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EFFECT OF EXTERNAL FORCE AND TEMPERATURE INFLUENCES ON DYNAMIC MATERIAL FRACTURE

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Contents

Introduction

The relevance of the thesis topic

The study of the material reaction to dynamic and static loads plays an important role in ensuring the safety of technological processes and the engineering structures operation. There are many experimental studies showing that the material response to static and dynamic loads is different. Under dynamic loads, a number of specific effects are observed that cannot be explained using classical interpretations of strength and traditional failure criteria. For example, the dependence of dynamic strength and fracture toughness on the time parameters of the impact – duration, rate and energy input history. In relation to statics, a strength inversion effect can be observed, whereby a material with high static strength compared to another material becomes less strong under high-speed loads.

External factors such as temperature, pressure, moisture content, and environment have a significant impact on the strength characteristics. In some cases, material becomes less durable, in others, on the contrary, they acquire a higher load-bearing capacity. Experimental studies show that when calculating the strength of structures, it is not enough to use tabular values of strength characteristics determined for static loads. It is necessary to consider the dynamics of the process, the external factors influence, and the loading history of materials.

Taking into account the effects of hydrostatic pressure and temperature is crucial for a number of applied problems. The action of additional external factors changes the properties of natural and structural materials. For example, when restoring buildings, structures and materials after a fire, it is essential to consider changes in the material mechanical characteristics. Since, as experimental results show, the load-bearing capacities of structures deteriorate after heat treatment. In addition, when developing rock deposits and exploiting geothermal sources, to ensure work safety, it is necessary to calculate the strength and crack resistance of materials taking into account the preliminary temperature effect and the influence of hydrostatic pressure.

Classical approaches to strength calculations do not allow us to describe the above processes, since in the conventional strength theory of the deformable solid state mechanics the inertial forces contribution is not considered. The material fracture is considered as an instantaneous process. In fact, the material fracture is a process that occurs in time and space. The dynamic strength values are determined by entering additional parameters depending on the impact form and the problem formulation, which complicates engineering calculations and leads to the need to increase the experiments number to determine these parameters.

The development and implementation into engineering practice of a general approach to determining the material strength characteristics for static and high-speed impacts, which allows us to take into account the external factors influence, is the most crucial task. The fracture theory should be based on a minimum number of parameters, which are not problem characteristics, but physically based quantities. Prediction of the structures behavior in real conditions under dynamic and static loads will reduce the number of failures and unexpected fracture, ensuring operational safety.

The degree of development of the thesis topic

Experimental studies of the strength and fracture toughness of materials have shown significant differences in fracture as a result of static and dynamic loads [1], [2], [3], [4]. Various approaches have been proposed to describe the dynamic material fracture. Today, the theory of strength is being actively studied, and classical failure criteria are being developed [5] in which it is believed that fracture in a material begins when stresses (strains, energy) reach a critical value.

Kinetic fracture criteria have been proposed, which take into account the formation of defects at the microlevel and their development as a result of external influences [6]. Both to determine the limiting values in the traditional criteria of the strength theory, and to find the constants and functions included in the kinetic fracture criteria, it is necessary to conduct experiments. Limit values for classical strength criteria are determined during standard strength tests, but these criteria do not allow describing the material fracture under high-speed loads. On the other hand, kinetic fracture criteria make it possible to describe the strength curve dynamic branch, but determining the parameters and functions introduced in these approaches requires a large number of complex experiments. Another difficulty of these approaches is the impossibility of describing the material fracture using one condition for the entire range of loading rates.

Tests on the material fracture under intense loads have revealed that upon fracture process, thermal, mechanical and structural transformations are observed in materials [7]. During structural transformations, the formation, movement and merging of defects, phase transitions, deformation and crack at the micro level occur. For a large number of materials, before the formation of main cracks, microdamages are formed, the further development and merger of which leads to the material separation into parts.

When considering the material response to impulse loads in the continuum fracture mechanics of solids, ductile, brittle fracture, as well as fracture with the formation of adiabatic shear bands are distinguished. These processes are accompanied by the formation of a large number of different defects types (pores, microcracks, shear bands). In order not to consider each damage separately, in continuum fracture mechanics models, special parameters are introduced that characterize the material damage at the micro level. This approach was proposed by L. M. Kachanov [8] and Yu. N. Rabotnov [9]; a scalar damage parameter was introduced to describe the creep processes in materials.

Another commonly used fracture model within the framework of continuum mechanics was proposed in the works of J. Lemaitre [10], [11], in which a damage parameter was introduced, characterizing the processes of formation and accumulation of damage in the material and structure degradation. U. K. Singh [12], to describe the anisotropic response of a brittle solid and the crack growth under the influence of general applied loads, proposed to introduce a set of damage vectors. These theories assume that once the values of these damage variables reach a certain level, the material can no longer bear the applied load and failure occurs.

The main disadvantage of the continuum fracture mechanics approaches described above is the absence of the fracture time characteristics consideration. Thus, it becomes impossible to consider the effect of changes in the external influences parameters (amplitude, duration, pulse shape, load application method) on the dynamic material fracture. Fracture models containing damage parameters can describe the impact of additional external factors (hydrostatic pressure, temperature, moisture content), but often the number of entered parameters is redundant, and determining these parameters through experimental tests is difficult. Thus, the development of a unified approach is required to describe the material fracture, which allows considering not only the specific effects of dynamic fracture, but also the additional external factors influence.

The aim and objective of this research is to develop a universal theoretical approach to describe the fracture processes, determine material strength characteristics (strength, crack resistance) in a wide range of loading rates, which allows taking into account the additional external factors impact.

To achieve the research aim, the following tasks were solved:

1. Development of a structural-temporal approach to describe the influence of external factors on the material facture. Verification by experimental results.
2. Construction of a model of wave propagation in rods located in an elastic environment.
3. Description of the rod fracture in an elastic environment based on the structure-time approach.
4. Study of the temperature treatment and deformation rate influence on the rock strength properties.
5. Study of the rocks strength properties at different strain rates and levels of hydrostatic pressure.

6. Study of the cement mortars (barite and standard) strength properties dependence on temperature treatment, strain rate and mix proportions.
7. Analysis of compressive strength and fracture toughness rate dependences curves of materials, taking into account the additional external factors impact.
8. Finding the incubation time dependence on the pretreatment temperature and hydrostatic pressure.

