for delivery) from production. Next, there is an increase by a certain percentage, then with an additional increase in the price column and a decrease in the multiplicity of the order, we add delivery from ourselves within the city where the branch warehouse is located. Among the columns with the delivery we pay for, we take into account the cost of the car (a small gazelle is more expensive than a truck in terms of 1 piece), the territory of the delivery, which we deduce from the actual costs of the company that deals with the delivery (this includes the cost of 1 km or 1 hour, the presence of a forwarder, the fact of loading / unloading products, the capacity of the car). With large and regular volumes, it may be worth considering buying your own transport to reduce the cost of delivery.

1.8.13. Distribution of variable costs

Many costs are known how to distribute them by nomenclature, because they are directly related to this. Raw materials and components are known from the specification and configuration of products developed by technologists (the supply department provides prices). Electricity costs can be measured for each conveyor (workshop, power consumption, conveyor output), the costs of s/n operators, loader drivers and auxiliary workers can also be distributed, since they are linked to their production sites and technological maps.

The distribution of logistics costs was described above. Managers' salaries can be distributed in proportion to revenue or according to piecework coefficients, if any, introduced in the company.

Whenever possible, it is necessary to try to find a direct relationship between the nomenclature and total costs, to look for a fair distribution of costs. Thus, the most accurate production cost will be determined, below which the price of products can never fall.

1.8.14. Distribution of fixed costs by assortment

At the moment, some of the costs are not directly related to the nomenclature. For example, the salaries of office managers do not directly relate to a specific product.

There is no single approach to cost allocation in the literature in such cases. There is an option to generally consider them an error and not take them into account (as in unit economics), and only constantly monitor the profitability of the entire company. But still, it is risky enough not to take them into account when producing goods.

There are such distribution approaches:

- by volume;
- by piece;
- by pallet;
- by product weight;
- by pipeline performance;
- at the average selling price;
- by production cost;
- by raw material cost.

The task is to choose a distribution based on economic feasibility and analysis of competitors' pricing policy. Intuitively, the distribution by pieces seems unfair, because regardless of the liter, weight, and price of products, we add a constant amount of costs (a conditional package of 5 liters will increase by 20% in price because of this, and 50 liters by 2%). There is a similar problem with the conveyor capacity (the difference between the capacity of 5 liters and 50 liters differs by no more than 2-3 times).

The distribution by liters looks fair at first glance, but in fact, if you consider that 0.25 liters and 50 liters are produced, then the cost per piece will differ by 400 times. This is a lot, and it turns out that the profitability of small packaging/litre is artificially overestimated, and the amount of overhead costs for large packaging is greatly unfairly increased. The situation will be similar with weights and pallets.

Perhaps the fairest (and many companies use it) is the allocation of costs in proportion to the production cost. In this version, the difference between volumes will be taken into account, but not very large, while the proportionality of price differences by liters and types of products will be observed.

Remark 1.8.1 If the products are heterogeneous, this approach will become unfair. If, in addition to relatively cheap and massive soils, the company starts selling gold (which is small but expensive), then a lot of overhead costs will be distributed unfairly on gold. This situation is of course fictional, but there is a less vivid example (in the agricultural sector, in addition to soils and mineral fertilizers, poisons are sold: usually these are small expensive ampoules that are diluted with a lot of water). In this case, product classifications should be clearly separated, and fixed costs should be divided between these nomenclature groups in some way, before applying the distribution of production costs within nomenclature groups.

1.8.15. Feasibility of Production

A material warehouse, as well as a finished goods warehouse, can potentially add profit if it has a high turnover and a small area. The supply Department regulates stock balances and timely deliveries. In the company in question, there is practically no such intermediate stage as work in progress, therefore, there are only two warehouses in production: a material warehouse and a finished product warehouse.

Sales analysis shows that some positions are in low demand, have low sales figures even in the season. A logical question arises: is it worth producing it further? If the quantity of finished products can be limited by the volume of output, then the stocks of the material warehouse are limited to minimum shipments of raw materials and supplies. Moreover, the smaller the batch, the more expensive it is per unit. So, by analyzing sales, raw material consumption and minimum/optimal batches of products, you can deduce the number of months for which this batch of raw materials will be consumed. Based on these data, a decision is made to extend or exclude/replace the product.

The important point is that it works well with long-manufactured products, however, if the product is new and is put on the market, then it takes several years to occupy its niche, to displace competitors. Therefore, poor sales of new products may be due to the long «addiction» of the buyer to the product.

1.8.16. Optimization of warehouse stocks of materials

In the company, the supply department deals with the inventory of the material warehouse. Recall that the warehouse of FP (finished products) has stocks for 2 weeks. The situation with the material warehouse is different. Suppliers from different parts of Russia supply raw materials, which means logistics can take up to two weeks. And for 90% of the raw materials, this is the only limitation. That is, for this type of raw material, it takes from one to three weeks from order to production. 10% of the raw materials have longer delivery times, including the most expensive component - the film for consumer packages, into which the finished peat is filled.

The specific situation of the company is that film manufacturers are asked to place an order before the 15th of each month for the next month. The order is carried out up to 20-30 days. Due to the risk of not receiving raw materials on time, the supply department is reinsured by ordering large batches of film, arguing that the batch is cheaper, and on the 15th the exact plan for the next month is not yet known, and even more so for 2 months ahead, i.e. deviations in the plan are possible in a big way. On the one hand, the human factor is clear: the supply department does not want to take responsibility for production disruptions, realizing the risks. On the other hand, even having products for 2 months in advance, rather than for 4 or more, the company has more working capital, which directly or indirectly will bring more profit. Therefore, the way out should be a continuous improvement in the accuracy of forecasting sales, inventory stocks and production, not only for the next month, but also with a horizon of 2 and 3 months.

1.8.17. Key Performance Indicators (KPIs)

The main «locomotive» of revenue in the company is the sales department. This and the next few paragraphs describe approaches to motivating managers, although these approaches, with some modifications, can be applied to tractor drivers, adjusters, operators, storekeepers, loader drivers, utility workers.

In the company, the salary consists of the constant and the bonus part. The bonus part takes up a significant share of the salary, so it is very important to identify the optimal method of awarding bonuses, i.e. motivating employees to sell as profitable products as possible.

At the moment, the KPI (key performance indicator) includes 4 positions:

- execution of the sales plan in pieces
- implementation of the sales plan in rubles
- overdue accounts receivable
- special tasks.

The introduction of the first point was the fact that the manager, having received a sales plan in rubles, is not interested in selling specific products, but only wants to achieve planned implementation. As already mentioned, the company sells products of different sizes and prices. Naturally, different profitability is planned for each product, that is, it is much more profitable for the company to sell one product than another. It is no secret that it is more profitable to sell the same product line in small packages than in large ones. There is an unofficial rule that products of one volume should be cheaper per liter than products of 2 times smaller volume. It is easy to verify this on the prices of juices of 1 and 2 liters in any grocery store chain. By entering this indicator, managers will be motivated to sell smaller packaging.

Special tasks here are understood, for example, to attract new customers, sign important contracts, promote new products and other tasks that are determined by the head of the sales department of the branch or the CEO.

These 4 positions occupy 100% of the bonus part, but the shares between them vary to increase managers' focus on one or another indicator.

1.8.18. Development of incentive programs

By motivational programs, we mean what will add income to managers in excess of the bonus (that is, eliminates demotivation), while motivational programs are compiled taking into account profit maximization (that is, so that the costs of the motivational program are covered by profits from products sold as a result of this program).

For example, a program is organized in a competitive form: among all branch managers, a competition is held on the percentage of growth of some type of product.

You can try a different format. The fulfillment of the first and second KPI indicators is the percentage of completion of the plan. That is, the number of units sold (rubles) is divided by the plan in units (rubles). If we select the necessary items of the nomenclature, and for each piece sold we will count not 1 piece, but, for example, 3 pieces, then managers will be motivated to sell more of these products, because it is easier to achieve and exceed the corresponding KPI. The task arises - to determine the size of the increasing coefficients. Again, they need to be made so that the profit is greater than the cost of this type of motivation. To do this, you need to make a preliminary calculation, possibly in several scenarios at once.

By analogy, you can make not only a motivational program, but also review the 1st and 2nd points of the KPI. It was described above that 1 KPI point affects the reduction of the average pack's litre. On average, it is more cost-effective than large packaging. Then it is possible to make a constant coefficient from the previous paragraph for small packaging (or for profitable products). However, in order not to increase the premium fund, it will be necessary to enter a coefficient less than 1 for low-profitable positions (usually large packaging), i.e. for every 10 sold pieces, we count 5 if the coefficient is 0.5. When drawing up such an approach, it is important not to leave the manager at a loss, that is, initially choose such coefficients so that if he did not pay attention to them and sold the assortment according to his regular distribution, he would receive as much as usual. If the coefficient is linked to profitability, then this will be more accurate, but it will have to be revised after each cost update. This can happen once a month or quarter. Then you will often have to make efforts to recalculate these coefficients. Therefore, if the company does not have automatic summing up and programmatic recalculation of motivational coefficients, then it would be more correct to link the coefficient to the number of liters.

On the one hand, this approach will save us from a drop in profitability and profit if the average volume of products increases for some reason. On the other hand, if this approach is used correctly, the manager will be able to receive more if he intensively sells products with a coefficient greater than 1.

It is clear that the customer is most important, and it is he who will make requests for the order and insist on exactly the products that he needs. But we expect that up to 20% of customers, with the help of professional manager skills, will additionally or instead of other products take products that are profitable for us.

1.8.19. Development of piecework sales coefficients

When using the current KPI method for plans, there is a significant disadvantage for the company and for managers:

- when setting a sales plan, seasonality is taken into account, so even in the season, even in the off-season, the manager receives the same salary (with 100% KPI fulfillment), although he spends different amounts of effort at different times of the year. At the same time, the company has little revenue and profit in the off-season, that is, the risk of cash gaps increases;
- when setting up a sales plan, sales from previous months are taken into account. If the manager had sales growth for the year, then the plan will not be set based on sales of the same month last year, but first of all the sales of the previous month will be taken into account. This means that the plan will be higher than last year, and the salary will remain the same;
- The basic salaries and bonuses of managers were formed on a contractual basis and they differ by branches. Moreover, salaries are not proportional to sales volumes.

These and other similar reasons push us to change the methodology of the basic calculation of premiums. The specific coefficients approach is a direct approach, when for each sold piece of each item of each price column, the manager receives a certain amount of rubles and kopecks. The higher the price column, the higher the specific coefficient, the more profitable the position, the higher the coefficient among the price column.

In the initial version, in order not to greatly complicate the understanding of managers about the calculation of wages, piecework coefficients can be entered for each manager of their own, divided by literages. But without dividing into price columns, because they are rigidly defined by the price list: if the customer starts buying more than a certain amount, he goes to the price column below.

The heads of the commercial departments of the branches can also be transferred to a piecework coefficient, which can be calculated from the total sales of each branch.

This approach will allow us to take into account the shortcomings of the planned approach. There is a main disadvantage of this approach: due to significant sales in the season, managers will receive a lot, that if they immediately pay this amount, they may not have enough to produce new products and the situation of a cash gap will arise again. In the off-season, the manager will have a salary slightly higher than the salary, while everyone has personal fixed expenses. From the manager's point of view, the solution is simple: do not spend the entire amount from seasonal sales, since managing personal finances is the problem of each employee, not the company. Considering that the total salary for the year has not changed, then there can be no claims against the company. From the point of view of the company and the legislation, there is also a way out: unlike the salary, the bonus part does not have to be paid immediately, but it can be partially issued for a certain period, including off-season months.

Summarizing the manager will understand that his salary will grow proportionally from sales growth (while maintaining the size of piecework coefficients at least for the season), and the company will have working capital at the beginning of the production season, that is, when big sales have not yet begun and money from customers has not begun to arrive.

1.8.20. Making a Profit and Loses (PnL) plan and fact

The PnL (Budget of Profit and Loses) shows the company's profit and profitability. However, even in such a well-studied, formalized approach to accounting for income and expenses, there may be various variations.

The standard (aka accounting and tax) approach is the implementation strategy. That is, only those expenses that participated in the sale of products are included in income and expenses. If the output was greater than the sales, then only the share of direct costs equal to the ratio of sales to output is written off to the cost of PnL. The rest of the costs incurred for the issue are set aside in the balances. In those months when output is less than sales, sales costs are written off according to the ratio of sales to output. Costs are written off more than in the real issue, because everything in excess of the output is deducted from the costs in the balances (in fact, it is: if the output is less than the sales, then part of the sales must have been from the balances).

This approach is mandatory for accounting and tax accounting, and is convenient primarily for companies whose monthly output and sales do not differ much from each other. In this company, in some months, output may be 2 or more times more than sales (for overstocking before the season, because according to forecasts there will not be enough production capacity), that is, less than half of the real consumption is written off. On the other hand, in the off-season, there may be scheduled equipment repairs for almost a month, when there is only a week of release, and almost all sales come from leftovers. Thus, according to this approach, we will have to take the cost of production, divide by the amount of output, and multiply by the number of products sold. The PnL will need to record a figure ten times higher than the cost of production. As a result, profit or loss by month looks completely meaningless from a managerial point of view. Only the financial result (profit or loss) for the year and the remaining cost of the cost in monetary terms looks relatively indicative.

To solve the problem of adequately displaying the financial result on a monthly basis, you can apply such a speculative management approach: given the fact that there are few balances regarding output and sales in the company (ideally for 2 weeks), it can be assumed that the implementation of the issue is the real implementation, but there are no balances. That is, the business is presented not as a model of output, sales of products and residues, but as the sale of services or release strictly for sale, realizing that in fact there is no profit now in that volume, but it will appear in 2 weeks. But this is probably the only disadvantage. There are more advantages:

- does not need to «twist» costs, that is, how much was actually spent on the release, so much goes into costs. An implementation is an implementation of a release. That is, we see the real costs, there is no need to find out why this or that figure is very small or huge.
- the financial result is visible on a monthly basis, and the seasonality of profit is also visible, correlating with the seasonality of sales

This approach is called «PnL by production». This is how we make a plan (we take data from statistics and forecasts of responsible departments). We record the fact in the next column at the end of the month and analyze the deviations.

1.8.21. Making a Cashflow Plan

The structure of the Cashflow Budget has sections:

1. Operational activity.
   These are income from basic sales, cost of production, commercial activities, management costs, Payroll and taxes
2. Financial activity.
   These are income from deposits and deposits, expenses on loans, debts, leasing (both for the body and for interest).
3. Investment activity.
   These are income from the sale of equipment, shares in a subsidiary, and dividends. Expenses, for example, are the purchase of equipment.

The preparation of a Cashflow Budget plan is made up of a ready-made PnL, articles that are not in the PnL (filled in by those responsible, for example, leasing) and an auxiliary Shift model - for each Cashflow Budget article, a payment offset relative to the PnL is set.

For management control, budget adjustments should be made several times during the year. The company uses an even more precise approach - a rolling budget for 24 months ahead, updated every month.

Each section has subsections, for each of which a responsible person is responsible for planning. The Finance director collects budget adjustments from all those responsible. Taking into account the adjustments, the rolling budget is updated. The fact is entered as payment is made. At the end of the month, the deviation of the plan from the fact is analyzed, work is carried out with those responsible for articles with large deviations.

It is important to note that the fundamental pair of articles - the implementation and receipt of money is planned as carefully as possible. There is a report on accounts receivable in the accounting database, which takes into account payment schedules under deferral agreements and additional agreements (in the hobby peat market, it is customary to stock up customer warehouses before the season, but receive payment during the season according to the schedule, and not according to a fixed small delay). Based on this report, it is possible to plan the receipt of money without involving the commercial department, or at least calculate the weighted average shift of receipts relative to sales. This report becomes especially accurate when Overdue accounts Receivable is one of the KPIs of sales managers.

1.8.22. Compilation of report on current company status

The management report on the status of the project includes stocks in warehouses in sales prices, stocks of raw materials and materials in purchase prices (in its warehouse and in the warehouse of the manufacturer awaiting full payment of raw materials), accounts receivable, balance on accounts and in cash, loans paid, the value of property and own raw materials. From this, we subtract the accounts payable, loans and credits with interest As a result of the monthly compilation of such a certificate, we see how the state of the company is changing. In fact, this is a simplified analogue of the company's balance sheet form.

1.8.23. Calculation of payments for loans and deposits

The company has investments from the holding company, issued in the form of a loan, there are bank loans that we take from the bank, there are deposits. For each loan and deposit, the company's economists calculate and coordinate payment schedules, analyze the terms of loan agreements with banks, lending rates and deadlines. If it is possible to determine the volume of payments themselves during the year, an optimal payment schedule is drawn up in terms of the availability of funds, i.e. the seasonality of sales, taking into account the delay.

1.8.24. Analysis and optimization of raw materials and components delivery methods

Peat is the main raw material and cost item in the delivery of goods to production. It is supplied from the peat extraction site. In order to minimize shipping costs, it is necessary to choose the best delivery method with the available resources. If it were possible to choose from any type of transport, then the choice would fall on railway transport delivering by single-track railway. That's what some companies do. The capacity is large, the fuel costs are small, the only question is whether you have your own railway and locomotive. In this case, there is neither a road nor a diesel locomotive (because this is a significant investment). As well as there is no own transport. However, there is a boiling area (peat pressing). Peat is a fairly light mineral with a low density, easily compressible 2-3 times of the original volume. Traditionally, peat is transported in bulk in a dump truck of 35 m3, alternatively it is a truck with a capacity of 18 pallets of 4 m3 of compressed peat in each. It remains to compare the costs associated with packing and choose the best method. There is an alternative to a dump truck - a woodchipper truck. At least 2 times more peat fits into it, but this is not a common type of transport for rent, i.e. it is necessary to consider the option of purchasing transport. In addition, a dump truck in the cold has the problem of freezing part of the peat to the body, in a woodchip truck the same problem will be on an even larger scale.

The rest of the raw materials are purchased from suppliers, which they either supply to production themselves, or their own machine takes from suppliers. In the second option, you can analyze and optimize the transport route by suppliers, i.e. we have a typical traveling salesman [15] problem, the solution of which is well known.

1.8.25. Analysis and optimization of delivery methods to customers within the city

With the help of hired transport, products are delivered to customers several times a week. Tariffs depend on the load capacity of the car, the number of hours in the city, the number of kilometers in the region (outside the city), the number of points, the availability of shipment, the availability of unloading.

Of course, the cargo carrier controls exactly how the car will move, but it is possible to contribute to helping the cargo carrier determine the optimal route (again, we are talking about the traveling salesman's task). This will be beneficial to the company as a customer, since it takes less hours/km on the road, but the freight carrier will be able to get free faster and accept a new order, increasing the turnover of one car.

1.8.26. Optimization of product layout

Own store does not generate much revenue during production. However, the store can be used as an experimental platform where you can analyze consumer behavior when prices change, goods, their location on the shelf, types of promotions and demand for products, including in dynamics.

It is not possible to lay out all the nomenclature items in the store, because in addition to the main products produced, the company is a distributor of related products from other suppliers: mineral fertilizers, insect and rodent protection products, poisons, seeds, garden tools. Together with related products, the number of SKUs increases several times. With a large number of types of products, it is important to make a division by zones and think about where it would be most appropriate to exhibit products, that is, to prioritize what should be in the most prominent place and why.

1.8.27. The concept of discounts

If for some reason (reloading of the main warehouse, approaching the expiration date, loss of the presentation of the package, testing hypotheses) discounts are needed, you need to consider their size. In one of the previous paragraphs, the concept of discounts was discussed in detail. However, the growth data is predictable. To collect statistics, the store helps with production, as well as chain stores and other end customers with whom they directly negotiate a discount. To stimulate even more growth, we can arrange an artificial shortage of products of a larger volume so that the customer buys the products we need. There is a possibility that the customer will turn around and leave, but the discount amount may make him change his mind. For example, if there are products in 50 liters and 25 liters, it is necessary to sell 25 liters of products as much as possible. Then we remove 50L products from the store, and get a discount on 25L products such that 2 packs of 25 liters cost the same as one 50L. For most customers, it does not matter if 2 packs are small or one is large, if the price per liter is the same.

1.8.28. Optimization of store space

The number of SKUs, taking into account the sales of related products, becomes so large that it is impossible to put all products on the trading floor. At first glance, it is worth expanding the warehouse and laying out everything at least one piece at a time. In fact, not every product has one piece in the trading floor right now. The store is divided into a sales area and a utility room - the store's warehouse. Products are brought from production to the store's warehouse. For sure, they are not delivered in optimal quantities, i.e. not in proportion to demand. As part of optimization, it is important to set restrictions, for example, that products in the size of 1 prefabricated pallet are brought from the warehouse to the store. Observations show that the turnover of products can vary from 2 days to 4 months. If you optimize the goods in the sales area and in the warehouse of the store, you can significantly reduce the area. And only if there is not enough space in this case, then make an extension, but of a smaller size. It is also never superfluous to conduct an ABC analysis of sales [16], and isolate only liquid products.

1.8.29. Optimization of salesperson working hours

By observing the time of receipt issuance during the day and the dynamics throughout the year, it is possible to determine the optimal number of sellers by determining the standard for the number of products sold per hour. In this case, we will save on the salaries of sellers and the electricity spent on lighting and heating the store. Thus, we will increase the profit and profitability of this outlet.

1.8.30. Implementation of project management system

It can be noted that many tasks overlap with other departments: marketing, logistics, commercial, production, supply department, technologists, accounting. In general, tasks can be divided into operational ones, i.e., which an employee does on an ongoing basis and project ones, i.e., which are done on a one-time basis as part of the project implementation, and then the project is completed or transferred to operational tasks. In a constantly developing company, the number of project tasks is increasing and a system is needed to analyze the speed of execution of these tasks without compromising quality.

The traditional project management system has been a cascade model for many years (model «Waterfall»). Within the framework of it, the transition to the next stage took place only when the current stage was fully ready, so as not to return to the previous one. It used time, resources, and quality inefficiently.

Therefore, a different approach is now being used in the world. Initially, IT companies, and now in many other areas, use a flexible project management system - Agile. Its two main branches, scrum and kanban, are used in approximately equal distribution between companies [43, 44].

Software is needed to implement a flexible methodology. There are both free versions and paid platforms with many features. Although sometimes only physical «task boards» are used, and just as effectively as electronic ones. The company's experience has shown that it is convenient to use this system not only in IT. At the same time, time planning, prioritization of tasks, standardization of meetings made it possible to reduce the project execution time while maintaining quality.