Scientific novelty

The work presents a number of new theoretical scientific results. The structural-temporal approach was used to qualitatively and quantitatively consider the external factors influence on the dynamic fracture of cement mortars and rocks. Interpretation of the material response to shock wave loads in a wide range of external influences based on incubation time differs from classical theories of static fracture. Incubation time is a material property that does not depend on the loading history and the sample geometry. The incubation time can be determined experimentally. This approach allows us to describe the impact of structural changes resulting from the external factors action (hydrostatic pressure and preliminary temperature treatment) on the strength material properties (fracture toughness and strength) in a wide range of speed impacts.

The dynamic crack resistance and compressive strength for pre-heat-treated rocks and cement mortars under high-speed impacts were calculated. The strength and fracture toughness inversion effects for materials processed at different temperatures are shown. The incubation time dependence on treatment temperature was revealed. Based on the structural-temporal approach, the strength and fracture toughness rate dependences of rocks were found for various hydrostatic pressure levels. A relationship between the incubation time and hydrostatic pressure was studied.

The waves propagation in a rod located in an elastic environment was simulated. For the first time, based on the structural-temporal approach, this rod fracture was analyzed. The possibility of increasing the initial pulse amplitude was shown. It was revealed that there is a range of optimal impact durations when the rod is broken down with a minimum loading pulse amplitude. It is shown that the rod fracture can occur both as a result of spall and when a wave passes in the forward direction along the rod. External influence frequencies at which the rod can bear maximum loads were identified.

Theoretical and practical significance

The scientific results obtained in this work contribute to the methods development for analyzing the materials strength exposed to various external influences. The use of a structural-temporal approach to predict the material fracture in a wide range of loading rates makes it possible to substantiate the experimentally identified temporary effects of fracture, taking into account the influence of external force and temperature factors.

The method proposed in this dissertation for calculating the strength characteristics of materials is applicable to solving applied problems that require calculation of the load-bearing capacity and reliability of structural and natural materials that have been exposed to thermal effects, for example, in the fire case. The incubation time criterion allows determining the strength and fracture toughness of rock under the influence of static hydrostatic pressure. Incubation time is a measured material property. This approach allows us to avoid the use of additional parameters in the fracture model when solving applied problems.

The proposed model for the rod fracture located in an elastic environment can be used to calculate the optimal impact frequencies when driving piles into the ground. A number of durations are also shown at which the rod fracture can occur with minimal effort.

Methodology and research methods

To find the strength characteristics of materials taking into account the additional external factors influence, a structural-temporal approach is used. Limit values of compressive strength and crack resistance are calculated based on the incubation time criterion, presented in the structure- temporal approach. This approach allows solving a wide range of problems in the deformable solid mechanics. For example, prediction of brittle and plastic fracture of solids, calculation of material limit states (structural and natural) in the case of dynamic, static and combined impacts, as well as modeling of fracture and structural transformations in media as a result of a wide range of high-speed impacts. In the incubation time fracture criterion, quantities invariant with respect to the loading history are used.

The uniqueness of the structural-temporal approach lies in the fact that, unlike other known approaches to describing the material fracture, using this approach, a wide range of mechanical problems are considered from a unified point of view, namely brittle fracture, plastic flow and structural transformations under the influence of pulsed and quasi-static loads and their combinations. Also, with slow impacts, the incubation time criterion is consistent with the classical strength theory criteria.

Dynamic fracture in media occurs at various scale levels. This fact is partially confirmed by the presence of a large scatter in the threshold characteristics of dynamic strength, critical stress intensity factor, critical temperatures, pressures and other parameters of structural transformations in the experimental results. The presence of significant scatter in the results indicates that in reality the data obtained may relate to different scale levels of observation, thus, the scale level at which the transition process is observed is of fundamental importance. The structural-temporal approach solves the problem of large-scale discrepancy between strength characteristics determined experimentally and finds the large-scale level of medium fracture. Thus, the structural-temporal approach to the analysis of dynamic fracture processes allows not only to predict the time dependences of critical deformation and strength characteristics and the effects of their unstable behavior, but also to take into account scale effects.

Experimental studies show that the action of various external force and temperature factors leads to structural transformations, the formation of microdamages, chemical reactions and, as a consequence, changes in the dynamic strength characteristics of materials. The structural-temporal theory takes these changes into account and makes it possible to predict the dynamic material fracture.

Analytical and numerical analysis is used to model the wave propagation in a rod surrounded by an elastic medium and to calculate displacements, deformations and stresses arising in the rod under an impulse load.

Numerical calculations to describe the material fracture and find the limiting values of strength characteristics were carried out in the software «Wolfram Mathematica 11.3»

Approbation of results

The main conclusions and results of the dissertation work are presented at conferences:

1. School-conference «Mechanics, chemistry and new materials». Saint Petersburg. 2022.
2. First Virtual European Conference on Fracture – VECF1. 2020.
3. International conference on natural sciences and humanities – «Science SPbU – 2020». Saint Petersburg. 2020.
4. National conference on natural sciences and humanities with international participation «Science of St. Petersburg State University – 2020». Saint Petersburg. 2020.
5. Seminar «Computer methods in continuum mechanics». Saint Petersburg. 2018.
6. XIV International Scientific and Practical Conference «Comprehensive Security and Physical Protection». Saint Petersburg. 2018.
7. International youth scientific conference «XLIV Gagarin Readings». Moscow. 2018.
8. International summer school of Harbin Institute of Technology «Summer School on Land-Sky-Ocean». Weihai, China. 2017.
9. Twelfth student conference-competition «Chemistry, physics and mechanics of materials». Saint Petersburg. 2017.

The main results of the dissertation research are presented in 6 works, of which 1 article was published in journal indexed by Scopus and Web of Science databases, 3 articles were published in journals recommended by the Higher Attestation Commission of the Ministry of Science and Higher Education of Russia.

1. Igusheva L. A. Influence of preliminary heat treatment on the dynamic strength characteristics of cement mortars / L. A. Igusheva, Y. V. Petrov // Physics of the Solid State. – 2024. – Vol. 66. – No. 3. – P. 481-489. (in Russian)
2. Igusheva L. A. Dynamic compressive strength of thermally treated sandstone / L. A. Igusheva // Processes in geomedia. – 2024. – No. 1(39). – P. 2400-2405. (in Russian)
3. Igusheva L. A. The preliminary heat treatment influence on the rock fracture toughness / L. A. Igusheva // Ecological Bulletin of Research Centers of the BSEC. – 2024. Vol. 21. – No. 1. – P. 26-33. (in Russian)
4. Igusheva, L. A. Effects of dynamic deformation and fracture in the Klein – Gordon stress field / L. A. Igusheva, Y. V. Petrov // Procedia Structural Integrity. – 2020. – Vol. 28. – P. 1303-1309.
5. Igusheva, L. A. Dynamic fracture of a rod in a Klein-Gordon wave field / L. A. Igusheva // Proceedings of the seminar "Computer methods in continuum mechanics". 2018-2019 – 2019. – P. 21-38. (in Russian)
6. Igusheva, L. A. Shock wave deformation and fracture of a rod interacting with the environment / L. A. Igusheva, Yu. V. Petrov // Integrated safety and physical protection. Proceedings of the VII Memorial Seminar of Professor B. E. Gelfand XIV International Scientific and Practical Conference. – 2018. – P. 361-376. (in Russian)

Main scientific achievements

1. Wave processes simulation occurring in a rod of finite length surrounded by an elastic medium was carried out. The equation describing the longitudinal vibrations of the rod is the well-known Klein – Gordon equation [79, P. 24-25], [108, P. 363-364]. As a result of the environment influence, depending on the characteristics of the medium, the rod and the loading pulse, different patterns of wave propagation are observed. Three possible options are identified: wave dispersion, wave damping, and an increase in the initial pulse amplitude upon reflection from the

free rod boundary [79, P. 27-31]. The results were published in [79, 108]. Personal participation of the author in obtaining these results: literature analysis, numerical and analytical calculations, interpretation of results, writing the articles.

2. Dynamic fracture analysis of a finite length rod of located in an elastic environment was presented. To predict the rod fracture, the incubation time criterion was applied [79, P. 32-33]. The dependences of the impact threshold amplitude on the loading pulse duration were constructed [108, P. 372-373], [109, P. 1307-1308]. The possibility of both failure during wave direct passage along the rod and as a result of spall when the wave is reflected from the free rod boundary is shown [108, P. 373-374]. The results were published in [79, 108, 109]. Personal participation of the author in obtaining these results: literature analysis, numerical and analytical calculations, interpretation of results, writing the articles.
3. The correspondence of the calculated strain profiles in a PMMA rod located in silicone with the experimental data results is shown [109, P. 1306]. The results were published in [109]. Personal participation of the author in obtaining these results: literature analysis, numerical and analytical calculations, interpretation of results, writing the article.
4. The effect of external force and temperature influences on the dynamic compressive strength of some materials (sandstone [133, P. 2403], barite and standard cement mortars [118, P. 483-484]), as well as on the dynamic crack resistance (granite [117, P. 30], barite and standard cement mortars [118, P. 487]) was studied. It is shown that with an increase in hydrostatic pressure, an increase in strength and crack resistance occurs, and with an increase in treatment temperature, a decrease in the strength properties of materials is generally observed [118, P. 483, 484, 487], [117, P. 30], [133, P. 2403]. The results were published in [117, 118, 133]. Personal participation of the author in obtaining these results: literature analysis, numerical and analytical calculations, interpretation of results, writing the articles.
5. Calculations were carried out and an explanation was given for the compressive strength inversion effect [118, P. 485], [133, P. 2403] and the crack resistance

inversion effect [117, P. 31], [118, P. 488], when comparing two material samples processed at different temperatures, one sample demonstrates higher strength (fracture toughness) under quasi-static loads, but has lower strength (fracture toughness) under dynamic impacts. The results were published in [117, 118, 133]. Personal participation of the author in obtaining these results: literature analysis, numerical and analytical calculations, interpretation of results, writing the article.

6. The applicability of the incubation time criterion is shown to explain the influence of external factors and the loading rate on the dynamic fracture of specific materials, in particular granite [117, P. 29-30], sandstone [133, P. 2401-2403], barite and standard cement mortars [118, P. 482-484, 485-488]. The resulting theoretical calculations were verified by known experimental data. It was revealed that temperature treatment and hydrostatic pressure affect the static ultimate strength characteristics and the fracture incubation time. The dependence of the incubation time on external factors was found [117, P. 30], [118, P. 486, 488], [133, P. 2404]. In addition, to consider the additional factors influence, a minimum set of material constants (static tensile strength (fracture toughness) and incubation time) is sufficient [117, P. 30], [118, P. 483, 486], [133, P. 2402-2403]. The results were published in [117, 118, 133]. Personal participation of the author in obtaining these results: literature analysis, numerical and analytical calculations, interpretation of results, writing the articles.

Results submitted for defense

1. Wave propagation model in a rod located in an elastic environment is constructed.
2. The effect of a significant influence of the elastic environment on the amplitude of the initial pulse when reflected from the stress-free edge of the rod is studied.
3. The structural- temporal effects of the rod fracture in an elastic environment are investigated.

4. Modeling of the rock fracture toughness and compressive strength rate dependences subjected to preliminary heat treatment in a wide temperature range is carried out.
5. A model of the hydrostatic pressure influence on the rock dynamic fracture toughness and compressive strength under high-speed impacts is proposed.
6. The strength characteristics of cement mortars (standard and barite) subjected to preliminary heat treatment are determined.
7. A relationship is established between the incubation time and the additional external factors values (temperature treatment, hydrostatic pressure).

The reliability of the results is ensured by the correspondence of theoretical calculations with the experimental results presented in the scientific literature. The structural-temporal approach used in this work to predict the material fracture allows to describe the dynamic fracture effects observed in experiments.

Author's personal contribution

This thesis is a summary of the author's research results on the material behavior under high-speed loads. The problem statement and the study goals were formulated together with Professor Y. V. Petrov. The author personally under the guidance of Professor Y. V. Petrov carried out all analytical and numerical results presented in the work.

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Thesis structure

The dissertation consists of the introduction, four chapters, the conclusion, and the bibliography. The work is presented on 109 pages, contains 31 figures and 8 tables. The bibliography includes 136 items.