1.8.31. Analyzing the speed of task completion

Within the framework of the project, employees receive tasks prioritized and with a description of the approximate time to complete them. To analyze the performance of tasks and fix weaknesses, you need a tool for visualizing the analysis. Traditionally, this is the Gantt chart [18]. It is also possible to use the PERT [19] evaluation and analysis method. The goal is to minimize project time while respecting quality and costs. For example, within the framework of achieving the set goal, it will be possible to adjust the composition of teams and the number of people involved in solving a specific task based on the personal speed of completing tasks of each project participant.

1.8.32. Description and optimization of business processes

Often in large companies, the tasks of an analyst, or rather a business analyst, include a description of the company's business processes for subsequent analysis and optimization. First, the analyst goes through all the responsible persons, interviews them, what tasks they are engaged in, with whom they interact. This is a collection of data to describe processes «as is». Next, visualization flowcharts and descriptions of Business processes (or departments) are compiled. Some of the well-known formats are BPMN and IDEFx [20, 21]. The analyst, together with the superiors and heads of departments participating in the scheme, determines what can be changed, which blocks are superfluous, which should be handled by another department, which are not in the right order. As a result of the coordination, the model is obtained as it «to be». According to her, departments are starting to work. One way or another, the speed and quality of work increases, and the execution time of the process decreases. That is, profits will increase directly or indirectly. At the same time, the structure begins to be clearly traced for each department and process. No actions are done by intuition, for new employees, the described business processes can be included in the instructions.

1.8.33. Implementation of management reports

It is important for senior management to understand the state of affairs in their company. However, the manager does not have enough time to download tables from the database every day and do a comparative analysis. Responsible people in each department help him in this.

In the financial and economic department, these are daily registers of payments, receipts of money and Cashflow budget, monthly plan and fact of PnL budget.

The analyst provides the above-described information about the state of the company. The analyst also makes a conclusion about the plan/fact of sales by branches for each product in order to understand exactly how planning takes place, and monitor revenue growth and other indicators. The same report also includes production (with production by conveyors and shifts), warehouse operation (shipment of pallets, machines, number of crews and production), warehouse balances by branches, and dynamics of changes in indicators to the previous period.

The accounting department issues a report on the results of work for the month/quarter.

The sales department makes an analysis by customers, by managers, by revenue growth.

Almost all departments have similar reports. It takes a lot of time to look at all these tables. Ideally, you need to visualize all the reports in one place, which will be shown in the next paragraph.

1.8.34. Implementation of the report visualization system

Basically, reports in the company are collected from the accounting system. A small amount of data is stored locally, but it is possible to organize their entry into the main accounting system. However, the Accounting System provides information in the form of tables and reports. The figures are accurate, but not always visual, unlike graphs and diagrams.

To visualize management reports, companies are implementing a report visualization system that takes data from the accounting database (in general, it can be data from the database of call centers, CRM systems, Excel files, and from Google analytics or Yandex.Metrica - the main sites that track information about site traffic). In this visualization system, you can set up graphs, for example, by department. And all reports can be seen dynamically in all indicators, by clicking «sinking» deeper if necessary, that is, such «dashboards» helps to place information of any level of detail compactly and visually. Visually, information is often perceived faster, it is easier to draw conclusions and make management decisions.

1.8.35. Forecasting peat extraction

Peat extraction takes place from May to the end of August. This is the optimal time for mining, because in addition to the fact that there is no snow anymore, the peat has dried up, that is, its humidity, and therefore its density decreases, mining equipment can enter the fields. This means that eventually more products will fit into the car/railcar than with wet peat, which means that the cost of delivery one piece will decrease.

However, in order for the peat to be dry and of high quality, it is necessary to prepare the fields of extraction: uproot and collect stumps, flush drainage pipes, deepen ditches, level the profile of the field. All together, this is called swamp preparatory work (SPW).

In addition, throughout the year, there is a transfer from the peat extraction site to peat production. From September to November, peat is exported from swamp fields to dry valley.

The peat extraction site is divided into fields, and the fields are divided into maps. At the beginning of the calendar year, the amount required for peat extraction is determined. The head of the peat extraction site determines the fields and maps from which it will be possible to extract it, whether the current peat harvesting equipment is enough. Peat extraction begins in May. During this time, you need to understand the average number of sunny days, because peat is not mined in the rain. The task of predicting the amount of peat is to determine the standards of extraction by technique, by the volume of peat extracted per hectare, the number of people involved with a limit on the number of days for extraction, the remoteness of fields from the base and the condition of the quality of the conducted SPW.

1.8.36. Cost analysis and optimization

Within the peat extraction process, many processes occur suboptimally. This can be the number of fields for mining, and their choice, optimization of diesel fuel costs, the number of people involved, optimization of movements and operations. Upon closer examination, many weaknesses can be identified that can be fixed without additional investment and without changing the technology. It was at this site that a pilot lean manufacturing project was implemented with the search for losses and the use of solution tools. It was possible to reduce the time of the process from the beginning of the SPW to the end of peat extraction by 30%, increase the volume of production by 20% with the available equipment. The main solutions for improvement were the organization of a summer canteen and a rest point in the fields, which reduced an hour per day of working time on the road, optimizing the distribution of queues for blowing equipment and gas stations through scheduling, as well as the introduction of an additional unit of vehicles for transporting employees. In addition, the purchase of transport for the inspection of fields by the site master, the introduction of shift tasks in the form of an operational production analysis, a report on it, as well as the standardization of workplaces helped to improve quality control of extracted peat and reduce the number of defects.

1.9. Chapter conclusions

This chapter describes the mathematical, economic, financial and analytical tasks of the enterprise. The tasks of inventory management, distribution of the production plan along conveyor lines, creation of a recommendation system, calculation of pricing of marketplaces and the task of delivery to nonresident customers are given in a mathematical description and their solution algorithms are proposed, the mathematical task of creating a new product according to specified characteristics is set. It is important to recall that this chapter discusses a specific full-cycle enterprise with its own specifics. Therefore, both the tasks themselves and the proposed solutions may be suboptimal or not at all suitable for another enterprise. On the other hand, it is worth saying that many tasks in one form or another exist in any business, including in the digital economy, so at least their review and designation, and maybe the proposed solution ideas can be applied at other enterprises. The value of this chapter is the analysis of the application of mathematical models of various levels of complexity to real tasks of a manufacturing enterprise. Moreover, by generalizing the formulations of many problems, researchers can build universal practical models using a wide range of mathematical theory. In addition, the set of task descriptions helps to understand standard enterprise management systems and their relationship.

The second chapter will be more fundamental in nature, however, its results also have high practical significance, including with some limitations they can be applied in business tasks from the first chapter, for example, in sales forecasting.

CHAPTER 2. Development of analysis of nonstationary stochastic processes

2.1. Introduction

Stochastic processes play an important role in the analysis and forecasting of phenomena in various fields, such as economics and finance (exchange rates, oil prices, stock prices, inflation, GDP of the country), climatology (temperature, atmospheric pressure, wind speed, precipitation), engineering (vibrations of machinery and equipment, acoustic time series, deformations, stresses), biology (populations, clinical parameters: blood glucose level, blood pressure, pulse, body temperature) and others. An important characteristic of stochastic processes is their stationarity, that is, the statistical properties of the process do not change over time. However, in real-world applications, they often encounter processes that do not meet the conditions of stationarity. Such processes are called nonstationary.

The purpose of this chapter is to study the development of nonstationary stochastic processes and to develop methods and models for their analysis and forecasting. One of the key aspects is the task of forecasting non-stationary processes. In the process of studying nonstationary processes, the task of approximating models arises, that is, choosing an adequate mathematical description of such processes. This may include the development of new models or modification of existing ones to adapt to non-stationary conditions. An important aspect of the analysis of non-stationary processes is their reduction to stationary ones. This makes it possible to apply classical time series analysis methods developed for stationary processes. The separation of a piecewise stationary process from a nonstationary one will be considered in this chapter.

The tasks of non-stationary processes were considered in the works of Magnus J. R. [45] (he identifies short-, medium-, and long-term forecasting horizons, which will be discussed later), deals with issues of heteroscedasticity. Berezin I. S. writes about approximation methods, including systems of Chebyshev trigonometric polynomials [46]. Long-term forecasting models are considered by T. Anderson [47]. Smoothing methods are present in the work of Slutsky E.E. [48].

2.2. Time series trend analysis

2.2.1. Introduction

Let several definitions be introduced for the same understanding of terms.

Definition 1. A random (stochastic) process is a one-parameter family of random variables that have finite variance.

Definition 2. The implementation of the process (sampling function, sampling, trajectory) is the corresponding family of realizations of random variables with sequential iteration of all parameters.

Definition 3. A trend is a line of mathematical expectations of random variables of a stochastic process.

Definition 4. A time series is one of the implementations of a random process.

Definition 5. The term «forecast» will denote a trend model of a random process based on a time series and bands of possible deviations around the trend (model). If necessary, the deviation band may either not be considered at all, or any part of it may be taken into account.

Definition 6. The time series forecasting horizon is the interval in the future for which a forecast is made for the past.

Definition 7. The depth of the forecast of a time series is the interval in the past, according to which the forecast horizon is determined.

Regarding the above definitions, there is a detailed textbook by Ostrem K. Yu. [49].

There are two main models that are used to predict time series: an autoregressive model and a trend-highlighting model. The problem with the first model is that, due to its construction, it is possible to predict only for short-term periods, since the variance for each subsequent time node increases significantly, that is, the prediction result becomes less reliable. Meanwhile, the second model is actively used in long-term forecasting. Such models are considered, for example, by T. Anderson [47].

Often, the authors make a forecast for any intervals — short-term and long-term, without thinking about how far they can predict. In the literature, this problem was dealt with by Prasolov A. V., Khovanov N. V. and Wei K. [50–52].

2.2.2. Problem statement

It is required to identify the trend of the time series by the Fourier series decomposition method. It is necessary to take into account the inaccuracy of calculating the coefficients of the Fourier series and minimize it by optimizing the coefficients [6]. Next, the task is — based on the selected trend to obtain an algorithm for constructing a forecast and a time series horizon [4]. The algorithm will be tested on time series of various nature: Brent crude oil prices, the ratio of euro to dollar exchange rates, temperature data in Moscow and St. Petersburg, as well as world grain prices. As a result, it is necessary to compare the estimates obtained for compliance with real life.

2.2.3. Construction of time series trend

It is necessary to understand that the trend is the sum of a deterministic (linear $Y(T_i)$ and the segment of the Fourier model $F(T_i)$) and the random (speculative) component $\omega(T_i) \neq 0$. In this section, the deterministic part is built in three steps. For clarity of the results, all calculations were made using the prices of Brent crude oil taken in the period from January 2009 to September 2014 with an interval of one week [53].

The time series $A(T_i), i = \overline{1, n}$, is considered, where T_i — are equidistant time measurements. In the context of this example, $T_i - T_{i-1} = 1, i = \overline{2, n}$; $n = 299$; $A(T_i)$ — the price at each point in time.

2.2.4. Allocation of Linear Component

First, the linear component of the trend is highlighted using the least squares method. The approximation $A(T_i), i = \overline{1, n}$, occurs by a polynomial of the first degree $Y(t) = at + b$.

For the time series shown in Figure 5, the linear function will look like

$$Y(t) = 0,19t + 68.$$

Substituting the discrete values $T_i, i = \overline{1, n}$ in $Y(t)$ instead of the continuous variable t, we get (Figure 6)

$$Y(T_i) = 0,19T_i + 68, \quad i = \overline{1, n}.$$

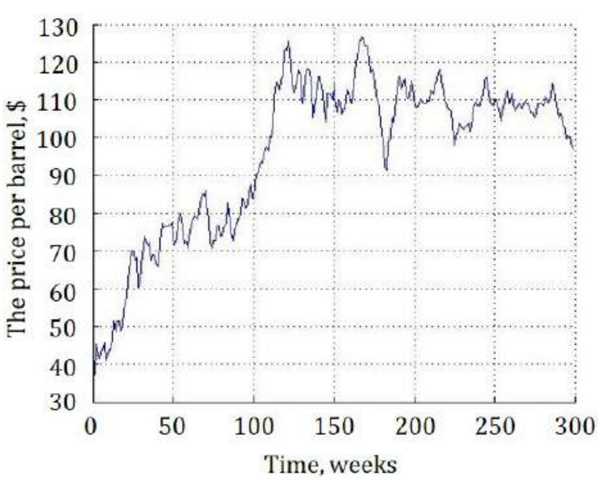

Figure 5 – The price of crude oil «Brent»

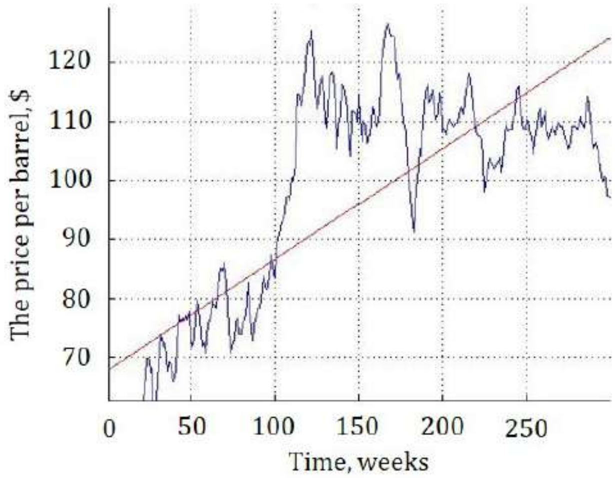

Figure 6 – The linear component of $Y(T_i)$

Before proceeding to the next step, the values of $Y(T_i)$ are subtracted from $A(T_i)$ and new values of the residuals are obtained (Figure 7)

$$R(T_i) = A(T_i) - Y(T_i), \; i = \overline{1, n}.$$

2.2.5. Decomposition into a segment of a Fourier series

At the second stage, a new time series $R(T_i), i = \overline{1, n}$, is approaching by the Fourier series decomposition method.

The trigonometric Fourier series of the function $Z(t) \in L^2([0, T])$ is called a functional series of the form:

$$Z(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left(a_k \cos\left(\frac{2\pi\, k}{T} t \right) + b_k \sin\left(\frac{2\pi\, k}{T} t \right) \right).$$

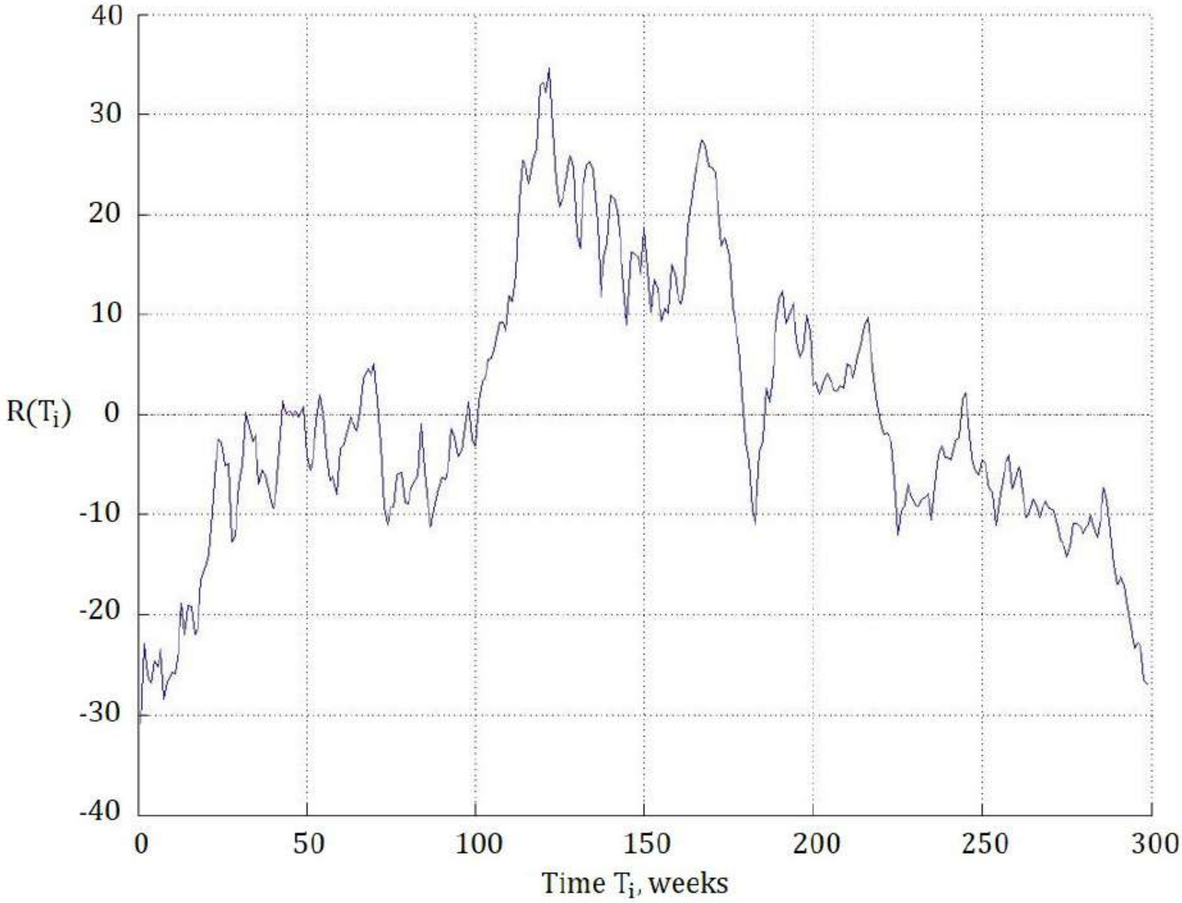

Figure 7 – $R(T_i)$ — Time series without linear component

Since there is a discrete time T_i, $i = \overline{1, n}$, then the coefficients a_k and b_k will be searched approximately, with the replacement of integrals

$$a_k = \frac{2}{T} \int_0^T R(t) \cos\left(\frac{2\pi\, k}{T}\, t\right) dt; \quad b_k = \frac{2}{T} \int_0^T R(t) \sin\left(\frac{2\pi\, k}{T}\, t\right) dt$$

the areas of the corresponding rectangles [47]:

$$a_k \approx \frac{2}{n} \sum_{i=1}^{n} R(T_i) \cos(\frac{2\pi\, k}{n} T_i); \quad b_k \approx \frac{2}{n} \sum_{i=1}^{n} R(T_i) \sin(\frac{2\pi\, k}{n} T_i).$$

Due to the fact that the function is approximated numerically, the number of terms (harmonics) in the Fourier series will be assigned by the researcher himself by entering the variable $\boldsymbol{c}$. Then, given the fact that a_0 will turn to 0 (this is obtained after the linear component is subtracted), it turns out

$$F_c(T_i) = \sum_{k=1}^{c} \left(a_k \cos\left(\frac{2\pi\, k}{n}\, T_i\right) + b_k \sin\left(\frac{2\pi\, k}{n}\, T_i\right)\right), \quad i = \overline{1, n},\ c \in \mathbb{N}.$$

For clarity, the result is given for $\boldsymbol{c} = 3$ (figure 8) and $\boldsymbol{c} = 15$ (figure 9).

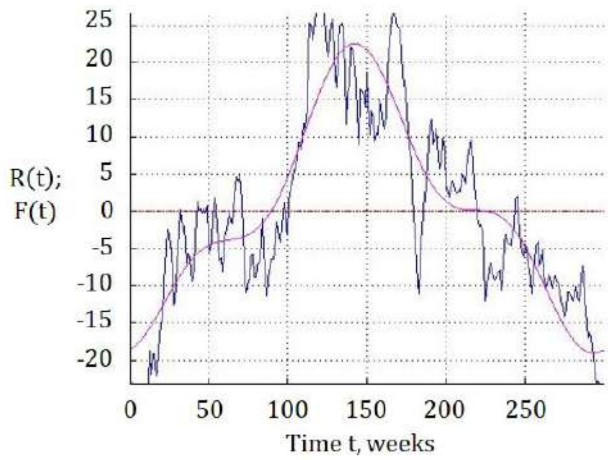

Figure 8 – The periodic component of the time series at c=3

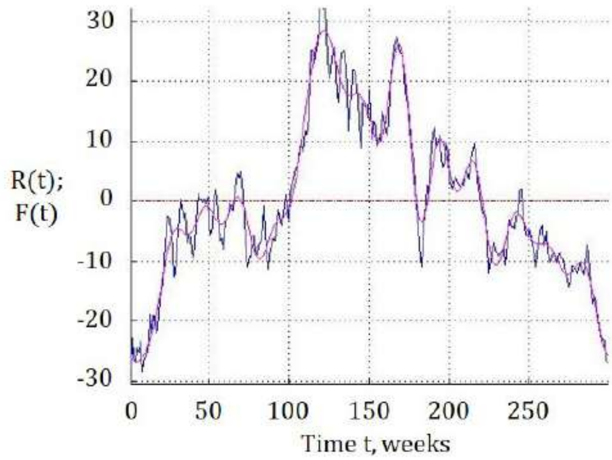

Figure 9 – The periodic component of the time series at c=15

The squares of the differences $R(T_i)$ and $F_c(T_i)$ are considered for various $\boldsymbol{c} = \overline{1,200}$ (figure 10)

$$D(c) = \sum_{i=1}^{n} \left(R(T_i) - F_c(T_i)\right)^2, \quad c = \overline{1,200}.$$

Remark 2.2.1 When $\boldsymbol{c} = 149$ functions $F(T_i)$ and $R(T_i)$ are actually match. In this case, the values of $D(c)$ for $\boldsymbol{c} = 148$ and $\boldsymbol{c} = 150$, $\boldsymbol{c} = 147$ and $\boldsymbol{c} = 151$,... match. Then we can assume that the optimal number of harmonics $\boldsymbol{c}$ of the time series corresponds to $N = \frac{n-1}{2}$ for an odd and $N = \frac{n}{2}$ for an even number of nodes of the time series. This assumption logically follows from the fact that time is discrete, which means that you need to approximate the function of time at n points. But N — is the number of harmonics of the Fourier series and they are enough to approximate the time series with n nodes [47,54].