Chapter 1. Literature review

As a result of rapid technological progress, grandiose buildings and structures are constructed, complex mechanisms are produced that are used in different environments: on land, in the air, in water. The strength and reliability of these structures during operation plays an important role, since the life and safety of people depends on it. Unexpected various structures failure can lead not only to a large number of human sacrifices, but also to global environmental disasters. An example of such a disaster is the damage in 2011 of several reactors of the Fukushima-1 nuclear power plant [13], which occurred as a result of an earthquake of magnitude 9. As a result of the nuclear accident, radioactive materials were released, a large contaminated area was formed, and many people lost their homes. Another recent tragic example is the severe earthquake in Turkey in 2023, which led to an emergency situation and the destruction of a huge number of buildings and structures [14].

When designing structures, engineers often rely on material strength values calculated for the case of standard static loading conditions. In fact, as practice shows, the structures failure occurs more often as a result of intense dynamic loads: explosions, impacts, spalls caused by technogenic and natural influences, floods, earthquakes, fires. These loads can be approximately considered as quasi-static only in a limited number of cases.

In the case of static loading, the critical stress values are considered constant for the material and can be obtained from standard tests. Under high-speed loading, the ultimate strength value differs from the static influences case [2], [15]. An significant increase in ultimate strength is observed with increasing loading rate. It is worth noting that the speed dependence observed for each material is characterized by unstable behavior and changes depending on the load application conditions and the impact time parameters. In the fracture mechanics classical theory, the material strength is considered a material constant, regardless of the type of external influences, therefore the above effect of changing the ultimate strength a in the case of dynamic loading cannot be described within the framework of this theory. In addition, the dynamic material strength is significantly influenced by external factors such as temperature [16], [17], pressure [18], [19], and water saturation [20], [21]. Therefore, when designing structures it is necessary to consider the behavior of materials under impulse loads, taking into account the external factors influence on the material properties, since structures are incessantly subject to dynamic influences, both in normal operation and as a result of emergency situations.

When calculating the load-bearing capacity of structures, it is important to use universal failure criteria, which make it possible to take into account the material response for the entire range of time and force parameters of external influences. The load limiting value leading to fracture is not a material constant, but depends on the load parameters and other external factors, therefore it is not enough to calculate the material strength by introducing a safety factor to the static strength value [1]. The dynamic strength of materials is affected by the pulse shape, duration, amplitude, loading rate, as well as additional factors [22]. In this regard, it is required to conduct a large number of experimental tests to determine the dynamic material strength with varying both impact load parameters and external factors, since standard experiments on static tests do not provide a full picture of the material behavior.

It is significant to develop fundamental approaches that use a minimum number of parameters to describe fracture processes, since this will reduce the necessary experiments number to determine the material strength. In this case, it is crucial that the entered parameters do not depend on the process itself and loading conditions, being material constants. Also, the fracture criteria must be valid for the entire range of external influences, both static and dynamic. Thus, it can be concluded that the introduction of a universal approach into engineering practice to describe the dynamic material fracture in a wide range of loads, taking into account the influence of external factors, is an urgent task.

1.1 Strength criteria

There are many works that provide reviews of the fracture mechanics development and approaches to determining the structures strength [23], [24]. The difficulty in describing fractures under high-speed loads previously lay in the lack of experimental results. With the technology development, the science of material strength has made great strides, and now the literature contains a large number of experimental works (for example, [25], [26], [27], [28]) devoted to the dynamic material failure under various high-speed external influences. The main task today is the correct analysis of the results obtained, based on universal, physically justified fracture models. There are a large number of mechanical failure criteria and phenomenological fracture theories [29], as well as strength criteria in the case of high-velocity impacts [2], [23], [24].

The first failure theories, also called limit state theories, analyzed static and quasi-static cases. The task was to search for a limiting state, upon reaching which the fracture process began. The term «fracture» in this case can be understood as both the solid brittle fracture and the onset of plastic flow.

The most well-known criterion is the greatest normal stresses criterion:

$$|\sigma| \le \sigma_c, \tag{1.1}$$

where σ – the largest normal stress in the sample, σ_c – critical stress value for a given material, above which fracture occurs. Similarly, the maximum deformation criterion, the maximum shear stress criterion and the maximum energy criterion were formulated, depending on the parameters that were observed [30]. However, as practice shown, such criteria are valid only for the case of «slow» impacts. Criterion (1.1) is not applicable for shock loads, since with large amplitudes and short durations of impact, the limiting stress value changes compared to the static one σ_c. The impact duration, the time, and external force application method are not consider in any way in criteria of the form (1.1). These criteria postulate that failure will occur when the limit situation conditions are met. Such criteria are usually called classical static criteria, since they are valid only for the case of static loading.

There are a number of dynamic loading case effects that cannot be described within the classical strength theory framework. Since the strength classical theory considers all loads as quasi-static. In 1974, N.A. Zlatin, S.M. Mochalov, G.S. Pugachev and A.M. Bragov [31], considering spall fracture, first described the phenomenon of «dynamic branch». It turned out that under high-speed loads the material can withstand loads significantly exceeding the static critical stress, and unstable behavior of the threshold load amplitude was revealed. Similar results were later obtained by other authors [2], [32], [33], [34]. Classical strength criteria do not allow us to describe these effects, so the discovery of these phenomena led to the development of new material fracture criteria. A large number of different strength criteria have emerged, a description of which is presented in [35], [36], [37], [38]. The presented criteria cannot be applied to all loading rates and are often correct to use only for a limited number of experiment.

The second feature observed under short-term loads is the fracture «delay» effect. This effect was obtained in [39], [40], [41], [42], it was shown that the material fails not when local stresses reach a maximum value, but in the post-peak period [43], on the dynamic load curve part, when the stresses in the sample decrease over time. This phenomenon is difficult to describe based on the traditional strength theory (1.1), therefore it was proposed to determine the material strength using curves obtained experimentally in order to take into account the critical stress values dependence on the characteristics of the loading pulse. Some application packages still use this approach. However, such the material response description to dynamic loads does not reflect the applying the load method and the impulse shape influences. Moreover, conducting experiments to take into account all forms of loading force is an impossible task. Therefore, it is crucial that the dynamic material strength must be described by a failure criterion containing a limited set of parameters that are not process characteristics, but material constants determined from a minimum number of standard experiments.

The work [40] proposed an explanation for the fracture «delay» effect. The authors argued that with a decrease in the loading force duration, the process time becomes one order of magnitude small compared to the defects development time in the material, which leads to the influence of the characteristics of the external load on the strength.

Dynamic fracture is a complex process that consists of several stages. First, microdefects appear in the material, then they develop and merge, forming germinal microcracks. As a result of the growth and association of microcracks, macrocracks are formed, due to which the material fracture [30].

Let us present some of the main fracture criteria from the entire variety of approaches to describing dynamic strength, which made a significant contribution to the development of the approach to the material fracture that used in the dissertation work.

Neuber – Novozhilov criterion

G. Neuber [44] and V.V. Novozhilov [45], [46] proposed a failure criterion in which the structural size was introduced. Neuber considered the fatigue strength of a specimen with a corner cutout [47]. Novozhilov used a similar criterion to predict brittle fracture [45]. The Neuber–Novozhilov criterion for an elastic solid weakened by a cut has the following form:

$$\frac{1}{d}\int_0^d \sigma_y dx \le \sigma_c, \tag{1.2}$$

where it is assumed that the crack-cut is located along the axis Ox, σ_c is the static strength limit, and d is the structural scale parameter.

In this criterion, the geometry of the medium determined the structural size; it was associated either with the grain size or with the interatomic distance in the crystal lattice [24]. If the stress critical value σ_c and the stress intensity factor limiting value for the static load case K_{Ic} are known for the material, then the d is determined by the following formula [48]

$$d = \frac{2K_{Ic}^2}{\pi\sigma_c^2}. \tag{1.3}$$

The Neuber – Novozhilov criterion well describes the material fracture under quasi-static loads; in addition, it is applicable to fracture problems of bodies with corner cutouts, but does not explain the effects observed when the material reacts to high-speed loads.

Nikiforovsky – Shemyakin criterion

V. S. Nikiforovsky and E. I. Shemyakin studied the dynamic material fracture [49], [50]. It was shown that with increasing the loading rate there is a significant increase in the limiting material strength. To describe the resulting effect, an integral criterion was proposed that takes into account the history of local stress. It is consider that fracture occurs when the total force impulse becomes equal to a certain critical value. The Nikiforovsky – Shemyakin criterion has the following form

$$\int_0^{t_*} \sigma(t)dt = J_*, \tag{1.4}$$

where t_* is the moment of time at which fracture occurs, J_* is the critical force impulse (material constant, determined experimentally).

Criterion (1.4) is applicable only for cases of dynamic loading of «defect-free» media. For long-term impacts, the use of this criterion is impossible, since with increasing the external force duration the critical stresses tend to zero. In this regard, the static part of the curve of the strength dependence on loading rate cannot be described using criterion (1.4). However, the key point that contributed to the material dynamic strength theory further development was the introduction the time parameter as one of the properties of the fracture process.

Tuler–Butcher criterion

In 1968, F. R. Tuler and B. M. Butcher [38] proposed the following integral failure criterion to describe the effect of the dependence of spall stress on the duration of external influence under dynamic loading

$$\begin{gathered} \int_0^{t_*} (\sigma_0 - \sigma(t))^{\lambda} dt = K, \\ |\sigma(t)| \geq |\sigma_0| \geq 0, \end{gathered} \tag{1.5}$$

where t_* is the fracture time, σ_0 is the threshold static stress, λ, K are constants.

In the mentioned failure criteria, based on the theory of damage accumulation, stress is considered as a function of time, so these criteria are able to take into account the loading history. In criteria (1.3)–(1.5), the local force impulse determines the stress state, this allows us to qualitatively consider the increase in ultimate strength with increasing loading rate. However, the disadvantage of these approaches is the impossibility of describing the entire curve of the material strength speed dependence, since with an increase in the impact duration and the transition of loads from high-speed to static, the previously presented criteria did not correspond to the traditional strength criteria.

There are a large number of failure criteria that are valid only for static or only for dynamic loads. Moreover, there is no possibility of transition from one type of load to another within the framework of these criteria; this imposes restrictions on the scope of their application. As a result of changes in external load parameters and the additional external factors influence, it is difficult to determine in advance the material mechanical response. In addition, to correctly select the fracture criterion, it is necessary to know whether external influences will be slow or fast for a given material, which is also difficult to determine in advance. To take into account the previously mentioned issues, based on the approach of Y. N. Rabotnov [51], the impulse fracture criteria (1.3)–(1.5) can be rewritten in the form:

$$\int_0^t \sigma(s) K(t-s) ds \le \sigma_c. \tag{1.6}$$

This approach is based on the theory of memory decay. Instead of a force impulse (as in criteria (1.3)–(1.5)), criterion (1.6) uses a convolution operator with a damping function.

1.2 Structural-temporal approach

Nowadays, the literature presents a large number of experimental works devoted to the study of the material fracture in a wide range of speed impacts [1], [52], [53]. Experiments are carried out both for the case of static loading and under dynamic loads. In the classical theory of fracture mechanics, it was believed that the ultimate strength is a material constant and does not depend on external loads. In fact, it turned out that the mechanical response of materials to dynamic loads differs from the response to static loads [54]. Classical failure criteria, such as the maximum stress criterion for defect-free media failure and the Irvine fracture toughness criterion for cracked media, which are widely used to predict fracture under static and quasi-static loads, were unable to describe the effects observed under impact loads [55]. Distinctive features of dynamic fracture are, confirmed experimentally, the dependence of the limiting values of fracture parameters on the type and shape of the applied load [31], as well as the effect of «delay» of fracture [41], in which fracture occurs after reaching the maximum stress value in the sample, but not at peak load. The material strength is influenced by the characteristics of the load applied to the sample (loading speed, amplitude, pulse duration), as well as other additional external factors (pressure, temperature, water saturation). These features are valid not only for dynamic fracture (the beginning of crack growth in a sample, spallation), but also for a number of other physical processes, such as cavitation in liquids and electrical breakdown in dielectrics [22].

The listed features and a number of other experimentally observed effects of dynamic fracture are explained within the framework of the structural-temporal approach based on the concept of incubation time of fracture (considered as a material constant, depending on the scale level), which was first proposed by Yu. V. Petrov [56], [57]. Within this approach, incubation time is the main indicator of the response of materials and environments to dynamic loading.