2.2.6. Optimization of Fourier series segment coefficients

In the resulting model $F(T_i)$ there is a calculation error that arises from two sources: an approximate calculation of integrals for a given function $R(T_i)$, which is decomposed into a series, and the absence of this function, since the integral contains realizations of random variables, and not the values of a deterministic function. In addition, the decomposition does not go into a Fourier

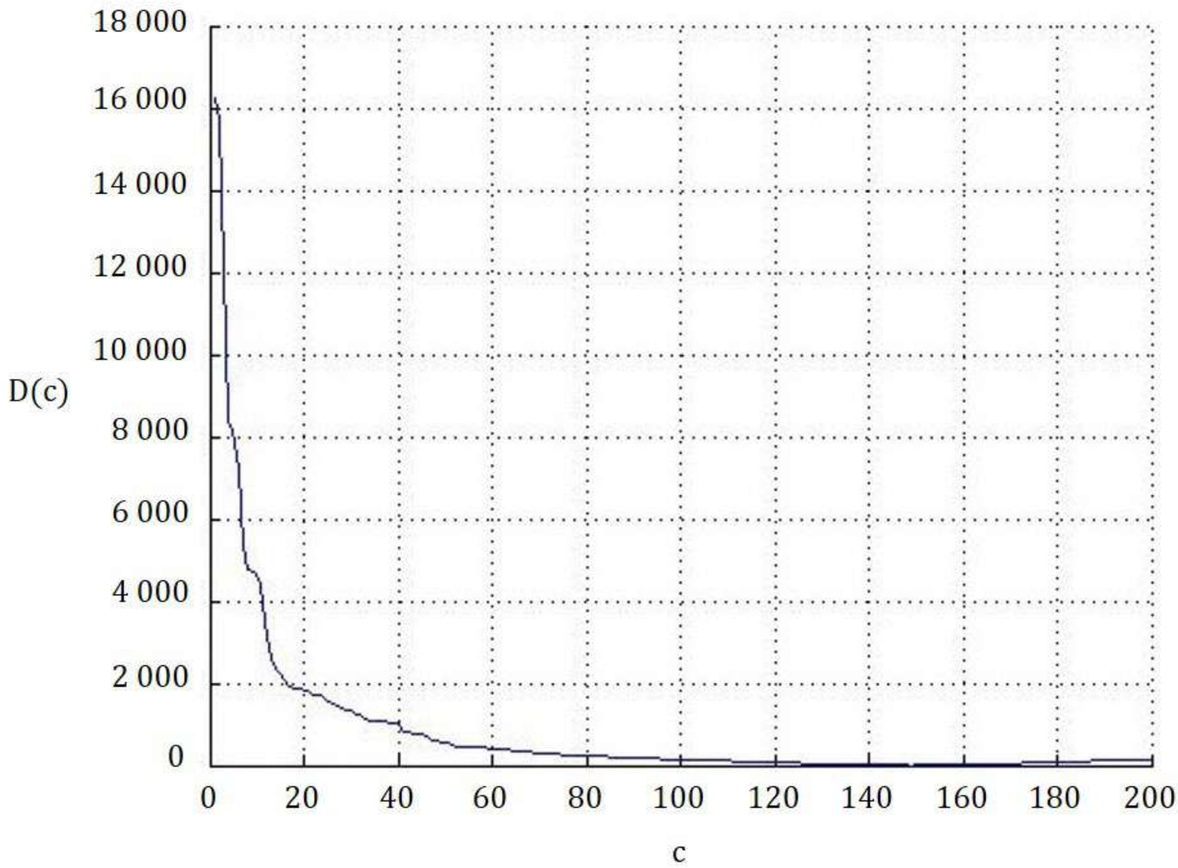

Figure 10 – Plot of $D(c)$

series, but is limited to a segment of $\boldsymbol{c}$ harmonics.

These errors will be taken into account in the coefficients $a_k + \varepsilon_k$, $b_k + \delta_k$ of the new segment of the Fourier series:

$$\sum_{k=1}^{N} \left((a_k + \varepsilon_k) \cos\left(\frac{2\pi\, k}{n} T_i \right) + (b_k + \delta_k) \sin\left(\frac{2\pi\, k}{n} T_i \right) \right), \quad i = \overline{1, n}.$$

The function $S(\varepsilon_k, \delta_k)$ is introduced:

$$S(\varepsilon_k, \delta_k) = \sum_{i=1}^{n} \left(R(T_i) - \sum_{k=1}^{N} \left((a_k + \varepsilon_k) \cos\left(\frac{2\pi\, k}{n} T_i \right) + \right.\right.$$
$$\left.\left. + (b_k + \delta_k) \sin\left(\frac{2\pi\, k}{n} T_i \right) \right) \right)^2 .$$

It is minimized by unknown parameters ε_k and δ_k. The function S is the sum of squares, so there is a minimum of this function at least zero, that is, $\min\limits_{\varepsilon_k, \delta_k} S(\varepsilon_k, \delta_k) \geq 0$. The difference is transformed by grouping the known values in one bracket, and the unknown ones with their known coefficients in another, we get:

$$S(\varepsilon_k, \delta_k) = \sum_{i=1}^{n} \left(\left[R(T_i) - \sum_{k=1}^{N} \left(a_k \cos\left(\frac{2\pi\, k}{n} T_i\right) + b_k \sin\left(\frac{2\pi\, k}{n} T_i\right) \right) \right] - \right.$$
$$\left. - \left[\sum_{k=1}^{N} \left(\varepsilon_k \cos\left(\frac{2\pi\, k}{n} T_i\right) + \delta_k \sin\left(\frac{2\pi\, k}{n} T_i\right) \right) \right] \right)^2 .$$

But then, from the fact that ε_k, δ_k were introduced by us, it is possible to achieve $\min\limits_{\varepsilon_k,\delta_k} S(\varepsilon_k, \delta_k) = 0$, i.e. each term under the sign of the external partial sum must be zero. And from the fact that each of the terms is squared, it follows

$$R(T_i) - \sum_{k=1}^{N} \left(a_k \cos\left(\frac{2\pi\, k}{n} T_i\right) + b_k \sin\left(\frac{2\pi\, k}{n} T_i\right) \right) =$$

$$= \sum_{k=1}^{N} \left(\varepsilon_k \cos\left(\frac{2\pi\, k}{n} T_i\right) + \delta_k \sin\left(\frac{2\pi\, k}{n} T_i\right) \right).$$

Let

$$X(T_i) = R(T_i) - \sum_{k=1}^{N} \left(a_k \cos\left(\frac{2\pi\, k}{n} T_i\right) + \right.$$
$$\left. + b_k \sin\left(\frac{2\pi\, k}{n} T_i\right) \right), \quad i = \overline{1, n}.$$

Then, substituting all known values, we get a system of linear equations with $(n \times 2N)$-matrix that is resolved using the least squares method.

$$\begin{pmatrix} cos\left(\frac{2\pi 1}{n} T_1\right) & sin\left(\frac{2\pi 1}{n} T_1\right) & \cdots & cos\left(\frac{2\pi N}{n} T_1\right) & sin\left(\frac{2\pi N}{n} T_1\right) \\ cos\left(\frac{2\pi 1}{n} T_2\right) & sin\left(\frac{2\pi 1}{n} T_2\right) & \cdots & cos\left(\frac{2\pi N}{n} T_2\right) & sin\left(\frac{2\pi N}{n} T_2\right) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ cos\left(\frac{2\pi 1}{n} T_n\right) & sin\left(\frac{2\pi 1}{n} T_n\right) & \cdots & cos\left(\frac{2\pi N}{n} T_n\right) & sin\left(\frac{2\pi N}{n} T_n\right) \end{pmatrix} * \begin{pmatrix} \varepsilon_1 \\ \delta_1 \\ \vdots \\ \varepsilon_N \\ \delta_N \end{pmatrix} = \begin{pmatrix} X(T_1) \\ X(T_2) \\ \vdots \\ X(T_n) \end{pmatrix}$$

Denoting by Q the matrix of this system, y — the vector of the desired coefficients, X — the vector is a column of values, in these designations, the system will take the form

$$Qy = X.$$

Multiplying the system by Q^T on the left, we get: $Q^T Q y = Q^T X$ and, already having a square non-degenerate matrix $Q^T Q$ of dimension $(2N \times 2N)$, there is a vector y.

So, the coefficients $\varepsilon_1, \delta_1, ..., \varepsilon_N, \delta_N$ are obtained. It is being introduced

$$\varepsilon(T_i) = \sum_{k=1}^{N} \left(\varepsilon_k \cos\left(\frac{2\pi\, k}{n} T_i\right) + \delta_k \sin\left(\frac{2\pi\, k}{n} T_i\right)\right), \quad i = \overline{1, n}.$$

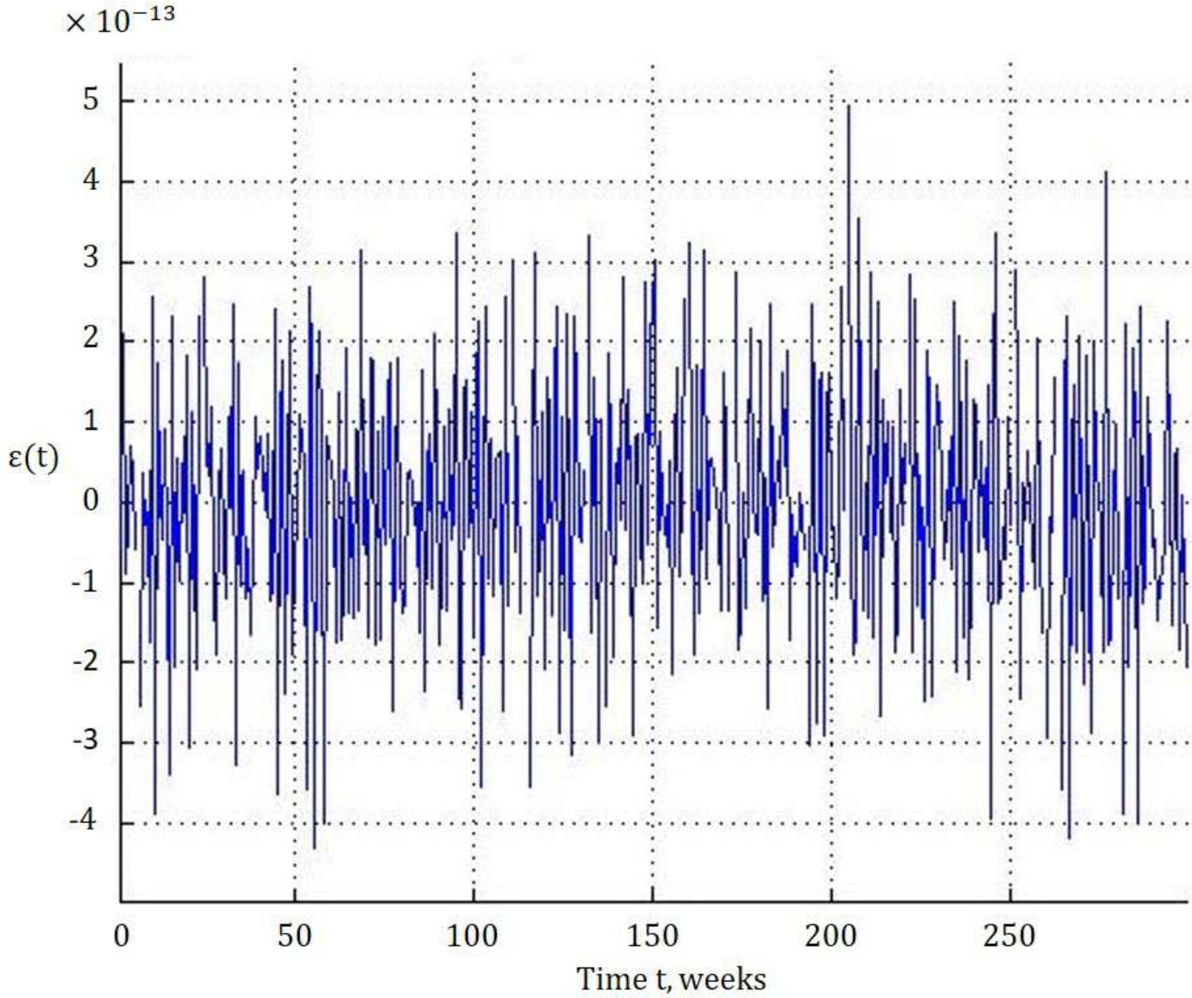

Figure 11 – Error $\varepsilon(T_i)$

It can be seen from the graph (Figure 11) that the introduction of new coefficients at $\boldsymbol{c} = N$ almost did not affect the approximation (the error error in calculating the coefficients does not exceed $5 * 10^{-13}$), which is logical, because $D(N) \approx 0$. However, as you know, time series are used for forecasting, where it is necessary to make a forecast based on a trend, rather than interpolation. This means that such good accuracy over a known interval may be unnecessary. Therefore, it is reasonable to choose $c < N$ in the forecasting task. The functions $\widetilde{X}_c(T_i)$ and $\widetilde{\varepsilon}_c(T_i)$ are considered:

$$\widetilde{X}_c(T_i) = R(T_i) - \sum_{k=1}^{c} \left(a_k \cos\left(\frac{2\pi\, k}{n} T_i\right) + b_k \sin\left(\frac{2\pi\, k}{n} T_i\right)\right) =$$

$$= R(T_i) - F_c(T_i), \quad i = \overline{1, n}.$$

$$\widetilde{\varepsilon}_c(T_i) = \sum_{k=1}^{c} \left(\varepsilon_k \cos\left(\frac{2\pi\, k}{n} T_i\right) + \delta_k \sin\left(\frac{2\pi\, k}{n} T_i\right)\right), \quad i = \overline{1, n}.$$

Then the function is introduced

$$E(c) = \sum_{i=1}^{n} \left(\widetilde{X}_c(T_i) - \widetilde{\varepsilon}_c(T_i)\right)^2.$$

From the graph (Figure 12) it can be seen that $E(c) < D(c), \quad c = \overline{1, N}$.

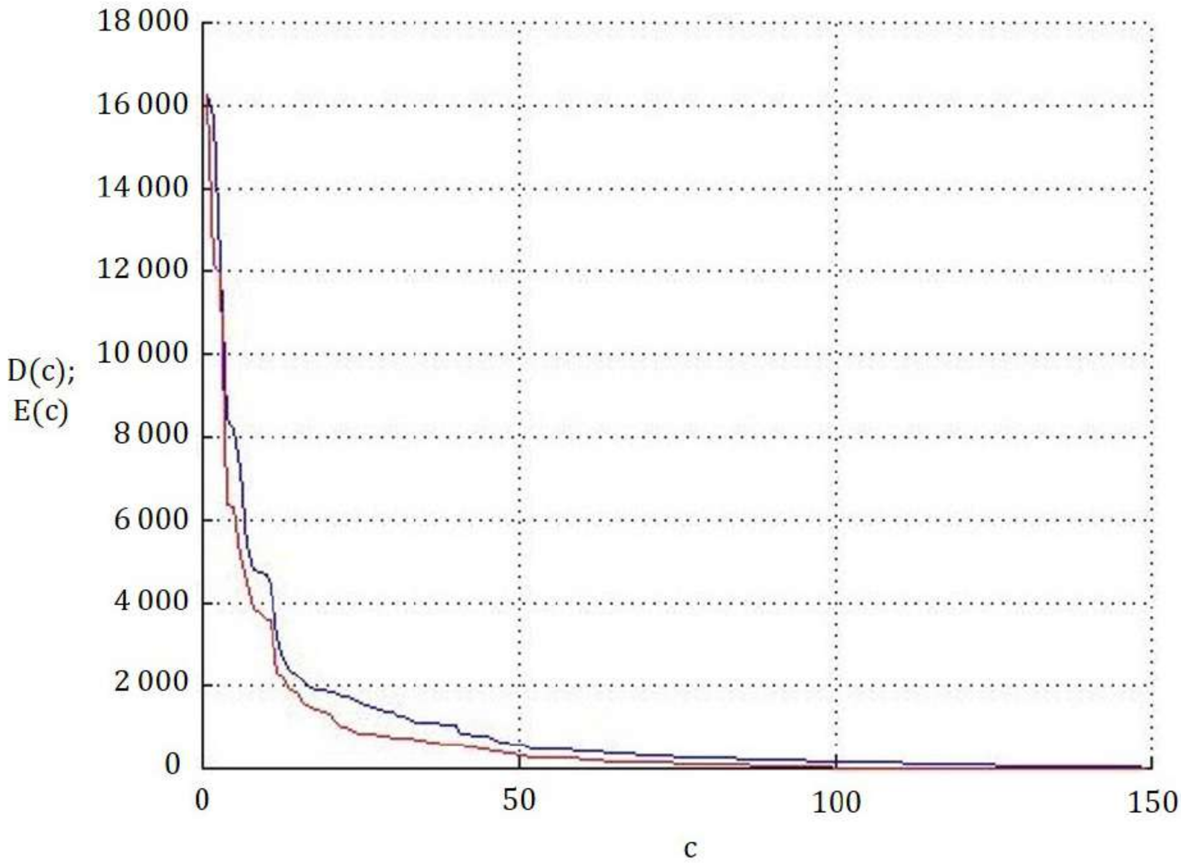

Figure 12 – Plots of $D(c)$ - (blue) and $E(c)$ - (red)

The arguments above proved Theorem 2.1:

Theorem 2.1 If the row is $A(T_i)$ is approximated not by the linear component $Y(t)$ and the Fourier series $Z(t)$, but by the linear component $Y(T_i)$ and only by a segment of the Fourier series expansion, if the coefficients a_k and b_k are calculated not by integrals, but by the areas of the corresponding rectangles, then for a more accurate approximation it is necessary to introduce the correction coefficients ε_k and δ_k and calculate the trigonometric component in the form

$$\sum_{k=1}^{N}\left((a_k+\varepsilon_k)\cos\left(\frac{2\pi\,k}{n}T_i\right)+(b_k+\delta_k)\sin\left(\frac{2\pi\,k}{n}T_i\right)\right),\quad i=\overline{1,n}.$$

2.3. Empirical analysis of the time series forecasting horizon

2.3.1. Introduction

The purpose of this paragraph is to estimate from above the forecasting horizon, i.e. the interval in the future for which the forecast is made for the past. The data for empirical analysis are taken from open sources on economics and meteorology [53, 55, 56]. Along the way, the task of finally determining the deterministic part of the trend of the time series will be solved, namely, determining the optimal number of harmonics c of the function $F_c(T_i)$. Redefine $F_c(T_i)$ in this paragraph as the sum of the linear component and the Fourier series with optimized coefficients

$$F_c(T_i)=Y(T_i)+\sum_{k=1}^{c}\left((a_k+\varepsilon_k)\cos\left(\frac{2\pi\,k}{n}T_i\right)+\right.$$
$$\left.+(b_k+\delta_k)\sin\left(\frac{2\pi\,k}{n}T_i\right)\right),\quad i=\overline{1,n},$$

Fourier series as a tool for approximating a function will converge fairly quickly to a function based on a sample. The purpose of the simulation is to construct an approximating function that tends to trend rather than sample. However, it is possible to build a band around the Fourier model, which will be an estimate of the trend location. To do this, it is proposed to construct such a part of the Fourier series, the residuals band relative to which will be no less than the band of the first differences of the original time series. This will be shown in more detail later.

However, $F_c(T_i)$ is only a deterministic component of the model, and we must not forget that there is a random component of $\omega(T_i)\neq 0$. Thus, at the moment, the $A(T_i)$ sample is approximated by the Fourier series, but not the trend. The task of sampling a stochastic process is to make a forecast for some interval ahead using empirical data.

The analysis of time series of various nature is carried out: data on prices for Brent crude oil, the ratio of euro to dollar exchange rates, daily temperature data in St. Petersburg. Some types of series may have a periodicity, others do not. Time series reflecting different processes in the economy and, more broadly, in nature, are characterized by varying degrees of conservatism: stationary stochastic processes have a time-independent distribution of random variables, this is the easiest case to predict. Processes with variable mathematical expectation are more difficult to predict, which requires trend modeling. In this section, modeling using Fourier series expansions is used, assuming that the random variables of the process have a limited domain of definition, and, in addition, that the trend is continuous and Lipschitz.

2.3.2. The algorithm for constructing the forecasting horizon

Let's consider one of the most inconvenient economic parameters for forecasting — oil prices «Brent», taken in the period from January 2009 to September 2014 with an interval of one week [53]. This time series contains economic characteristics of a complex nature: an economic component, which is a consequence of the equilibrium between oil supply and demand, as well as a speculative component, which is subject to a huge number of different disturbances. The consequence of this is that forecasting oil data is difficult.

Let's continue to consider the time series for «Brent» $A(T_i)$ crude oil. We make the assumption that for every two adjacent implementations of a random variable of the time series $A(T_i)$ the variances are finite and, moreover, not very different. Then, as suggested by Box J. and Jenkins G. [57], we construct a time series of the first differences $D(T_i)$, for example, on 2/3 of the data: $n_1 = 2/3n$; $D(T_i) = A(T_{i+1}) - A(T_i)$, $i = \overline{1, n_1 - 1}$.

Remark 2.3.1 If in the previous paragraph $D(c)$ meant the sum of the squares of the residuals, then in this paragraph we redefined $D(T_i)$ as a series of first differences.

In the figure 13 «asterisks» indicate the maximum and minimum values of the series. Half of the range of the series $D(T_i)$, i.e. max($D(T_i)$)-min($D(T_i)$) and will be the width of the band that is built around the trend. Therefore, the task of building a trend arises. When building $F(T_i)$ it is necessary to select the number of harmonics c.

Remark 2.3.2 By virtue of their construction, the nodes of the time series are $D(T_i)$ is an implementation of a random variable with a variance twice as large as that of the series $A(T_i)$. In order to use the span of the row $D(T_i)$ in the future, it is necessary to bring the variance to the same size as that of the series $A(T_i)$, therefore, half of the range of the series $D(T_i)$ is selected for the width of the band around the trend.