In the structural-temporal approach, the problem of creating a universal failure criterion, valid for both static and dynamic loads, was solved. The incubation time criterion was proposed [57], [58], [59]. which for the case of static loads turns into a classical failure criterion. This approach is based on a set of parameters invariant to the loading history, which are properties of the material. The incubation time fracture criterion can be used to calculate the material dynamic characteristics obtained in tests with various loading pulse shapes [60]. This criterion allows us to describe the specific effects observed during high-speed fracture.

The incubation time criterion, which describes the material fracture, has the form:

$$\frac{1}{\tau}\int_{t-\tau}^{t}\left(\frac{F(t')}{F_c}\right)^{\alpha}dt' \le 1, \qquad (1.7)$$

where $F(t)$ is the intensity of the local force field; F_c – static limit of local force field; τ – incubation time. When the equal sign in condition (1.7) is satisfied, the material is fractured. Parameter α characterizes the sensitivity to the level of the force field intensity that causes failure (in the elastic formulation of the problem $\alpha = 1$).

The incubation time criterion of fracture takes into account the fact that fracture in a material does not occur instantly, but is a process that happens over time. Macroscopic material fracture for both static and dynamic external influences is preceded by an incubation period, which is an important factor in the fracture process, the presence of which explains the specific effects observed under fast, impact loads, for example, changes in the strength of the material with varying strain rates. In the structural-temporal approach, a fundamental concept was introduced: the incubation time τ, which is associated with preparatory relaxation processes occurring in the material, preceding macrofracture. Incubation time is a material constant that characterizes the material response to dynamic influences. The method of applying the load, the pulse shape, and the sample geometry do not affect the incubation time values [55]. The incubation time of the material can be determined experimentally.

As a simple example showing the physical meaning of the parameter τ, consider the following problem. Let the failure of the sample occur as a result of the action of a slow ($t_* >> \tau$) linearly increasing tensile stress, determined by the formula $\sigma(t) = \dot{\sigma} t H(t)$, where $\dot{\sigma} = const$, $H(t)$ is the Heaviside function [61]. Substituting the stress into the fracture criterion (1.7), we obtain the fracture time $t_* = \sigma_c / \dot{\sigma} + 0.5\tau$, as well as the value of stress at the fracture moment $t = t_*$: $\sigma_* = \sigma(t_*) = \sigma_c + 0.5\dot{\sigma}\tau$, where $\sigma_c = F_c$ is the static material strength. When the increase in stress occurs «slowly» ($\dot{\sigma}\tau / \sigma_c << 1$), and the load can be considered as quasi-static, the critical stress becomes approximately equal to the static strength ($\sigma_* \approx \sigma_c$). These formulas show that, according to criterion (1.7), when the stress in the material reaches the value of the static strength limit at the moment of time

$t_c = \sigma_c / \dot{\sigma}$, the material still remains unbroken. Thus, in this example the incubation time characterizes the time required for complete failure of the material after the stress in the sample reaches the static strength limit (Figure 1.1). The incubation time can be determined during dynamic tests [62] or in static experiments (by measuring the time of stress drop at the front of the unloading wave, using interferometric or polarization-optical methods) [63].

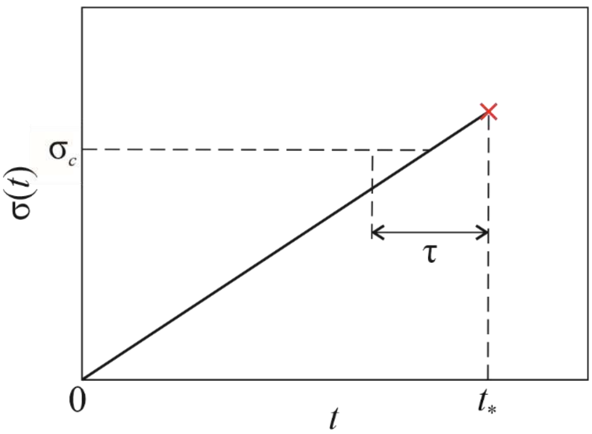


Figure 1.1. Time dependence of stress under slow linear loading

The advantage of the incubation time criterion is the fact that it is valid for both dynamic and static loads. Under static influences, criterion (1.7) takes the form of a classical strength criterion.

The variety of corresponding critical conditions using the concept of incubation time, considered in the works of Y. V. Petrov and A. A. Utkin [58], Y. V. Petrov and N. F. Morozov [59], N. F. Morozov and Y. V. Petrov [64], A. A. Gruzdkov et al. [65], V. V. Bratov and Y. V. Petrov [66], Y. V. Petrov and E. V. Sitnikova [67], Y. V. Petrov et al. [68], [69], made it possible to successfully study a large set of different phenomena: brittle fracture, dynamic plasticity, electrical breakdown, phase transformations, detonation, multi-scale, etc. The structural-temporal approach allows us to describe failure processes in a wide range of dynamic problems, which is why in this work it was chosen to describe the material fracture.

1.3 Models of rods in various environments

Rods are the basic elements of a huge number of engineering structures. Often, rod structures in different environments experience loads. Therefore, when calculating the strength of structures, it is necessary to take into account the environment influence on the deformations and stresses arising in the rods.

There is a large number of works devoted to static problems of the theory of rods. But one should always consider that in real conditions, dynamic loads also act on the rods, which lead to vibrations. It is known that vibrations in rods can have a significant impact on the reliability of structures containing rod elements [70]. Therefore, calculating the deformations and stresses that arise in rods under dynamic loads is a very important applied task.

One-dimensional models are often used to describe the longitudinal vibrations of rods. Despite the fact that these models cannot fully take into account all the processes occurring in the rods interacting with the environment, they have their advantages. One-dimensional models are distinguished by their simplicity and at the same time allow one to study the effects associated with rod structures. Therefore, the use of the assumption of one-dimensionality of the system for such problems is often found in the scientific literature.

At the moment, there are models describing the interaction of rods with various environments. For example, L. I. Slepyan [71] considers various non-stationary effects on rod structures. In particular, the problem of an infinite rod lying on a solid elastic base of known rigidity and loaded in one of the sections with a concentrated force was solved. It turns out that in the presence of an elastic base, significant deformations are localized near the point where the force is applied. In addition, L. I. Slepyan studies waves in rods immersed in a liquid, namely, he considers the influence of the environment – an ideal compressible fluid on the non-stationary waves propagation in an elastic solid. It is discovered that the liquid influences the wave propagation in the body. An additional normal load acts on the surface of the body – the reaction of the liquid, and pressure waves are emitted into the liquid, carrying away part of the elastic wave energy. Thus, the energy dissipation phenomenon is observed.