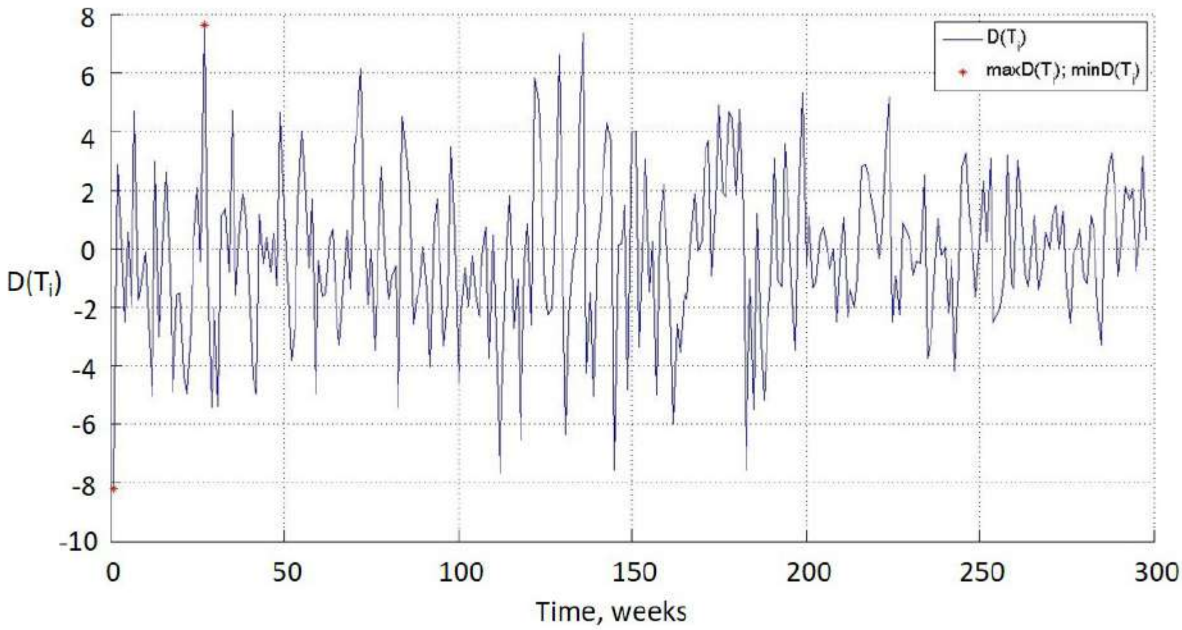

Figure 13 – $D(T_i)$ for «Brent» crude oil

Heuristic algorithm for constructing the horizon:

1. Building on n_1 data $F(T_i)$ one harmonic each
2. We consider a number of residues $X(T_i) = A(T_i) - F(T_i)$ on n_1 of data. From this series, we will need a scope.
3. We compare it with the span of the series $D(T_i)$: if half the range of the row is $D(T_i)$ less than the span of the series $X(T_i)$, then we increase the number of harmonics $\boldsymbol{c}$ to build $F(T_i)$ per unit and return to paragraph 1., otherwise we stop and build a model forecast.
4. Building a strip half the width of the range of the row $D(T_i)$ around the model data $F(T_i)$ both at the depth of forecasting and at the forecast part

of the series T_i.

5. We determine the output of the control data from the band surrounding the model data on the forecast part of the series T_i. This output will be the forecasting horizon.

Remark 2.3.3 Since there is no way to estimate the probability of leaving the band, the horizon can be considered a heuristic estimate, and the algorithm itself can be called a heuristic.

To illustrate the algorithm's operation, we will show an algorithm for constructing a forecast estimate using three examples.

2.3.3. Example 1. «Brent» Crude oil

As a result of using the algorithm, it is obtained that the smallest required number of harmonics is — 29. At the same time, the forecast horizon turned out to be equal to 1 (Figure 14). The linear component of the function $F(T_i)$ has the form: $Y(T_i)$=0.37 T_i + 53.15. Such a small horizon suggests that forecasting oil prices for long periods using this method will give the wrong result.

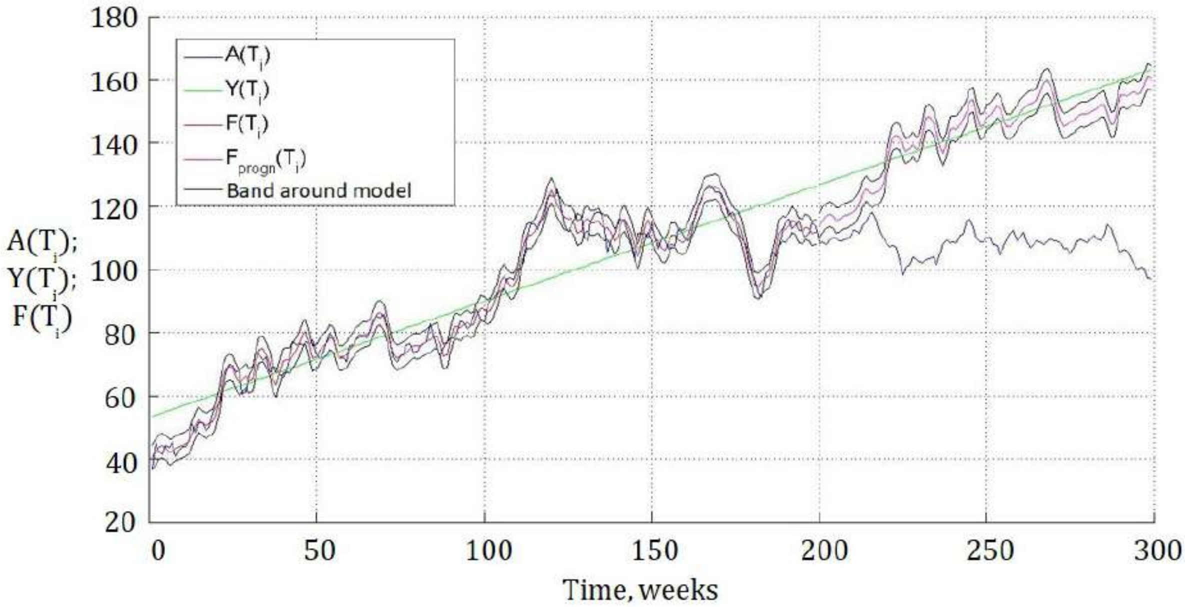

Figure 14 – $F(T_i)$, $Y(T_i)$ and $A(T_i)$ for «Brent» crude oil

2.3.4. Example 2. The ratio of the euro to the dollar

The second example is the time series of the daily relations of the euro/ dollar currency pair in the period from November 1, 2011 to December 31, 2013 [55]: $T_i - T_{i-1} = 1$, $i = \overline{2,n}$; $n = 543$; $A(T_i)$ — the ratio of the euro to the dollar at any given time. We choose a relatively quiet area on the stock market of the main world currencies. We determine the range of the series $D(T_i)$ at the identification interval (Figure 15). The algorithm showed that for $\boldsymbol{c}$=45, half the range of the series is $D(T_i)$ has become larger than the range of $X(T_i)$. As a result, we obtained a prediction horizon equal to 3 days (Figure 16). In this case, the linear component of the model is $F(T_i)$ has the form: $Y(T_i)$= - 0,000081 T_i + 1,31.

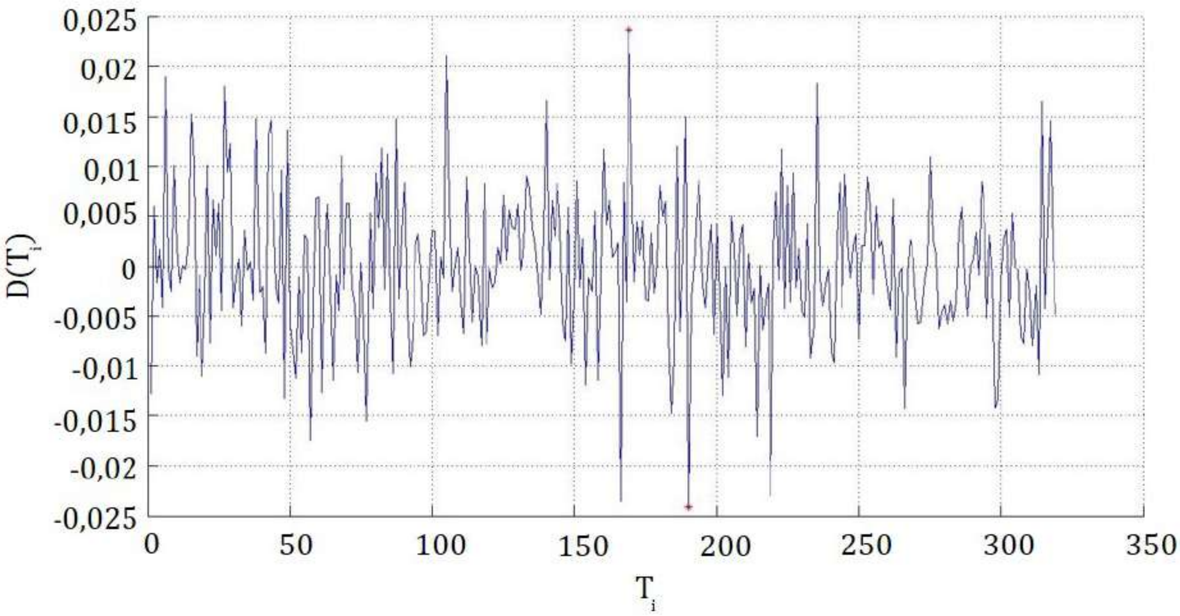

Figure 15 – $D(T_i)$ for Euro/dollar ratio

2.3.5. Example 3. Temperature in St. Petersburg

Let's give a number of daily temperatures in St. Petersburg in the period from January 2014 to September 2015 [56]: $T_i - T_{i-1} = 1$, $i = \overline{2,n}$; $n = 607$; $A(T_i)$ — temperature at any given time. The random component in these temperatures arises from climatic surges, which are caused by changes in the upper atmosphere, humidity and wind structure in the area being analyzed, and other natural factors. Taken together, these accidental impacts are much smaller

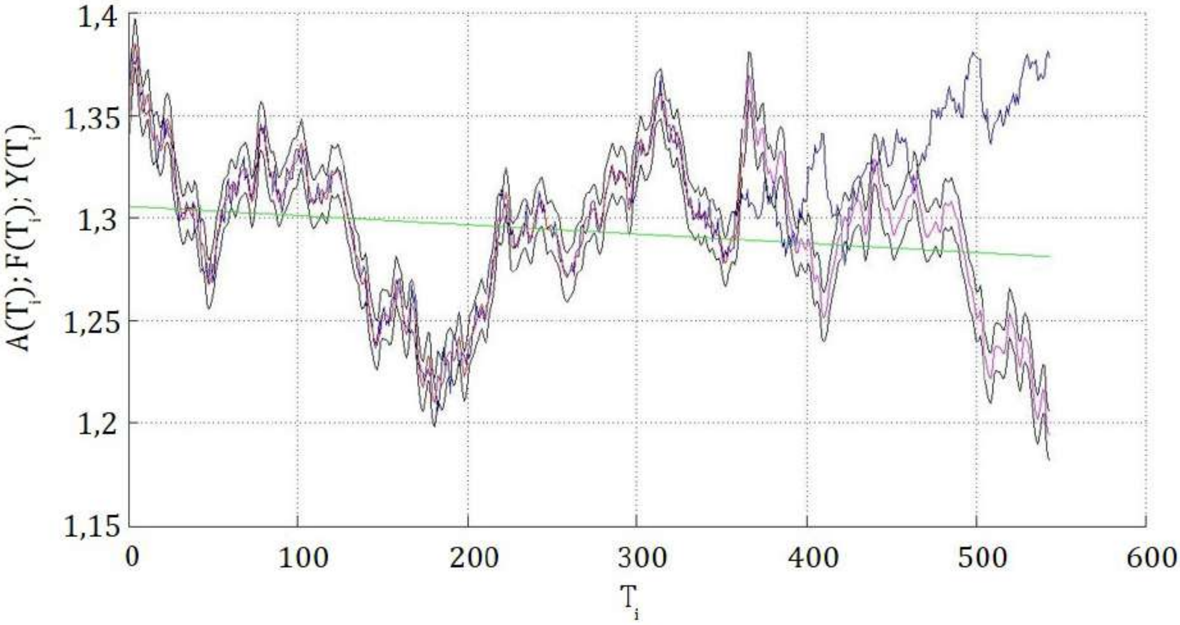

Figure 16 – $F(T_i)$, $Y(T_i)$ and $A(T_i)$ for Euro/dollar ratio

in their absolute magnitude than the influence of the Sun and the location of the Earth relative to the Sun. We determine the range of the series $D(T_i)$ for 2/3 of the data (figure 17), then proceed according to the algorithm.

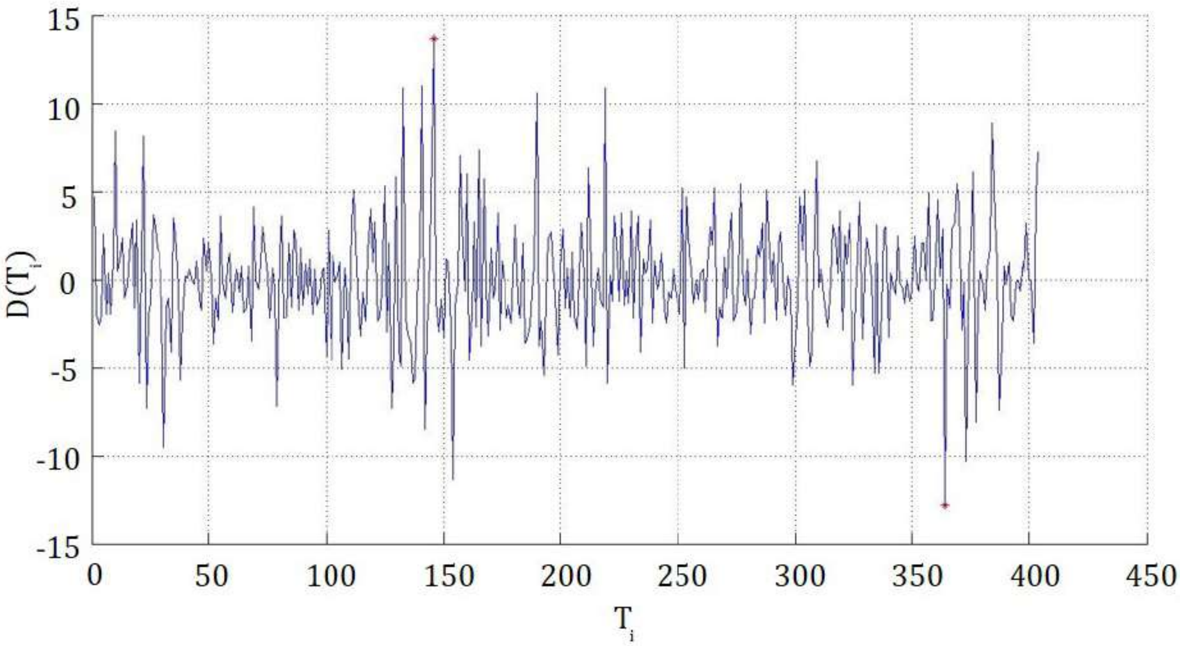

Figure 17 – $D(T_i)$ for temperature

As a result of the algorithm, it turns out that half of the range of the series is $D(T_i)$ at 50 harmonics $\boldsymbol{c}$ turned out to be larger than the range of the series $X(T_i)$. In this case, the horizon in this case is 6 days (Figure 18).

The linear component of the function $F(T_i)$ has the form: $Y(T_i) = -0{,}0023\ T_i + 7{,}85$. Such a result may mean that despite the fact that the series $F(T_i)$ approximates the sample, not the trend, in this case $F(T_i)$ turns out to be quite well approximated to a line made up of mathematical expectations, that is, the random component of the series is quite small, or it is well described by the series $F(T_i)$.

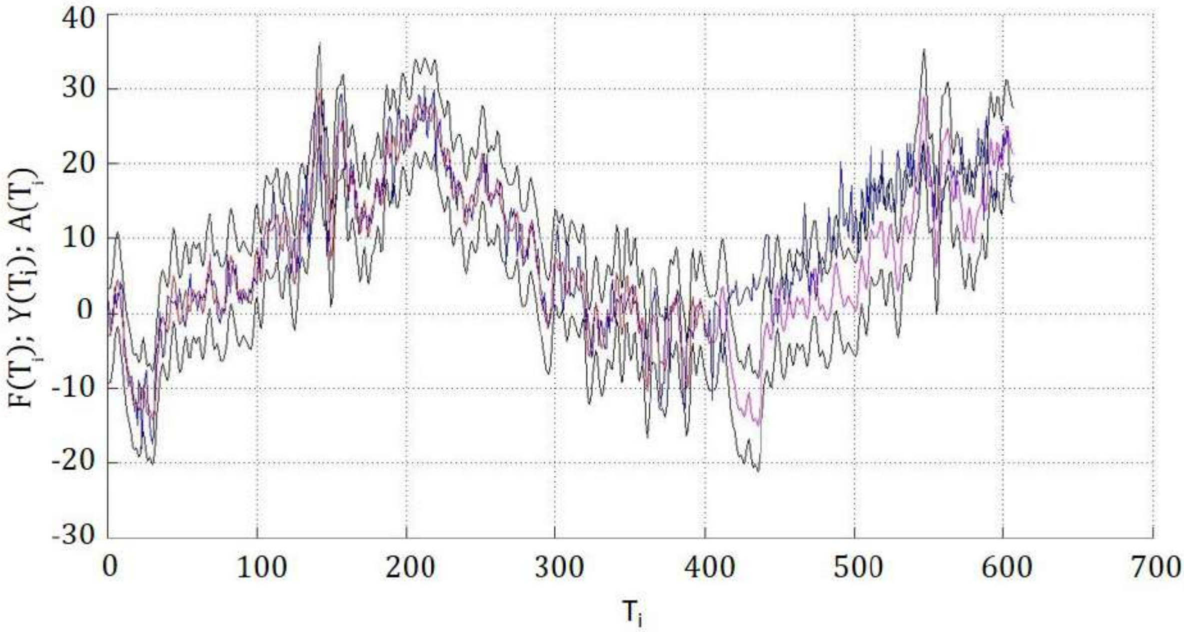

Figure 18 – $F(T_i)$, $Y(T_i)$ and $A(T_i)$ for temperature

Remark 2.3.4 With $\boldsymbol{c}$ equal to half the nodes of the time series $A(T_i)$, $F(T_i)$ best approximates the selection of $A(T_i)$. However, there is no need to take so many harmonics for two reasons:

1) It is necessary to approximate not the realization of random variables, but the trend;

2) At high harmonics, the band surrounding the model data will become too narrow, and then the real data will exit this band too early. In this case, the small size of the horizon will not reflect the essence of the ongoing process.

2.3.6. Algorithms for constructing the average forecasting horizon

According to the algorithm for finding the forecasting horizon, the question arises about the preservation of the horizon value if the identification interval is shifted or changed. In this section, two algorithms that solve this problem will be presented and their comparative analysis will be carried out.

The algorithm for constructing the average horizon by the shift method

1. From the time series $A(T_i)$ it is allocated, for example, the first third.
2. On a new section of the time series, the algorithm for constructing the horizon is implemented.
3. Shifting the new section to the right by a unit of time. (figure 19)
4. Until the right border of the new section has reached the end of the row $A(T_i)$, repeat steps 2-3.
5. We find the arithmetic mean of all the obtained horizons of the series and call this value the average horizon of the time series $A(T_i)$.

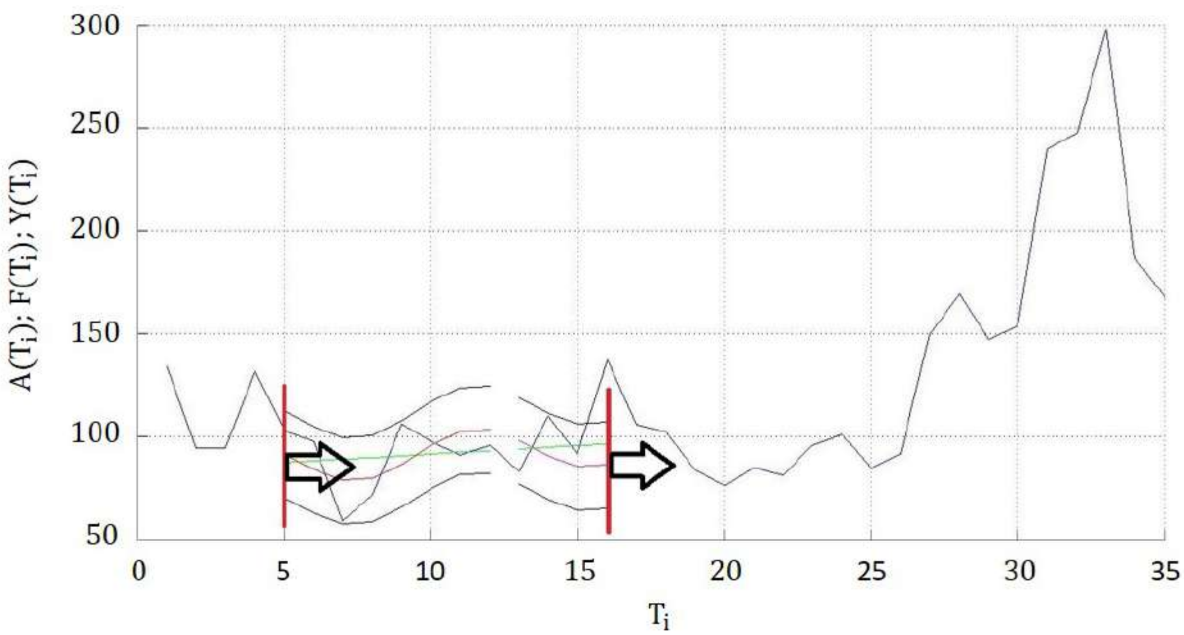

Figure 19 – The shift method

The algorithm for constructing the average horizon by stretching:

1. Considers some part of a time series, the beginning of which coincides with the beginning of the series $A(T_i)$.
2. On a new section of the time series, the algorithm for constructing the horizon is implemented.

3. We increase the volume of the new row by increasing the value of the right border (figure 20).
4. Repeat steps 2-3 until the new time series coincides with the original one.
5. The average horizon is found from all the received data.

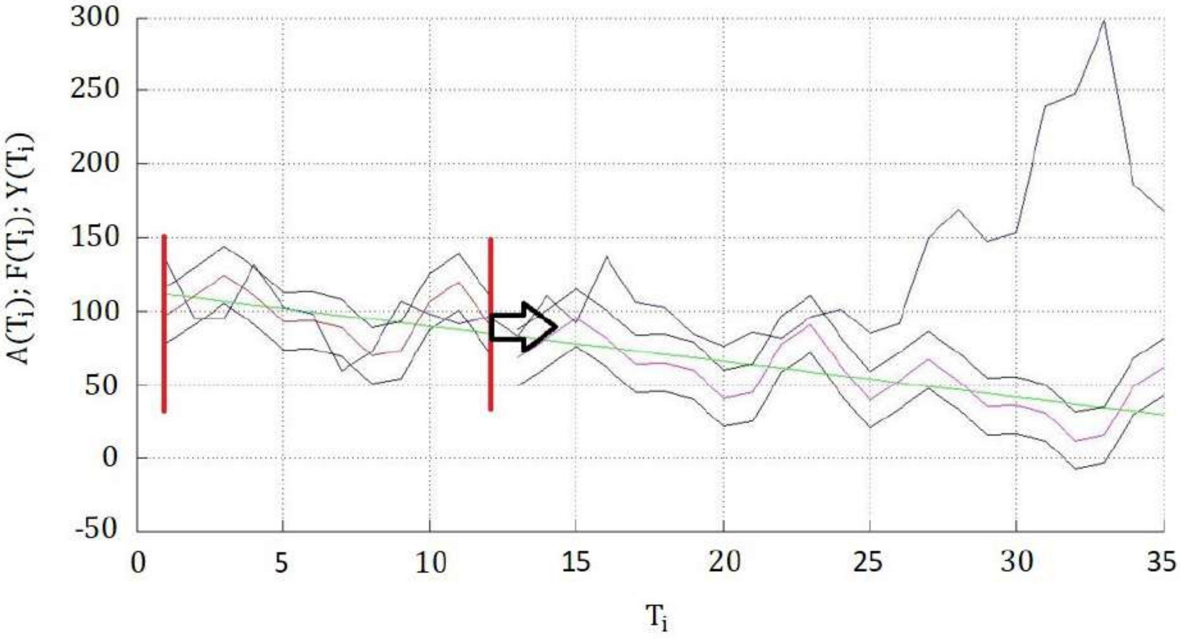

Figure 20 – Stretching method

A comparative analysis of the two presented algorithms was carried out using examples of world grain prices [58] by year, daily air temperature in Moscow [59] and weekly prices for «Brent» [60] crude oil. As a result, the average horizon was determined well enough for both methods, but the algorithm for constructing the average horizon by stretching has two significant drawbacks:

1. The operating time for large time series is hundreds of times longer than in the shift method (for example, the average horizon by stretching for temperature data in Moscow was considered to be about five hours, while the shift method took less than a minute on the same computer). This is due to the fact that with an increase in the identification interval, the trend building time increases, while in the shift method, the identification interval remains constant;
2. When the right boundary of the identification interval approaches the

boundary of the original time series $A(T_i)$ the interval of the forecast part decreases, that is, the forecast horizon may be smaller than it actually is.

Thus, we will consider the shift method as the algorithm for obtaining the average horizon. As a result of his work, the following results were obtained: for world grain prices, the average horizon is 2 years; for Brent crude oil prices, the result is 1.7 weeks; for the temperature in Moscow, the horizon is 2 days (in all examples, 1 unit of time corresponds to the period between each two adjacent measurements). Such results correspond to reality, which confirms the correctness of the algorithm.

2.4. Allocation the trend of a time series using the Chebyshev system

2.4.1. Introduction

In this section, we continue to formally represent the stochastic process as the sum of the deterministic part and the random part:

$$x_t = u_t + \varepsilon_t,$$

where x_t is the obvious value, u_t is the statistical, deterministic part of the observation, and ε_t is the random part of the finding (weak connection with zero expectation and constant variance). The t parameter in this paper attracts attention. It defines a probabilistic process. If, as provided, $E(\varepsilon_t) = 0$, then $E(x_t) = E(u_t) = u_t$. To apply the Gaussian-Markov theorem in the long term, it requires the use of nonresistance ε_t, i.e. $E(\varepsilon_t \varepsilon_\tau) = 0 \quad \forall\, t \neq \tau$.

The expression $x_t = u_t + \varepsilon_t$ does not exhaust all the variety of representations and is in some sense the simplest. For example, the case where the variance of $D(\varepsilon_t)$ depends on the value of u_t must be entered with a special assumption, for example, $\varepsilon_t = f(u_t)\delta_t$, where δ_t is Gaussian white noise. Many time series have such heteroscedasticity. Next, we will illustrate the operation of the algorithm, including observations of air temperature in cities around the world. Obviously, the random component of measurements has this property.

We need the form of representation of observations x_t only for simplicity of presentation.

Together with Prasolov A. V., the author obtained [4,6–8] the first results in solving the problem of estimating the forecasting horizon based on the allocation of the trend of a time series by decomposition into a segment of a Fourier series.

The trend model is built in two stages: a linear part is allocated to remove monotony, and then the remainder is approximated by a trigonometric polynomial of the best order.

Consider from paragraph 2.2. the time series $A(T_i), i = \overline{1,n}$, where T_i — are equidistant time measurements. Graphically, using the example of oil prices, we will show the allocation of the trend: $T_i - T_{i-1} = 1$ day, $i = \overline{2,n}$; $n = 299$; $A(T_i)$ — the price of oil «Brent» at any given time.

According to the same scheme, we get a series of $R(T_i)$ without a linear component (Figure 7).

$$R(T_i) = A(T_i) - Y(T_i),\ i = \overline{1,n}.$$

It is necessary to construct a trigonometric polynomial of the best order. Let's take trigonometric functions as Chebyshev's system:

$$\cos\left(\frac{2\pi 1}{T_n}T_i\right),\ \sin\left(\frac{2\pi 1}{T_n}T_i\right),\ldots,\cos\left(\frac{2\pi N}{T_n}T_i\right),\ \sin\left(\frac{2\pi N}{T_n}T_i\right).$$

We will assume that $n \geqslant 2N+1$. In this case, according to the general theory of [46], the trigonometric polynomial of the best approximation in the sense of the least squares method will be uniquely determined on the interval $[T_1, T_n]$

$$P(T_i) = \sum_{k=1}^{N}\left(a_k \cos\left(\frac{2\pi\, k}{T_n}\,T_i\right) + b_k \sin\left(\frac{2\pi\, k}{T_n}\,T_i\right)\right), \quad i = \overline{1,n},$$

The coefficients of the trigonometric polynomial are found by the Bessel formulas:

$$a_k = \frac{2}{T_n}\sum_{i=1}^{n} R(T_i)\cos\left(\frac{2\pi\, k}{T_n}T_i\right); b_k = \frac{2}{T_n}\sum_{i=1}^{n} R(T_i)\sin\left(\frac{2\pi\, k}{T_n}T_i\right).$$

In the figure 21 it can be seen that $(R(T_i) - P(T_i))^2 < 2*10^{-27}, \quad i = \overline{1,n}$ (the figure could show $R(T_i)$ and $P(T_i)$, however, visually it would look like overlapping graphs on top of each other and would not differ in any way from the figure 7). Such a small difference can be attributed to the computational error of the computer, and assume that $R(T_i) = P(T_i), \quad i = \overline{1,n}$. Therefore, the coefficients of the trigonometric polynomial could be found in another way:

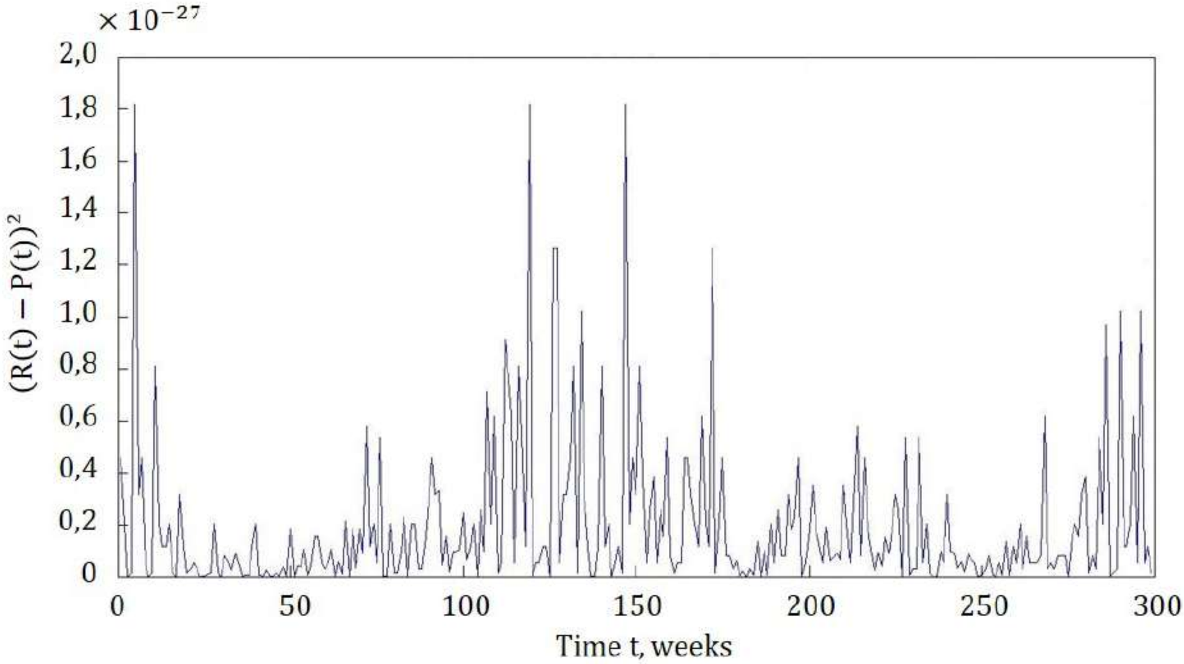

Figure 21 – The square of the deviation of the model data from the real ones

$$\sum_{k=1}^{N}\left(a_k \cos\left(\frac{2\pi\, k}{n}\, T_i\right) + b_k \sin\left(\frac{2\pi\, k}{n}\, T_i\right)\right) = R(T_i), \quad i = \overline{1,n},$$

Then, substituting all known values, we obtain a system of linear equations with $(n \times 2N)$-matrix, which we solve using the least squares method.

$$\begin{pmatrix} cos\left(\frac{2\pi 1}{n}T_1\right) & sin\left(\frac{2\pi 1}{n}T_1\right) & \cdots & cos\left(\frac{2\pi N}{n}T_1\right) & sin\left(\frac{2\pi N}{n}T_1\right) \\ cos\left(\frac{2\pi 1}{n}T_2\right) & sin\left(\frac{2\pi 1}{n}T_2\right) & \cdots & cos\left(\frac{2\pi N}{n}T_2\right) & sin\left(\frac{2\pi N}{n}T_2\right) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ cos\left(\frac{2\pi 1}{n}T_n\right) & sin\left(\frac{2\pi 1}{n}T_n\right) & \cdots & cos\left(\frac{2\pi N}{n}T_n\right) & sin\left(\frac{2\pi N}{n}T_n\right) \end{pmatrix} * \begin{pmatrix} a_1 \\ b_1 \\ \vdots \\ a_N \\ b_N \end{pmatrix} = \begin{pmatrix} R(T_1) \\ R(T_2) \\ \vdots \\ R(T_n) \end{pmatrix}$$

Denote by Q the matrix of this system, y — the vector of the desired coefficients, R — the vector column of values. In these designations, the system will take the form

$$Qy = R$$

Multiply the system by Q^T on the left: $Q^T Q y = Q^T R$ and, already having a square non-degenerate matrix $Q^T Q$ of dimension $(2N \times 2N)$, we find the vector y.

So, the coefficients $a_1, b_1, \ldots a_N, b_N$ and the series $P(T_i), \quad i = \overline{1,n}$ are obtained.

To build a trend model of the time series $A(T_i)$, is needed to find

$$F(T_i) = Y(T_i) + P(T_i), \quad i = \overline{1,n}.$$

2.4.2. Building algorithms for estimating the horizon

By the forecast of a time series we will understand the trend model of a random process based on a time series and the band of possible deviations around it.

To introduce a scatter band around the trend model, let's use the assumption that the trend satisfies the Lipschitz condition, i.e. $\mid F(t+\Delta t) - F(t) \mid < L\Delta t$, where L is some positive constant.

Let's continue to consider the time series $A(T_i)$. We make the assumption that for every two adjacent implementations of a random variable of the time series $A(T_i)$ the variances are finite and moreover not very different, that is, the trend is changing «smoothly». Then, as suggested by Box J. and Jenkins G. [57], we construct a time series of the first differences of $D(T_i)$:

$$D(T_i) = A(T_{i+1}) - A(T_i), \quad i = \overline{1,n}.$$

Remark 2.4.1 By virtue of their construction, the nodes of the time series are $D(T_i)$ is an implementation of a random variable with a variance twice as large as that of the series $A(T_i)$. In order to use the range of the row $D(T_i)$ in the future, it is necessary to bring the variance to the same size as that of the series $A(T_i)$, therefore, half of the range of the series $D(T_i)$ is selected for the width of the band around the trend.

The heuristic algorithm for finding the horizon of forecasting a time series, i.e. the interval in the future for which a forecast is made based on the past, as well as the algorithm for constructing the average horizon (using the shift method) fully correspond to the algorithms from paragraph 2.3.

2.4.3. Examples of practical application of the algorithm

Example 1. Temperature in Tokyo For an example of the algorithm's operation, consider a series of daily temperature data averaged over 24 hours in Tokyo for the period from 1.01.2016 to 06.02.2018 (total 767 days) [61]. Using a sliding window, we will select 154 days, of which 123 will be taken for identification and 31 days for forecasting. Then, averaging 615 values of the horizon and the band width, we get that the average horizon is 3.03 days, and the average band width is 12.34 degrees Fahrenheit. Figure 22 shows data for 154 days starting from April 11, 2016.

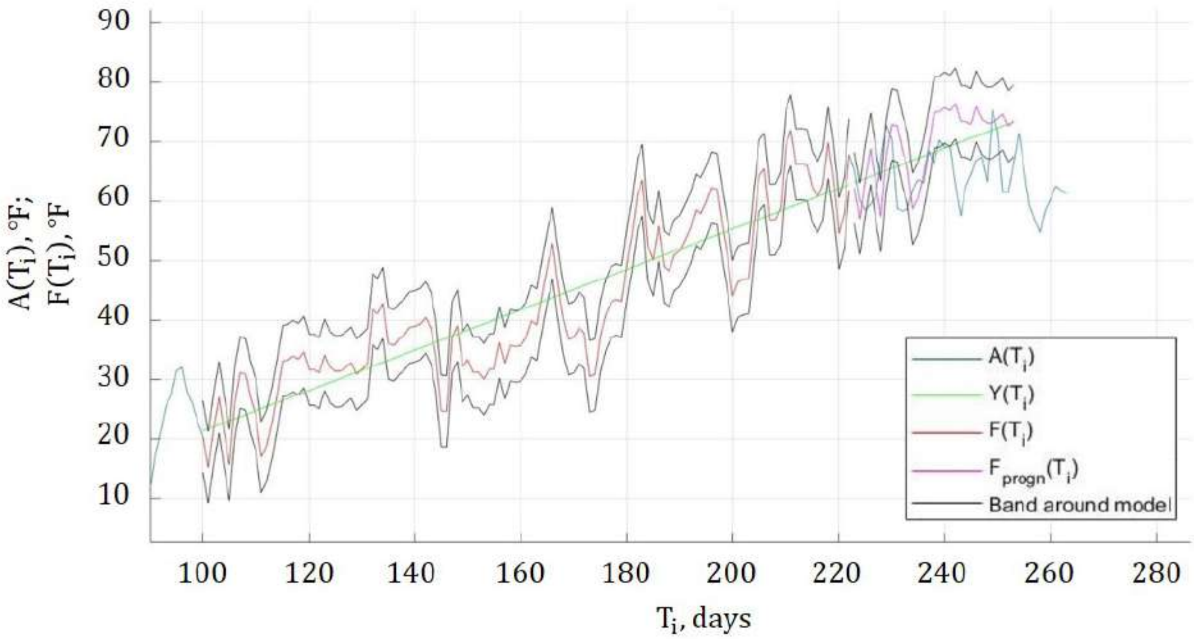

Figure 22 – The temperature in Tokyo. Horizon = 4

Example 2. Wheat prices Let's examine the time series of monthly daily average wheat prices from January 2005 to May 2017 [58]. For a sliding window, let's take 100 months, of which 80 are identification and 20 are forecast. As a result, we get an average horizon of 2.96 months, an average band width of 57.98 \$/ton. Figure 23 shows data for 100 months starting from October 2006.

Example 3. Euro to Dollar exchange rate We are testing the algorithm on one of the most volatile time series – the weekly (Friday values are taken) US dollar/euro exchange rate, taken from January 2, 2016 to February

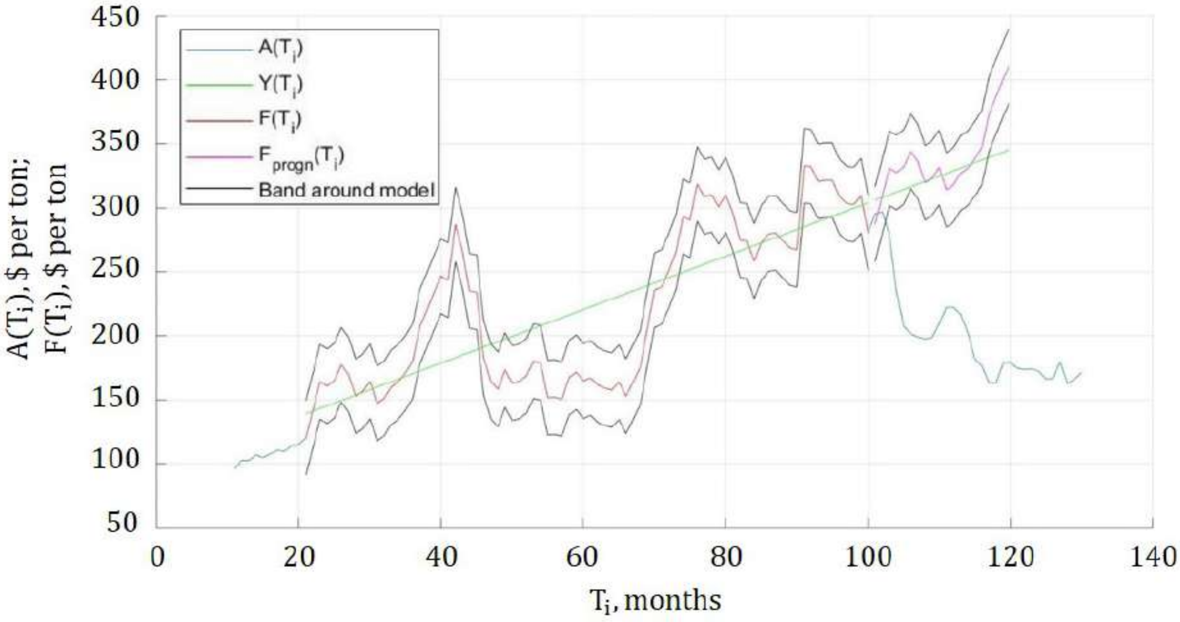

Figure 23 – The price of wheat. Horizon = 3

7, 2018 [55]. The sliding window will be 100 weeks, of which 80 are identification and 20 are forecast. We will get an average horizon of 2.29 weeks, an average width of 0.017 dollars per 1 euro. Figure 24 shows data for 100 days starting from the 16th business day of the stock exchange in 2005.

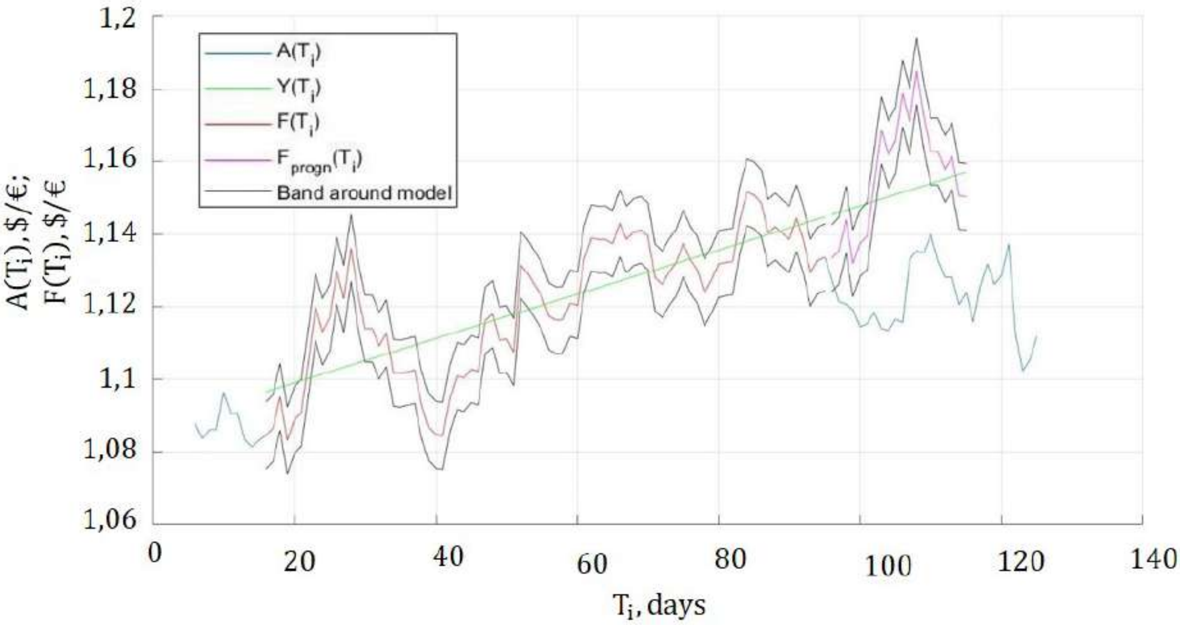

Figure 24 – The ratio of the Euro to the Dollar. Horizon = 2

Example 4. Brent crude oil Here are the results for the average daily price quotations for 1 barrel of Brent crude oil in US dollars from January 1, 2005 to February 5, 2018 (a total of 683 weeks) [60]. For the input parameters of the algorithm, we select 100 days – the value of the sliding window, 80 days – the identification part. As a result of 584 averaging, we get an average horizon of 1.27 days and an average band width of 7.41 $ per Barrel (Figure 25). This result confirms the speculative nature of this economic series.