L. V. Nikitin [72] considers many problems related to the statics and dynamics of solid state in the presence of external dry friction from the medium surrounding the rod. Particular attention is paid to one-dimensional problems. In particular, the stresses generated in a rod in contact with the environment were studied. Interaction with the environment is modeled according to the dry friction law. The distribution of stresses arising in the rod has been found. An impact with a constant force on an oscillator with dry friction is also considered. In addition, the behavior of a rod enclosed in a cage under dynamic load was studied. In this problem, it is assumed that the contact friction force is directly proportional to the longitudinal deformation in the rod. A solution to this equation has been found under the condition that a compressive stress is specified at the end of the rod.

The research of L. D. Akulenko and S. V. Nesterov [73] is devoted to the vibrations of a rod in an inhomogeneous elastic medium. The problem of finding the natural frequencies and shapes of plane transverse vibrations of a thin inhomogeneous rod in an elastic medium with a variable stiffness coefficient and arbitrary boundary conditions of elastic fastening is investigated. It is established that the presence of an external elastic medium, described by the Winkler model, can lead to an anomalous effect – an increase in the natural frequencies of lower vibration modes with a continuous increase in the length of the rod. This fact supports the idea that the environment has a special influence on the vibrations in the rods that should be taken into account.

A. N. Filippov [74] considers the propagation of longitudinal elastic waves in a semi-infinite rod of constant circular cross-section, which interacts with its Winkler-type environment according to the law of dry friction. The equation describing this model has the form $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + 2\frac{\partial u}{\partial x}$. To find a problem solution, the Laplace transform method is used. A solution to this equation is found. It is worth noting that in this formulation of the problem, it is believed that during dry friction the resistance force acting from the medium on the rod is directly proportional to the deformations occurring in the rod. But there are other variations in describing the impact of the environment, one of which will be discussed in this thesis in the second chapter.

Not many experiments are known to measure deformations in rods in contact with the environment under the impact influence. One example is the work of V. G. Bazhenov [75]. An experimental technique is proposed for determining the resistance force that arises when a deformable impactor is introduced into a soft soil environment. An inverted position is used: the striker changes roles with the target (the target strikes the striker). The parameters of the contact interaction process are recorded in a stationary rod (striker). During testing, a container with soil strikes a stationary head part of a corresponding shape attached to the striker. During the experiment, loading rates are chosen such that no plastic deformation occurs in the rod. Measuring forces is reduced to recording the pulse of elastic deformations in the rod using strain gauges.

As shown above, the environment can have a significant effect on the deformations in the rods, so it is necessary to develop various models to describe this effect.

The main goal of fracture studies is to obtain information and basic dependencies for the critical characteristics of external influences for material subjected to various types of loads. Loads can be static and quasi-static, as well as dynamic, which can be divided into high-speed and pulsed. Recent studies shown that dynamic strength depends on the relationship between the time characteristics of the material fracture process and the loading history (rate and duration of impact). Modeling the rate dependences of strength and fracture toughness under pulsed loads is currently one of the most crucial tasks in both experimental and numerical-analytical studies.

The dynamic strength of various materials, such as ceramics, glasses, metals, alloys, polymers under very rapid loads can be obtained experimentally by studying the effect of spall fracture. Recently, a large number of experiments was carried out that study the propagation of waves through materials under dynamic loading to expand knowledge about the strength and ductility of materials.

Spall in bodies occurs as a result of the reflection of a wave propagating through the material from the free boundary of the material. If the initial incident wave was a compression wave, which does not have a serious damaging effect on the material, then after reflection from the free boundary it becomes a tension wave and can cause fracture [76]. This phenomenon makes it possible to study the strength properties of media at very high strain rates up to 10^9 s^{-1}, when specific effects appear that fundamentally do not fit into the classical continuum fracture mechanics and physics approaches.

Methods for measuring spall strength are based on the analysis of wave interaction. In that part of the material where the tensile stress reaches sufficiently large values, spalling may occur. A study of the interference of direct and reflected waves gives the stress in the spallation plane σ^*, in a linear approximation $\sigma^* = \frac{1}{2}\rho_0 c_0 \Delta u_{fs}$, where ρ_0 is the density of the material, c_0 is the speed of sound in the material, Δu_{fs} is equal to the difference between the maximum value of the free surface velocity function and the value of this function at the first point of the local minimum, corresponding to the moment of arrival of the signal about the formation of a spall crack on the free surface [76]. The limits of this approach applicability to the analysis of spall strength were discussed in [77], and it was shown that the above formula is not always correct.

Typically, spall fracture experiments are carried out with plates. Let us pay special attention to spall fracture in rods; there are much fewer such experiments compared to plates. One of the works in which tensile stresses at which brittle fracture occurs as a result of spall in a rod was studied is the work of E. N. Bellendir [78]. This research examines an experimental study of the brittle fracture of the solid in a tensile stresses wave in a rod as a result of spall. It is known, that fracture occurs as a result of the combination of a large number of microdamages that arise and simultaneously grow as a result of the load action. It is found that under pulsed loading, many materials belonging to different classes of solids are capable of withstanding significant overstresses. Experimental data indicate that the process of material fracture under the influence of pulsed loads is significantly different from fracture under conditions of quasi-static force impact. In [78], the material fracture under dynamic loads with a duration of 10^{-4} – 10^{-6} s is considered.

Due to the influence of the environment, fracture in rods under dynamic loads, as shown in [79], can occur not only due to the reflection of a wave from the free boundary of the rod, but also during direct passage of the wave along the rod. Thus, it is necessary to calculate the deformations and stresses in the rod under high-speed loads before using rod structures in practice, in order to prevent unplanned failure.

Thus, the interaction of deformable solids with the environment is found everywhere in the modern world, so studying the influence of the environment on rods is an urgent task.

Chapter 2 examines the rod elements interaction model of engineering structures with an elastic environment. The model allows both to determine wave fields under dynamic influence and to predict the dynamic fracture of such rod elements. This model can be applied in many fields. One example is the installation of piles in construction; this model allows one to calculate the stresses and displacements occurring in the pile, thus the loads that the structure can withstand can be calculated.