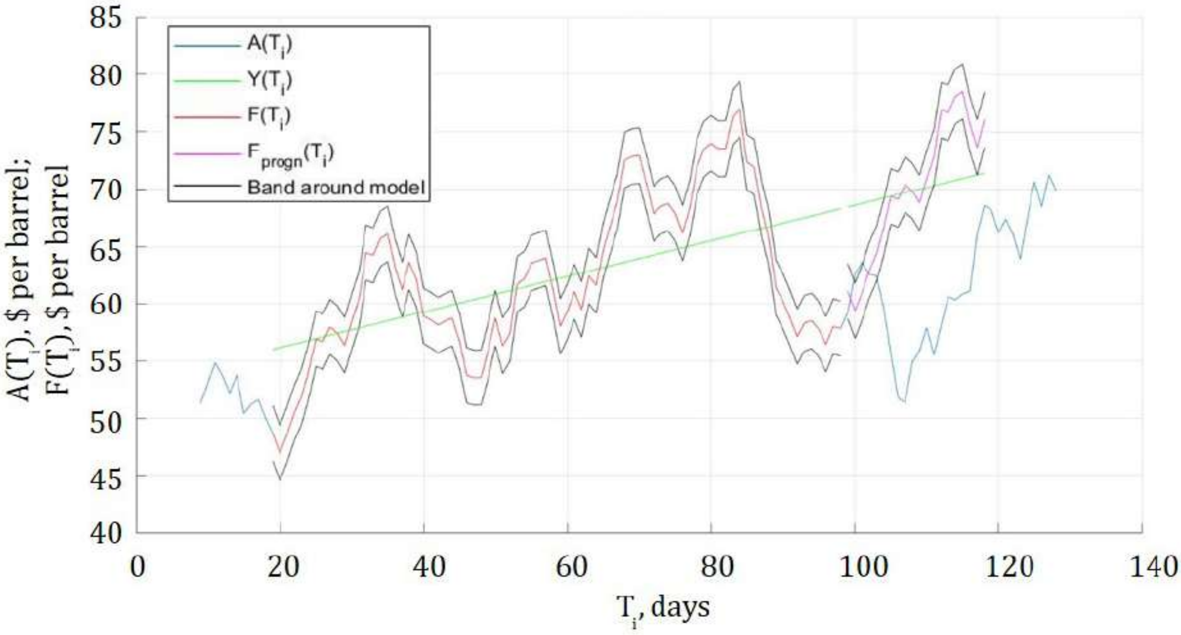

Figure 25 – Brent crude oil. Horizon = 2

Example 5. Weather in Moscow and comparison with weather forecasters Let's consider a number of daily temperature data averaged over 24 hours in Moscow for the period from 1.01.1995 to 5.03.2017 (total 8101 days) [62]. Using a sliding «window», we will choose 60 days. Then, averaging 8042 values of the horizon and the band width, we get that the average horizon is 3.54 days, and the average band width is 8.23 degrees Celsius.

It can be noted that the forecast horizon in Moscow is 0.51 days longer than in Tokyo from the first example. This is most likely due to the fact that Tokyo is located next to the ocean, from where the winds blow, changing the temperature and adding volatility to this time series.

Let's look at an example of forecasting from a weather site [63]. For seventeen days, weather forecasters' predictions for each of the thirteen future days were recorded. Then the deviation of the real data from the forecast data was considered. The result of deviation from real data depending on the forecast horizon is shown in Figure 26. It can be seen that from the first to the ninth day, the average deviation from the actual data does not exceed 4.1, which corresponds to the obtained model results. At the same time, the horizon is nine (after that, the average deviation increases sharply). It is quite large, because only seventeen days have been studied, among which there were no large temperature fluctuations.

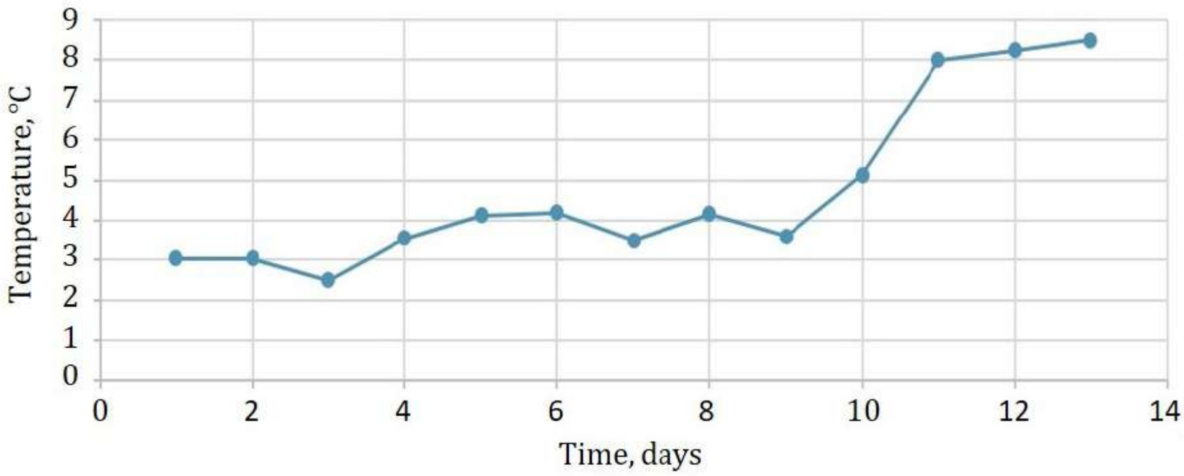

Figure 26 – Temperature in Moscow from March 1, 2017 to March 17, 2017, data from meteorologists

Example 6. The number of sales of peat soils Each of the previous examples shows the outfits of what nature an algorithm for estimating the forecasting horizon can be used. This paragraph shows the result of the algorithm based on real data on the number of sales of peat fertilizer from one commercial manufacturing enterprise. It is this point that shows the connection with Chapter 1. The task was set: based on monthly data from January 2013 to May 2017, to determine how far sales can be planned (Figure 27). As a result of the algorithm, where a period of 12 months was taken for identification, and for the forecast period of 6 months, an average horizon of 1.66 months and an average bandwidth of 20243 pieces were obtained. Such results corresponded to the real ones formed over 20 years of work. Thus, the efficiency of the algorithm in a real business task is shown.

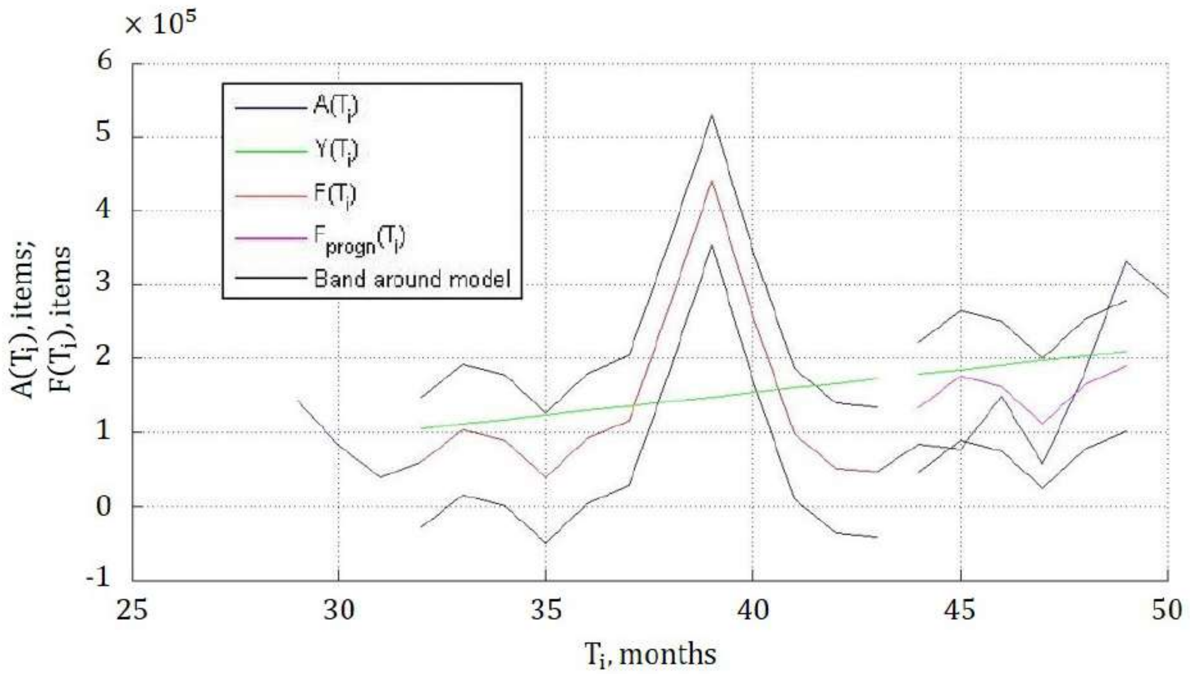

Figure 27 – Sales of peat soils. Horizon = 2

2.5. Analysis of algorithm modifications

2.5.1. Changing the distance between two measurements

In some previous examples, the horizon turned out to be small. Often, decision makers want to understand the situation much further. The question arises: is it possible to increase the forecasting horizon? Firstly, the horizon obtained by the algorithm is quite conservative, because it shows a 100% probability of hitting the band around the model. That is, it is possible to predict further with some degree of risk, bearing in mind that the deviation from the model will be greater. Secondly, it is possible to apply an algorithm for series with an increased step by smoothing method. Thus, the algorithm is considered based on the above-mentioned temperature data in Tokyo, averaged over 3, 7, 10, 14 days. Figure 28 and Figure 29 show the input data and the results of the algorithm. It is important that the number of measurements should be sufficient, otherwise you can get an incorrect horizon. Thus, the possibility of using measurement averaging to increase the forecasting horizon is shown.

Remark 2.5.1 The data smoothing method may give an incorrect result if you do not understand the nature of the series. During the smoothing process, the Slutsky-Yule [48] effect may occur, when seasonality can be lost if the amount of data for averaging coincides with the period. Also, during the smoothing process, the bursts go away, which are important from the point of view of their interpretation.

Start	End	Step	All steps	Moving window	Depth	Average horizon	Average bandwidth	Number of horizons for averaging	Average horizon, days
01.01.16	06.02.18	1 day	767	154	123	3,03	13,34	615	3,0
20.10.11	06.02.18	3 days (average)	767	154	123	1,57	20,07	615	4,7
27.05.03	06.02.18	7 days (average)	767	154	123	1,44	22,12	615	10,1
06.02.97	06.02.18	10 days (average)	767	154	123	1,26	22,31	615	12,6
07.01.97	06.02.18	14 days (average)	550	154	123	1,41	20,762	398	19,7

Figure 28 – Results for steps 1, 3, 7, 10, and 14 days

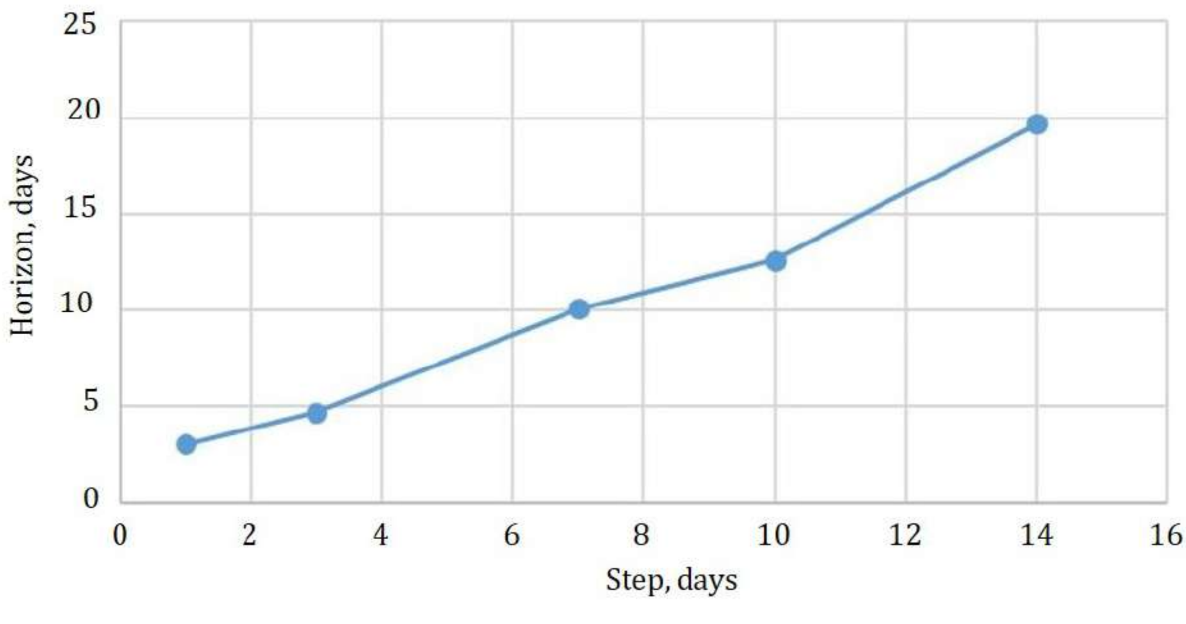

Figure 29 – Plot of the dependence of the average horizon on the step

2.5.2. Identification of periodicity

In the algorithm for estimating the forecasting horizon, the identification part is selected in an arbitrary way. One of the ways to find the identification interval is as follows:

1. We take the minimum allowed depth (by the nature of the data).
2. We are building an algorithm for estimating the forecast horizon, but we remember the deviation of the last depth value from the first forecast value. Using the sliding window method, we implement step 2 for all data and average the deviation
3. Increase the identification interval and repeat steps 2 and 3.
4. Plotting the dependence of the average deviations from the identification step

The essence of the algorithm is that if the depth coincides with the period, then the average deviation will be minimal.

Using the example of the temperature in Moscow (Figure 33), a period multiple of 1 year is clearly visible. However, for the euro/dollar exchange rate, no periodicity could be detected by this method: the average deviation increases monotonously (Figure 34). For both examples, the deviation is small for small identification intervals. This is because the first forecast value according to the horizon estimation algorithm copies the first identification value. Therefore, with a small identification interval, the first and last identification values are on average close to each other.

So, this algorithm has the right to exist, but it is not universal for determining the identification interval.

2.5.3. Revealing the depth of the forecast

Let's consider another example of searching for the depth of the forecast (identification interval).

1. We take the minimum acceptable identification interval (by the nature of the data).
2. We are building an algorithm for estimating the forecasting horizon, memorizing the horizon.

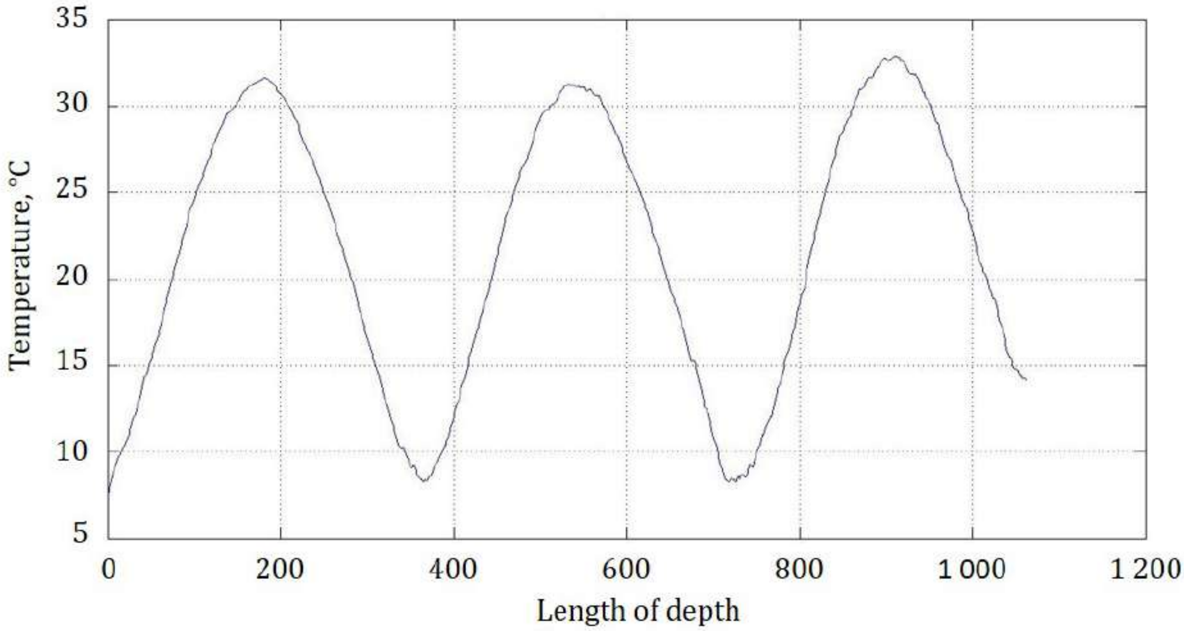

Figure 30 – The average deviation. The temperature in Moscow

3. We remember the module of the difference between the average values of the first and second halves of the identification interval. Using the sliding window method, we implement steps 2 and 3 for all data.

4. Averaging the values of horizons and module differences.

5. Increase the identification interval by 1 and repeat steps 2-5.

6. Select the minimum averaged difference modulus and the corresponding identification interval.

The idea is that the minimum modulus obtained as a result of the algorithm corresponds to the maximum horizon.

In practice, the algorithm was tested on daily temperature data in Tokyo (Figure 32). There was no relationship between the mean horizon and the mean deviation. This may have happened because the length of the identification interval did not reach the period, or as mentioned earlier, the calculation time increases exponentially as the sliding interval increases.

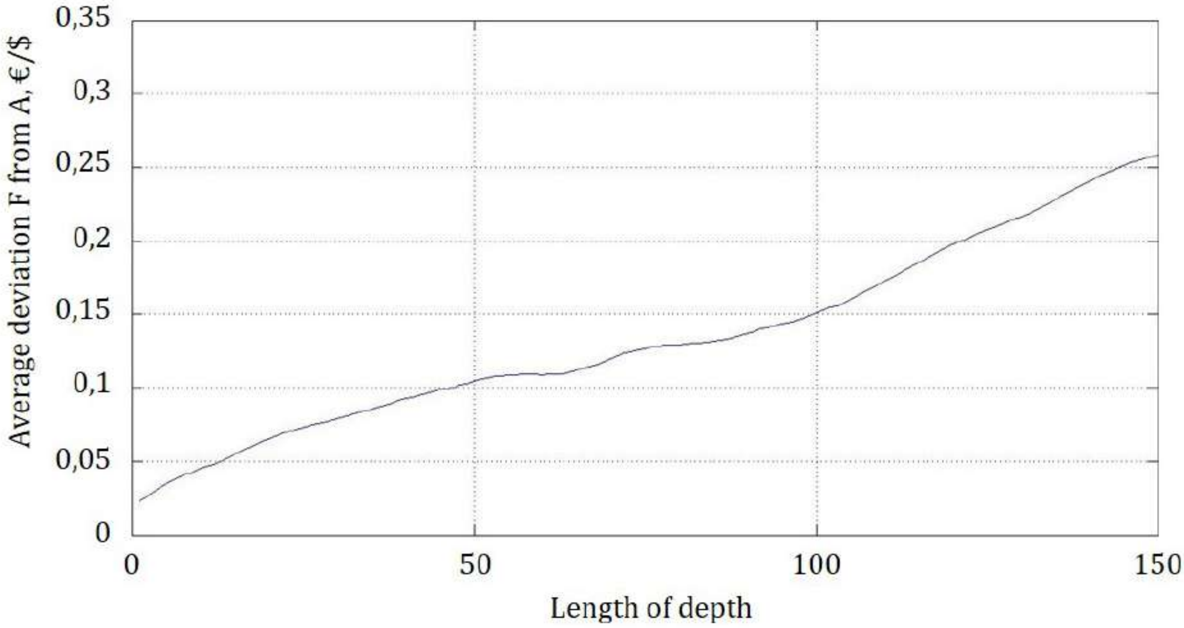

Figure 31 – The average deviation. Euro/Dollar ratio

2.6. A time series model as a piecewise stationary process

2.6.1. Introduction

The section is devoted to the construction of a space–time domain containing the trend of a time series as a line of mathematical expectations. The trajectory is an implementation of a stochastic process with discrete time. Therefore, any approximation of it is random and cannot claim to be an estimate of the trend. It is proposed to interpret an arbitrary time series as the trajectory of a piecewise stationary process. This will describe the algorithm for creating the area where the trend is located.

Consider the scalar time series $\{x_i\}_{i=\overline{1,n}}$. Since for each i the number x_i is an implementation of a random variable X_i with its own distribution, building a mathematical model means estimating the trend as a line of expected values $M(X_i)$. Ideally, to make a forecast, you need to know $M(X_i)$, $i = \overline{1,n}$. To complete the picture, it would be good to get the variance of $D(X_i)$. But this is not possible, because we only know the implementations of $\{x_i\}$. In practice, some approximation of the trajectory $\{x_i\}$ and a band around it are often built, inside which the trend is located with some probability. From our point of view, the latter statement, as a rule, is not proven. [4, 64]. The most studied and

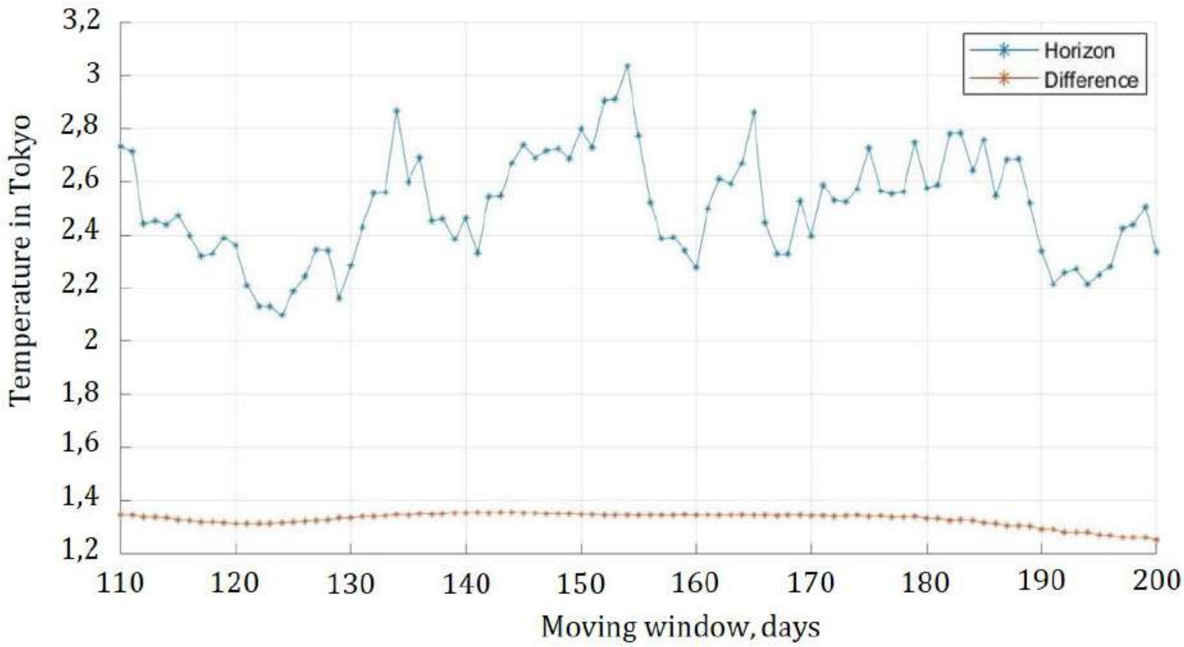

Figure 32 – The horizon and the difference in the deviations of the halves of the identification

widespread method of modeling is based on the assumption of stationarity of a stochastic process with discrete time, i.e. on the assumption of independence from the i probability distribution function of a random variable X_i. In this case, the formulas [65] are used: $X = X_i$;

$$M(X) \approx \frac{1}{n}\sum_{i=1}^{n} x_i = \overline{x};$$

$$D(X) \approx \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2 = \sigma^2$$

However, the assumption of stationarity is not realistic for most time series.

The second widely used method of constructing a model is the decomposition of observations into additive components: deterministic and random:

$$x_i = u_i + \varepsilon_i,$$

where ε_i perturbations require centering ($M(\varepsilon_i) = 0$), independence ($M(\varepsilon_i \varepsilon_j) = 0$) and homoscedasticity ($D(\varepsilon_i) = \sigma^2$). Then the trend of the time series (the line of expected values) becomes u_i:

$$M(X_i) = u_i.$$

If you specify a class of functions $U_i = u_i(b_1, \ldots, b_m)$ that depend on the parameters $\{b_1, \ldots, b_m\}$, then the task arises of searching in a given class U_i for such a representative (i.e. such a set of parameters) that σ^2 will be the minimum. The solution to this problem (the only one or not) it definitely exists. If the parameters $\{b_1, \ldots, b_m\}$ define u_i linearly, then we get the well-known linear regression problem. Obviously, the optimal parameters $\{b_1, \ldots, b_m\}$ are functions of random observed values $\{x_i\}$, i.e. random variables, and not deterministic, as we would like. In many sources [45,57,65–67] the class of functions is given based on some knowledge of the nature of the stochastic process. Usually, a monotonous component is distinguished (and called a trend), several types of seasonal fluctuations, the frequencies of which are specially selected, etc. The Holt-Winters [68] method works in approximately the same way.

This section is devoted to combining (in a sense) the two approaches described above. Time series modeling is carried out in two stages. At the first stage, the assumption of the stationarity of the stochastic process over a certain (short, but longer than 5 steps) time interval is considered. The sample mean and the sample mean-square deviation are calculated. The variance of the statistics is estimated. The following observation is added (the next element of the time series) and the hypothesis of belonging to its general population determined by observations at previous points in time is tested (more precisely, the general population whose sample was the previous time nodes). If the next implementation does not reject the hypothesis, then combine the sample and the last observation. Then we repeat the procedure until the new observation rejects the hypothesis of belonging to one general population. The fact of rejection of the hypothesis closes the procedure for lengthening the stationarity interval. For it, we calculate the sample average and estimate the spread for a given reliability guarantee.

The next stationarity interval is obtained by repeating the entire procedure until all observations with $i = \overline{1, n}$ are exhausted. As a result, we get a piecewise constant function and a zone around it, where the trend of the entire time series is necessarily located. Thus, the transition to a piecewise stationary stochastic process is carried out.

2.6.2. The algorithm for constructing the stationarity interval

Let's discuss in detail the algorithm for constructing the stationarity interval. Suppose that the previous interval ended with the number $i\,(i \in [1, n-5])$. Consider the observations $x_{i+1}, x_{i+2}, x_{i+3}, x_{i+4}, x_{i+5}$. Regarding these numbers, let's assume that they are the realization of a random variable X with expected value a and variance σ^2, i.e. on numbers $k = \overline{i+1, i+5}$, the time series behaves like a stationary one. The values of a and σ^2 are unknown, but assuming the distribution of X to be normal, we obtain unbiased and consistent estimates of the quantities a and σ^2 respectively [65]:

$$\overline{x} = \frac{1}{5}\sum_{k=i+1}^{i+5} x_k; \quad \hat{s}^2 = \frac{1}{5}\sum_{k=i+1}^{i+5} (x_k - \overline{x})^2$$

Note that for small sample sizes, $\overline{x}$ is not necessarily normally distributed, so we use the Student's distribution. Let's introduce statistics

$$t = \frac{\overline{x} - a}{s}\sqrt{n}.$$

According to the tables given in many sources, the critical value of $t_{\gamma,n-1}$ is determined, so that the probability that $|t| < t_{\gamma,n-1}$ is equal to γ

$$P\left(\left|\frac{\overline{x} - a}{s}\sqrt{n}\right| < t_{\gamma,n-1}\right) = \theta(t, n-1) = \gamma.$$

Note that if we limit ourselves to the reliability of γ=0.95 (95%), then we can approximate $t_{\gamma,k} \approx 2 + \frac{2.63}{k}$, where $k = 4, 5, 6, \ldots$ and thereby avoid numerous references to tables. (For reliability, γ= 0.9 (90%), the approximate formula has the form $t_{\gamma,k} \approx 1.64 + \frac{1.82}{k}$, where $k = 4, 5, 6, \ldots$. And, accordingly, for γ= 0.99 (99%), the approximate formula has the form $t_{\gamma,k} \approx 2,4 + \frac{8,15}{k}$, where $k = 4, 5, 6, \ldots$.).

The confidence interval for the general average a is found by the formula

$$\overline{x} - \Delta \le a \le \overline{x} + \Delta,$$

where $\Delta = \frac{t_{\gamma,n-1}s}{\sqrt{n}}$, or in the case of γ=0.95, we use the approximate formula

$$\Delta \approx (2 + \frac{2,63}{n-1}))\frac{\hat{s}}{\sqrt{n}}, \quad n = 5, 6, 7, \ldots$$

Next, let's consider another observation of the time series and check whether the assumption of the stationarity of the stochastic process persists. I.e., whether the next observation belongs to a sample of the general population, which was determined by previous observations? Thus, including all new observations in the consideration, we form the stationarity interval of the time series. The answer to the question posed above is given by well-known estimation algorithms, whether two samples belong to the same general population with a given reliability γ. In this paper, the increase in the stationarity interval stops as soon as an observation appears that «falls out» of the previously obtained sample, i.e. it is from another general population. In modern mathematical statistics, there are many algorithms for how to determine the «dropout» of [65]. One of them considers the new value as a second sample in the task of determining the uniformity of two samples with a given reliability. Let's describe it.

Suppose there is a sample of $x_1, x_2 \ldots, x_n$, from a normally distributed general population with unknowns a and σ^2. Let's evaluate them using $\overline{x}$ and $\hat{s}^2$. Let's add the following observation from the time series x^*. And we will consider it as a sample with volume 1. If we rely on the assumption that all distributions under consideration are normal, then even with small sample volumes, the statistics $t = \frac{\overline{x}-x^*}{s}\sqrt{n}$ has a t-Student distribution with $k = n - 1$ degrees of freedom. And, therefore, the hypothesis that x^* belongs to the same general population is rejected if $|t| \geq t_{\gamma,n-1}$. We believe that in this case the assumption of stationarity is rejected. Select the next set of observations (for example, from the next 5 members of the time series) and do the same procedure. As a result, the time series creates a piecewise constant function corresponding to the «piecewise» stationarity of the stochastic process. The band defined by the inequality $\overline{x} - \Delta \leq a \leq \overline{x} + \Delta$ around this function necessarily contains the trend of the stochastic process. And besides, the time series itself almost always lies inside this band with the reliability of γ. The intervals of stationarity and jumps (differences of neighboring values of averages) make up a random sequence.

2.6.3. Example

Let's consider the real weekly data on the price of Brent crude oil for 2015 (figure 33) [69].

	Week 1		Week 2		Week 3		Week 4		Week 5	
2015-01	01-02	$56,03	01-09	$49,47	01-16	$46,58	01-23	$46,44	01-30	$46,76
2015-02	02-06	$54,62	02-13	$56,57	02-20	$60,57	02-27	$60,63		
2015-03	03-06	$60,12	03-13	$56,51	03-20	$52,72	03-27	$55,01		
2015-04	04-03	$54,79	04-10	$56,51	04-17	$59,12	04-24	$61,41		
2015-05	05-01	$63,49	05-08	$65,01	05-15	$64,90	05-22	$64,31	05-29	$61,57
2015-06	06-05	$61,90	06-12	$63,24	06-19	$60,65	06-26	$60,84		
2015-07	07-03	$60,36	07-10	$56,63	07-17	$57,17	07-24	$55,75	07-31	$54,14
2015-08	08-07	$48,59	08-14	$47,94	08-21	$46,00	08-28	$43,53		
2015-09	09-04	$48,87	09-11	$47,60	09-18	$47,34	09-25	$47,33		
2015-10	10-02	$46,99	10-09	$51,33	10-16	$48,99	10-23	$46,81	10-30	$47,15
2015-11	11-06	$47,23	11-13	$44,75	11-20	$41,54	11-27	$43,65		
2015-12	12-04	$42,41	12-11	$38,76	12-18	$36,84	12-25	$35,90		

Figure 33 – Table of weekly brand price data «Brent»

It is known that these are bad data for forecasting: there are no periodic or monotonous components, pronounced heteroscedasticity, and so on.

The figure 34 shows the stationarity intervals obtained by the described algorithm. At each interval, a winding line depicts real oil prices, the horizontal line corresponds to a sample average, from which the limits of the confidence interval are postponed up and down (in this case with 99% reliability). The confidence band formed by confidence intervals along the entire length of the time series necessarily contains the trend (line of mathematical expectations) of the stochastic process. Note that if the confidence band were built around a time series, as the trajectory of a stochastic process, it would not guarantee the content of the trend.

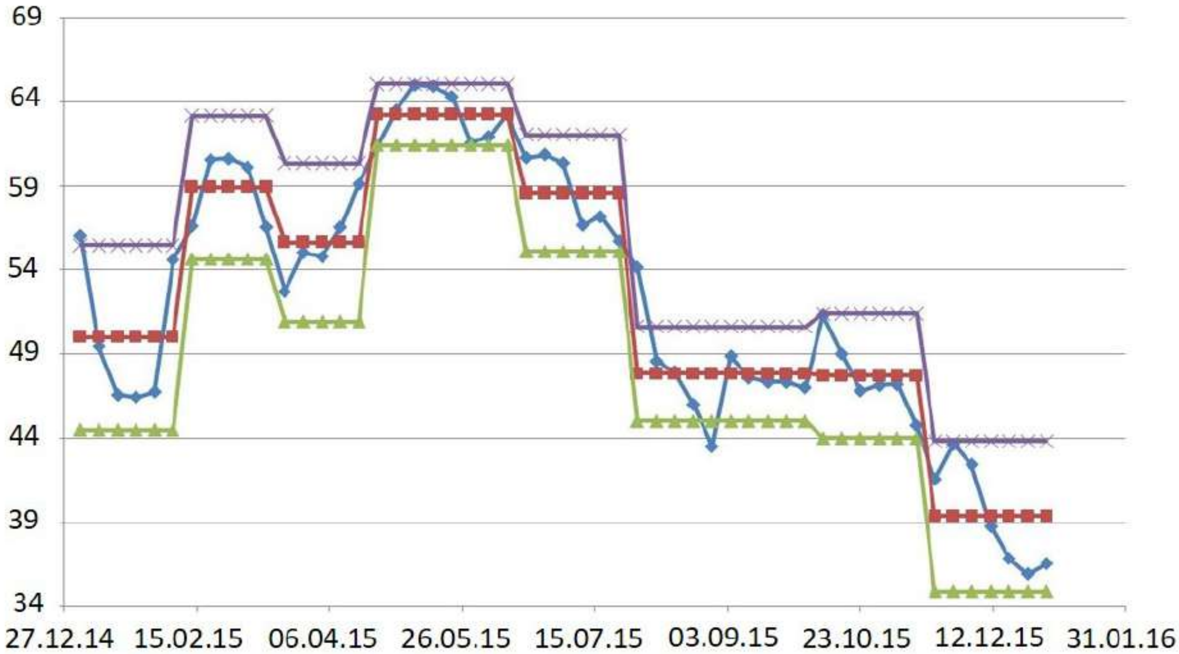

Figure 34 – Price «Brent» in USD per barrel

2.7. Estimation of the variance of a Weighted Least Squared (WLS) using a piecewise stationary process

2.7.1. Introduction

In this section, we will continue the research related to the obtained algorithm for separating a piecewise stationary process from a non-stationary one.

When analyzing time series in real life, researchers often face the fact that the series are non-stationary processes with heteroscedasticity. Recall that if the variance of the random error is not constant: $D(\varepsilon_i) = \sigma_i^2 \neq const$ and the errors are uncorrelated, then this situation is called heteroscedasticity.

To obtain effective (optimal) parameter estimates, you can use the so-called weighted least squares (WLS). To understand how it works, consider the case when the variance of random errors $D(\varepsilon_i) = \sigma_i^2$ is known [70]:

Let the model be considered

$$y_i = \beta_1 + \beta_2 x_i^{(2)} + \beta_3 x_i^{(3)} + \cdots + \beta_k x_i^{(k)} + \varepsilon_i,$$

for which all the prerequisites of the classical linear multiple regression model are fulfilled with one exception: the data shows heteroscedasticity $D(\varepsilon_i) = \sigma_i^2$.

In this case, you can divide the right and left sides of the regression equation by σ_i:

$$\frac{y_i}{\sigma_i} = \frac{\beta_1 + \beta_2 x_i^{(2)} + \beta_3 x_i^{(3)} + \cdots + \beta_k x_i^{(k)} + \varepsilon_i}{\sigma_i}.$$

After that, we'll do a simple variable replacement:

$$\widetilde{y_i} = \frac{y_i}{\sigma_i}; \widetilde{x_i}^{(1)} = \frac{1}{\sigma_i}; \widetilde{x_i}^{(2)} = \frac{x_i{}^{(2)}}{\sigma_i}; \ldots; \widetilde{x_i}^{(k)} = \frac{x_i{}^{(k)}}{\sigma_i}; \widetilde{\varepsilon_i} = \frac{\varepsilon_i}{\sigma_i}.$$

As a result of changing variables, we switch to a new model:

$$\widetilde{y_i} = \beta_1 \widetilde{x_i}^{(1)} + \beta_2 \widetilde{x_i}^{(2)} + \cdots + \beta_k \widetilde{x_i}^{(k)} + \widetilde{\varepsilon_i}.$$

The new model is useful because there is no heteroscedasticity in it, since the variance of the random error is a constant:

$$D(\widetilde{\varepsilon_i}) = D(\frac{\varepsilon_i}{\sigma_i}) = \frac{1}{\sigma_i^2} D(\varepsilon_i) = \frac{1}{\sigma_i^2}\sigma_i^2 = 1 = const.$$

Therefore, the LS (least squares) applied to the new model will give an effective result. Thus, the essence of a WLS is to make the correct substitution of variables so that the application of a conventional LS to a new model (with modified variables) leads to effective estimates of the coefficients. After that, you can return to the original variables to interpret the results.

Let's explain why the LS is called weighted. In the case of a weighted LS, we minimize with β_i the sum of the squares of the residuals of e_i^2 of the new model:

$$\sum_{i=1}^{n} \frac{1}{\sigma_i^2} e_i^2 \to \min_{\hat{\beta}}.$$

It turns out that we minimize the sum of the squares of the residuals, but each term is multiplied by the weighting factor $\frac{1}{\sigma_i^2}$, that is, we minimize the sum of the squares of the residuals with certain weights. The smaller the variance for the i-th observation (that is, the smaller the randomness factor for this observation), the more weight this observation has in the amount that we minimize. Thus, we attach the greatest weight to the most «reliable» observations. This makes it possible to improve the quality of the grades received.

Of course, in reality, the variance of the random error is usually not known to the researcher, which leads us to consider a more realistic case when we first need to estimate σ_i^2, and apply this estimate to a WLS.

Remark 2.7.1 A WLS is preferable to a standard one, because it makes it possible to obtain effective estimates of the coefficients, but only if the equation for the variance of the random error is correctly specified (that is, they correctly understood exactly how heteroscedasticity is arranged in the analyzed model). This fact can be seen in exercise 6.1. in the book of Magnus [45].

In the special case, when the variance of the random error is directly proportional to the square of the single variable b: $D(\varepsilon_i) = \sigma_i^2 = \sigma_i^2 b_i^2 > 0$, an algorithm for implementing WLS [45, 70] is given. If the variance of the random error is unknown, then some researchers suggest first evaluating the original model using LS and obtaining the residuals e_i, and then evaluating the auxiliary model for the residuals, where the regressors of the auxiliary model are the regressors of the original model as well as their squares (variables that affect the variance of the random error) [70]. However, the assumption that the residue model on all data is described by a single function does not agree well with the unsteadiness of processes in real data, when emissions may be different for different stages of the process.

2.7.2. Algorithm

Let's propose our own WLS algorithm with an estimate of σ_i^2 for the initial time series $X(T_i), i = \overline{1, n}$:

1. According to the nature of the data, we specify the class of functions $Y(T_i)$ to approximate the time series;
2. We apply to the series $X(T_i)$ the least squares method (LS), resulting in $y(T_i)$ - a representative of the class $Y(T_i)$;
3. Calculate the residuals $e(T_i) = X(T_i) - y(T_i)$;
4. For a number of residuals $e(T_i)$ we select a piecewise stationary process with the parameters: $M(T_i)$ is a time piecewise stationary series, $\Delta(T_i)$ is a

piecewise constant deviation for $M(T_i)$, which we will consider an estimate of σ_i;

5. We apply to the series $y(T_i)$ WLS with $\sigma_i = \Delta(T_i), i = \overline{1,n}$.
6. Getting a new model $\widetilde{y}(T_i)$. Doing the reverse substitution of variables: $z(T_i) = \widetilde{y}(T_i)\sigma_i, i = \overline{1,n}$. As a result, $z(T_i)$ is a model for approximating a series of $X(T_i)$ using a WLS.

2.7.3. Examples

In all the examples below, we will use the reliability level for a piecewise stationary process $\gamma = 0.95$. In the examples, the specification of the linear component and the seasonal one will be mainly carried out in the form of a pair of sine and cosine, and the period of the seasonal components is not equal to the length of the entire identification interval. If in the paragraphs where the Fourier series and Chebyshev systems were used there were problems with forecasting due to the period, then in this case the forecast will be a continuation of the function, and not a copy from the initial section.

Example 1. Consider a data series $X(T_i) = 0.25sin(T_i) + 0.75cos(T_i)$, $i = \overline{1,100}, T_i = \frac{i}{5}$. For approximation, we will look for a representative from the class of functions $Y(T_i) = \beta_1 + \beta_2 T_i + \beta_3 cos(T_i) + \beta_4 sin(T_i)$. Applying the weighted least squares algorithm, get a smaller sum of squared deviations for the WLS $(1,106 * 10^{-30})$, than for the standard $(1.2087 * 10^{-30})$ in the figure 35, and the cumulative sum of the differences is $e(T_i) - (X(T_i) - z(T_i))$ grows without strong jumps throughout $T_i = \frac{i}{5}$ (Figure 36).

Example 2. Now consider the straight line $X(T_i) = 2 + 0.3t, i = \overline{1,100}$, $T_i = \frac{i}{5}$. For approximation, we will look for a representative from the same class of functions $Y(T_i) = \beta_1 + \beta_2 T_i + \beta_3 cos(T_i) + \beta_4 sin(T_i)$. Figure 37 shows that even if the LS, due to the peculiarities of numerical calculations, mistakenly determined β_3 and β_4 to be nonzero, then the WLS will make them closer to zero, that is, it will significantly reduce the sum of the squares of the residuals relative to the standard LS.

Example 3. The following example shows what happens if the model

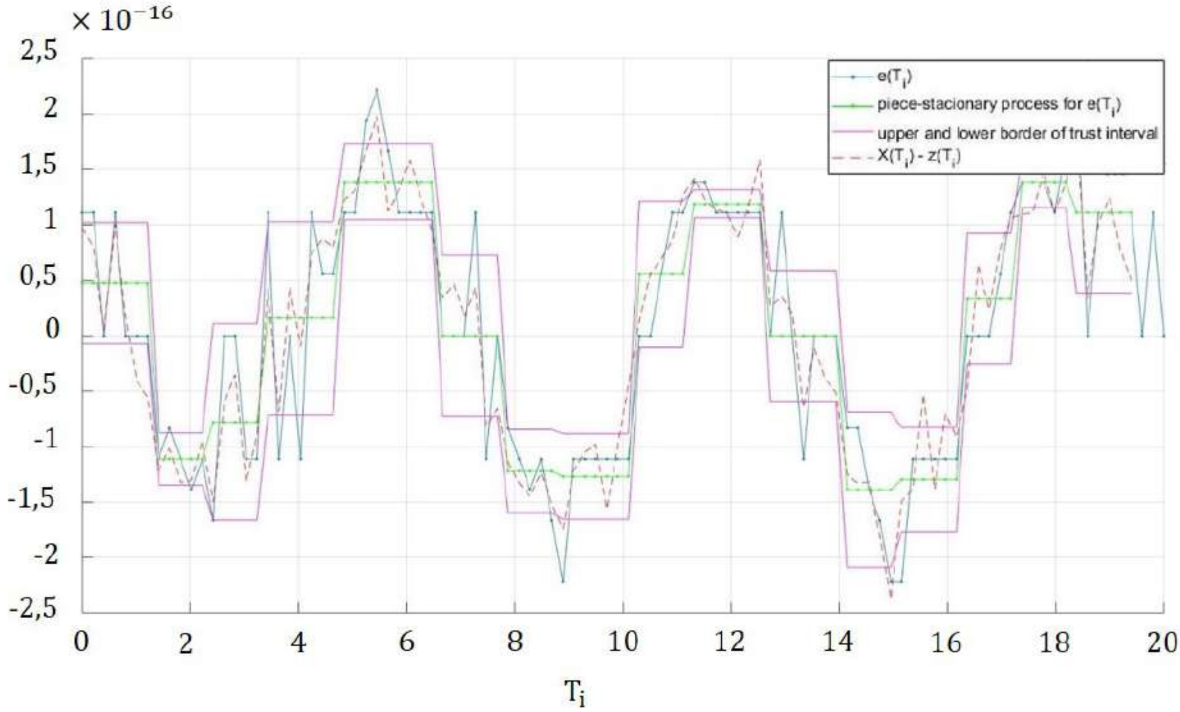

Figure 35 – Residuals of two models

specification is unsuccessful. For a series $X(T_i) = 5sin(0.3t^2), i = \overline{1,100}, T_i = \frac{i}{5}$ we will carry out an approximation from the class of functions $Y(T_i) = \beta_1 + \beta_2 T_i + \beta_3 cos(T_i) + \beta_4 sin(T_i)$.

Figure 38 shows that even a close approximation will not work due to an incorrect specification. However, in the figure 39 it can be seen that for the first T_i the approximation is close enough to the original series, then for both LS and WLS the deviations become larger. As a result, the WLS has a larger sum of squares of residues than the standard LS.

Example 4. Finally, let's give an example of real data. These are the data already given in the previous paragraphs on daily temperature measurements in Moscow [62]. To approximate this time series $X(T_i)$, $i = \overline{1,500}, T_i = i$ we will use a slightly different class of functions $Y(T_i) = \beta_1 + \beta_2 T_i + \beta_3 cos(\beta_4 T_i) + \beta_5 sin(\beta_6 T_i)$. This function is sufficient to approximate the annual seasonality. To do this, the amplitudes β_4 and β_6 are introduced. However, the standard MNCs can only define $\beta_1, \beta_2, \beta_3, and \beta_5$. To determine $\beta 1, ..., \beta 6$, the numerical nonlinear Levenberg-Marquardt method [71–73] is used to solve the least squares problem. The principle of the method is the descent method: initial coefficients

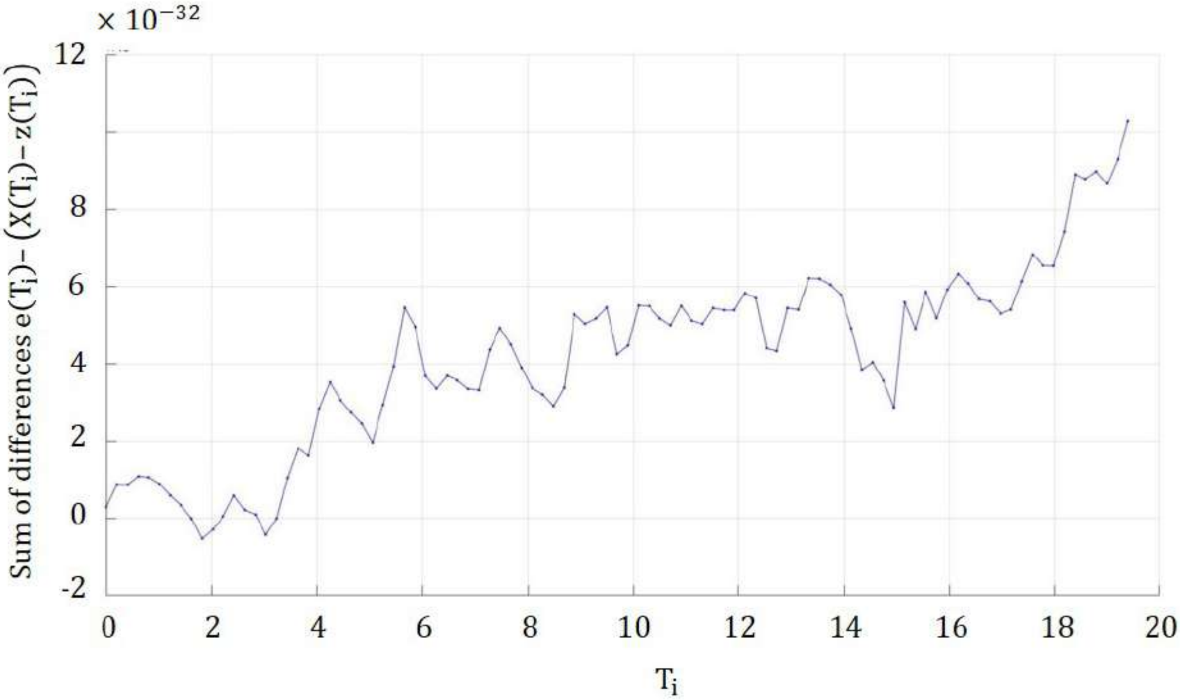

Figure 36 – Cumulative sum of residual differences between two models

are set, and the algorithm approximates the initial series until the stop condition is met or the maximum specified number of iterations is reached. As a result (Figure 40) it can be seen that the model specification is not accurate enough, the remnants of $e(T_i)The$ is quite large, so the WLS gave almost the same result as the standard LS.

2.7.4. Conclusions on the paragraph

Weighted LS gives an effective result by removing heteroscedasticity. However, in real life, σ needs to be evaluated. This method shows that for some series, the method really helps to make a more accurate approximation with proper specification of the model. At the same time, with a poor specification of the model and due to the peculiarities of the algorithm for constructing a piecewise stationary process, a WLS can give an even worse result than the standard one. Therefore, it is necessary to understand the nature of the series and apply this method wisely.

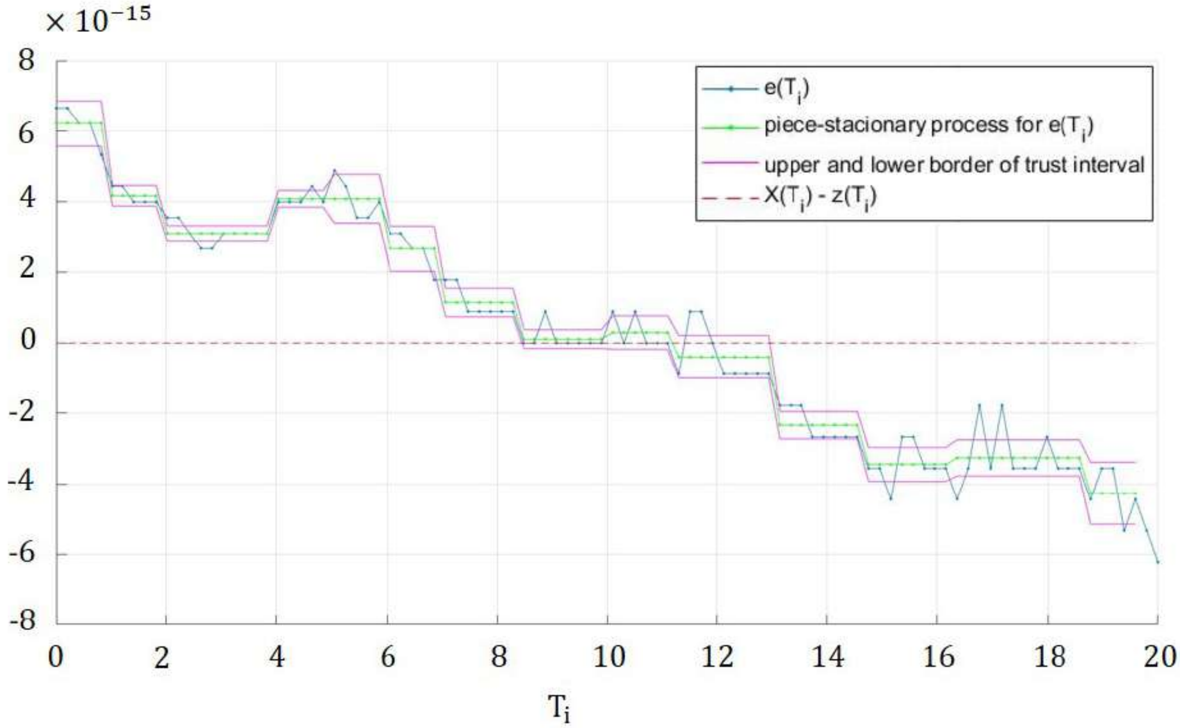

Figure 37 – Residuals of two models

2.8. Chapter conclusions

In the second chapter, models for approximating the trend of a time series using Fourier series and Chebyshev series are considered. An algorithm for estimating the forecast horizon from above has been created, then an algorithm for estimating the average forecast horizon is presented to increase statistical significance.

Modifications of the algorithm are considered to help identify the periodicity and length of the identification interval, artificial series for changing the distances between two time series measurements are considered.

Using the time series model as a piecewise stationary process, it was possible to identify stationary intervals in a non-stationary process. In addition, the algorithm for constructing a piecewise stationary process has become the basis for estimating the variance for the weighted least squares method.

The algorithms were tested both on artificial childbirth and on real data on temperatures in Tokyo and Moscow, wheat prices, the euro/dollar exchange rate, oil prices, and sales of peat soils.

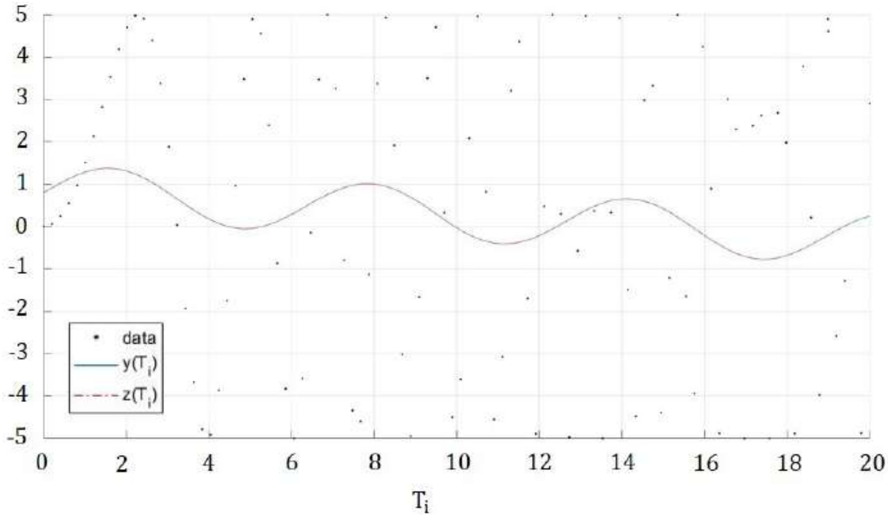

Figure 38 – $5\sin(0.3t^2)$ and approximations

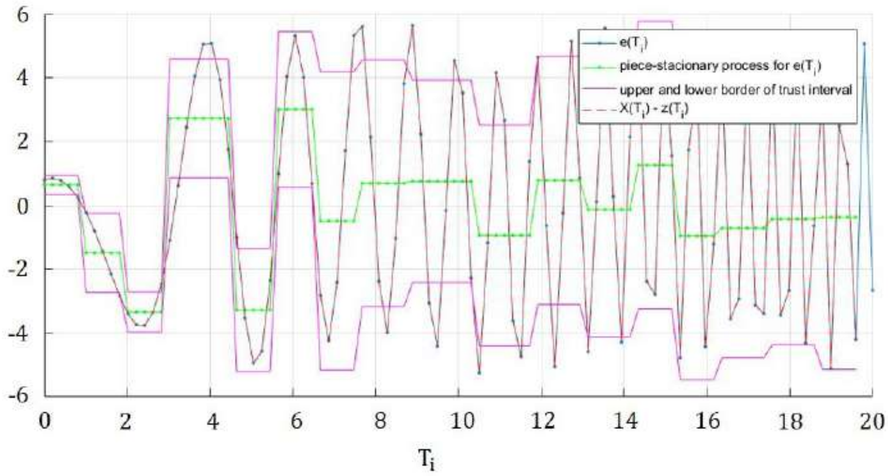

Figure 39 – Residuals of two models

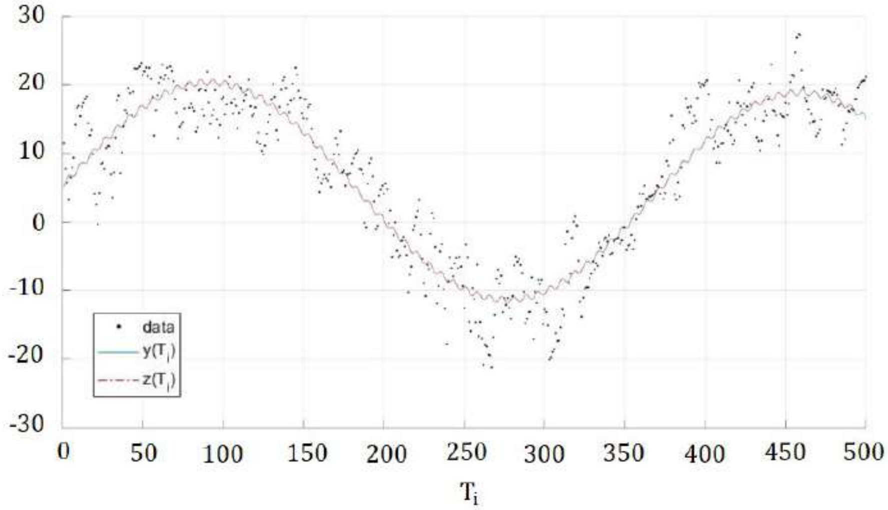

Figure 40 – Temperature in Moscow and approximations

CONCLUSION

The dissertation provides a comprehensive analysis of enterprise management systems for the extraction, production and sale of peat soils and organomineral fertilizers was carried out. The tasks of enterprise management are systematized into 12 blocks. For 5 tasks, a mathematical formulation of the problem and a solution algorithm are given: inventory management, distribution of the production plan along conveyors, a sales recommendation system, pricing when working with marketplaces, delivery to nonresident customers. For the task of creating a new product according to the specified characteristics, the problem statement is given. The example of these tasks shows where the analysis of non-stationary stochastic processes can be used and forecasting can be applied.

Algorithms for approximating the trend of a time series using Fourier series and Chebyshev polynomials have been developed. An algorithm for estimating the forecasting horizon has been created in the form of the horizon value itself and the confidence band, inside which the trend is contained. An algorithm is given for isolating a piecewise stationary time series from a non-stationary one. A band is defined for it, inside which any of the implementations of this process lies with a certain probability. An algorithm for estimating variance for the weighted least squares method, which removes the heteroscedasticity of the model, is proposed. The algorithm was tested on both artificial and real data. It is important to note a meaningful example of sales of peat soils, which shows the application of the constructed algorithms for enterprise management tasks.

All calculations are performed using MS Excel and the application computing package MATLAB.

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APPENDIX A. Software Code: Conveyor Allocation

Software implementation in the package MATLAB.

Calculation of the optimal distribution of the production plan along the conveyor lines: main file.

A monthly production plan (names, liters, quantity) is submitted as an input in the form of *.xlsx file. At the output, a quasi-optimal vector for the distribution of work shifts along conveyors and a corresponding matrix for the distribution of products along conveyor lines are formed.

```
clc
clear all
close all
tic
[A,B] = xlsread('Production Plan March 20.xlsx',4,'A2:F77');

A(:,3:5)=round(A(:,3:5),0);
Conv = zeros(13,9);
n = size(A,1);
m=size(Conv,1);
p=size(Conv,2);
%Volume of products (litres)
V=[0.25; 0.5; 2; 2.5; 3; 5; 10; 25; 50];
%productivity in 12h shift with 100% equipment operation
Conv(1,:)  = [0 0 0 0      0       0       0       0       0   ];
Conv(2,:)  = [0 0 0 0      0       0       0       6875    5625];
Conv(3,:)  = [0 0 0 0      0       11092   9787.5  0       0   ];
Conv(4,:)  = [0 0 0 0      0       11092   9787.5  0       0   ];
Conv(5,:)  = [0 0 0 0      0       12398   9787.5  0       0   ];
Conv(6,:)  = [0 0 0 12398  0       12398   9787.5  0       0   ];
Conv(7,:)  = [0 0 0 12398  0       12398   9787.5  0       0   ];
Conv(8,:)  = [0 0 0 12398  0       12398   9787.5  0       0   ];
Conv(9,:)  = [0 0 0 0      0       0       0       0       0   ];
```

```
Conv(10,:) = [0 0 0 0      0       0       0       0       0    ];
Conv(11,:) = [0 0 0 0      0       0       0       0       0    ];
Conv(12,:) = [0 0 0 0      0       0       0       0       0    ];
Conv(13,:) = [0 0 0 0      0       0       0       0       0    ];



%Assumption - there will be enough material
%remains or they will have time to buy
Plan = upak_palet(A,V,n);



R = zeros(n,m,p);
Flag = zeros(m,p);
RR = zeros(m,p);

%filling with specific products (1, 9-12 conveyors)
%this piece of code has been cut, because the example involves
%only in example 2-8 pipelines.

%calculation of the number of conveyors with the required volume
for k = 2:8 %film conveyors without granular
    for j=1:p
        if Conv(k,j)>0
            Flag(k,j)=1;
        end
    end
end

%the sum of the conveyors that can make a certain volume
%(only interchangeable <<bestroms>>)
CV=sum(Flag(1:8,:),1);
```

```
CV=CV';

%number of pieces of i type of product on the k liter conveyor j
for k=1:m
    for j=1:p
        for i=1:n
            if Plan(i,2)>0 && A(i,2)==V(j,1) && ...
             k~=1 && Flag(k,j)==1 && j>3
                R(i,k,j)=cell(Plan(i,2)/CV(j,1)); % rounding
                % up to more
            end
        end
        RR(k,j)=sum(R(:,k,j)); %number of pieces on conveyors
        % and literages
    end
end

sum(Plan(:,1))

P=zeros(m,p);
S=zeros(m,1);

% for each pipeline k is the sum of working days
% (from the sum of the pipelines with the necessary
% by volume)
for k=1:m
    s=0;
    for j=1:p
        if Conv(k,j)>0
         P(k,j) = sum(R(:,k,j)./Conv(k,j));
        end
        s=s+P(k,j);
```

```
    end
    S(k,1)=s;%number of working days for each conveyor
end


vpa([S(2:8) RR([2:8],[4,6:9])],5)

%calculate the maximum time and number
[MX,NO]=max(S([3:8],1));
NO=NO+2;

%calculate the minimum time and number
[mn,no]=max(S([3:8],1));
no=no+2;
for k=3:8
    if S(k,1)<mn && S(k,1)>0 && k>2
        mn=S(k,1);
        no=k;
    end
end
r=MX-mn; %difference of working days
r1=r; %variable for loop tracking
f=0; %number of iterations of the numerical method
Sf=S;% is entered to compare the current value
 %  with the previous one
eps=1*51*2/9785.5;

while r>eps && r<=r1
     %conveyor transfer function <<[MX NO]>>
     % of products on the conveyor <<[mn no]>>
    [MX mn S P R RR Sf NO no] = ...
    balans(S,Conv,R,RR,P,A,V,Flag,n,m,p,NO,no);
```

```
        r1=r;
        r=MX-mn;
        f=f+1;
        Delta(f,1)=r;
end

figure
hold on
grid on
plot(Delta(:,1),'b')
title('The difference between max(S) and min(S)')
ylabel('d(S)')
xlabel('Iteration number')

[B num2cell(Plan(:,1))]
 vpa(round([S, P],4),5)
ceil(S)
toc
```

Calculation of the optimal distribution of the production plan along conveyor lines: auxiliary function «balans()».

```
{function [MX mn S P R RR Sf NO no] = ...
balans(S,Conv,R,RR,P,A,V,Flag,n,m,p,NO,no,G,gr)
%search for maximum and minimum
Sf=S;

sht=1; % by how much to reduce the maximum working time
% and increase according to the minimum working time

L=zeros(n,p); %matrix of accounting units by nomenclature
%and volume, that cannot be reduced on the conveyor
```

```
if NO<no %the number of the maximum conveyor time
    %is less than the number of the minimum
    for k=G(gr,1):G(gr,2) %first we will meet the maximum number
        for j=1:p
            for i=1:n
                %if the minimum and maximum time pipelines are
                % can produce products of the same volume
                if k==NO && Flag(no,j)==1 && Flag(NO,j)==1 ...
                    && A(i,2)==V(j,1)
                 % and subtract 1 piece from it
                % (if after subtraction it is not less than 0)
                    if R(i,k,j) - sht>=0
                        R(i,k,j)=R(i,k,j)-sht;
                    else
                        L(i,j)=1; %subtract for this item
                        % and liters are not allowed
                    end
                %after that, we will meet the minimum number
                % and add on it 1 piece
                % (if you substract from the maximum)

                elseif k==no && Flag(no,j)==1 && Flag(NO,j)==1 ...
                        && A(i,2)==V(j,1) && L(i,j)==0
                    R(i,k,j)=R(i,k,j)+sht;
                end
            end
            RR(k,j)=sum(R(:,k,j));
        end
    end
elseif NO>no %the number of the minimum conveyor time is less
 % of the minimum number
    for k=G(gr,2):-1:G(gr,1) %first we will meet the max. number
```

```
        for j=1:p
            for i=1:n
                if k==NO && Flag(no,j)==1 && Flag(NO,j)==1 ...
                        && A(i,2)==V(j,1)
                    % and subtract 1 piece from it
                    % (if after subtraction not less than 0)
                    if R(i,k,j)-sht>=0
                        R(i,k,j)=R(i,k,j)-sht;
                    else
                        L(i,j)=1;
                    end
                % after that, we will meet the minimum number
                % and add on it 1 piece
                % (if subtracted from the maximum)
                elseif k==no && Flag(no,j)==1 && Flag(NO,j)==1 ...
                        A(i,2)==V(j,1) && L(i,j)==0
                    R(i,k,j)=R(i,k,j)+sht;
                end
            end
            RR(k,j)=sum(R(:,k,j));
        end
    end
end


% for each conveyor k is the sum of work shifts (out of the sum
% of stained glass windows for the conveyor)
for k=1:m
    s=0;
    for j=1:p
        if Conv(k,j)>0
            P(k,j) = sum(R(:,k,j)./Conv(k,j));
```

```
        end
        s=s+P(k,j);
    end
    S(k,1)=s;
end

%redefining the maximum and minimum time
% of the work of the conveyor number
[MX,NO]=max(S([G(gr,1):G(gr,2)],1));
NO=NO+G(gr,1)-1;
[mn,no]=max(S([G(gr,1):G(gr,2)],1));
no=no+G(gr,1)-1;
for k=G(gr,1):G(gr,2)
    if S(k,1)<mn && S(k,1)>0
        mn=S(k,1);
        no=k;
    end
end
end}
```