1.4 The influence of temperature and hydrostatic pressure on the material strength properties

Temperature effect

The mechanical properties of rocks play a crucial role in various engineering applications, especially in construction and mining (various rocks and coal) [80], [81], [82], [83]. Particularly important characteristics are the tensile strength and fracture toughness of the material, since they describe the strength properties. The material strength is a property that characterizes the material ability to resist fracture, as well as irreversible changes in shape (plastic deformation) as a result of the external forces action. The material strength is characterized by its tensile strength – a threshold stress value upon reaching which the material fails. Another important material characteristic is fracture toughness. Fracture toughness is a measure of a material's resistance to crack propagation and is the primary characteristic used to analyze the failure of rocks resulting from various types of external influences, including blasting, hydraulic fracturing, and impact penetration during rock drilling.

In various fields of science and technology, attention is paid to the mechanics of rock fracture; research in this area plays a key role in mining, civil engineering, and mechanical engineering [84], [85], [86]. The material strength properties are used for calculations that guarantee the safe operating conditions of geostructures and infrastructure.

Temperature has a significant impact on the fracture nature and on the strength properties of rocks. In a number of technological processes such as mining, oil, gas and shale production, nuclear waste disposal, and geothermal energy production, the influence of temperature is very significant [87], [88]. Prolonged exposure to high temperature changes both the physical and mechanical properties of rock. For example, tensile strength, elastic modulus, porosity, etc. change [89], [90].

The literature presents many works devoted to the study of the heat treatment effect on the mechanical properties of rocks. The effect of heat treatment duration on the fracture toughness of sandstone was studied, and it was found that with increasing heat treatment duration, the minerals in the rock expand, internal microcracks change, and therefore the tensile strength of the material decreases [91]. In [92], the change in the granite fracture toughness was considered depending on the change in processing temperature and the cut inclination angle. Fractures in mode I, mode II and a mixed mode of fracture (I+II) were realized; in all cases, a decrease in fracture toughness was observed with increasing temperature, and at temperatures below 600 °C the decrease was rapid, and above 600 °C the fracture toughness decreased, but more slowly . The study [93] examined mode I failure of thermally treated sandstone and identified three stages of failure depending on the temperature range. At all stages, a decrease in fracture toughness was observed with increasing temperature. In [94], it was shown that as a result of changes in the mineral components and internal structure of the rock, as well as a decrease in water saturation due to increased temperature (from 25 °C to 500 °C), changes occur in the physical and mechanical properties of the rock (sandstone and granite). When heated, dehydration of the rock and expansion of the mineral component are observed, as well as an increase in volume and a rapid increase in the number of microcracks, so the tensile strength, compressive strength and wave propagation speed decrease. In [95], a decrease in tensile strength with increasing temperature for sandstone is shown. The above results show that, on average, there is a deterioration in mechanical properties with increasing temperature, however, in [96] it was shown that fracture toughness decreases with increasing heat treatment temperature only up to the point of elastic-plastic transition.

At the moment, there is a large number of studies (for example, [92], [96]) devoted to the study of the temperature influence on the static properties of materials, as well as the search for threshold temperatures above which a rapid deterioration in the material properties is observed. However, it must be taken into account that in reality rocks are subjected to a huge number of dynamic loads caused by explosions, impacts, spalling, and seismic activity. With increasing depth of mining operations, it became obvious that the dynamic strength of rock depends not only on the rate of external influences, but also on preliminary heat treatment [97], [98]. In [17], using a system of split Hopkinson pressure bar system, the dynamic fracture toughness of gabbro and marble exposed to high temperature was studied. Two cases of temperature effects were considered: fracture of samples at high temperatures (100–330°C) and preliminary heat treatment of samples (200°C for marble and 600°C for gabbro) followed by failure at room temperature. In both cases, experiments showed that dynamic fracture toughness increases with increasing loading rate. As the loading rate increases, the values of dynamic fracture toughness become higher than the values for the case of static loads. These effects are valid both for the pre-heat-treated samples fracture, and for samples destroyed at high temperatures, as well as for samples that were not exposed to temperature. The part of the samples tested at high temperatures had many microcracks, probably caused by thermal cracking. It turned out that temperature has a significant effect on the static fracture toughness of gabbro and marble, for example, when marble is heated to 200°C and gabbro to 600°C, the static fracture toughness decreases by almost 50% compared to thermally untreated samples. At the same time, the effect of temperature on dynamic fracture toughness is much less, for example, for gabbro with the logarithm of the stress intensity rate equal to 5.6 MPa · $m^{1/2}$ s^{-1}, the difference in fracture toughness values for gabbro at room temperature and heat-treated gabbro is only 4%. The works [99], [100] consider the effect of heat treatment on the dynamic strength and dynamic fracture toughness for two cement mortars. It is shown that with increasing temperature there is a decrease in the dynamic strength characteristics for both solutions, due to microcracking and irreversible chemical changes in the samples caused by the temperature influence. To explain the observed effects of the strength characteristics dependence on temperature, it was proposed to introduce the parameter D to quantitatively describe thermally induced damage to the material. Thermal damage was divided into two types: damage caused by chemical changes (for example, dehydration and phase transitions of the components of the mortar) and microcracks resulting from heat treatment, each type has its own parameter D_c and D_p. Formulas are proposed for relating dynamic strength and dynamic fracture toughness to loading speed and static limit values using damage parameters. However, the authors note that calculating damage parameters is difficult for all possible processing temperatures.

Hydrostatic pressure effect

Rocks in natural conditions are under the influence of static hydrostatic pressure, while they are also subject to various dynamic influences caused by both natural phenomena, such as seismic activity, earthquakes, volcanic eruptions, and technogenic phenomena, for example, explosions and impacts produced in during drilling during mining operations. The literature presents works devoted to the study of the strength characteristics of rocks for the case of static loading at various levels of hydrostatic pressure. It has been shown that static fracture toughness [101], [102] and compressive strength tend to increase with increasing hydrostatic pressure [103], [104].

As dynamic experiments show, there is an increase in fracture toughness with increasing loading rate, which distinguishes these processes from fracture under quasi-static loads. In addition, it turned out that hydrostatic pressure has a significant effect on the dynamic characteristics of the material strength. For example, a study in [18] showed that the dynamic strength of various types of sedimentary rocks (limestone, shale, sandstone, tuff) increases linearly with increasing confining pressure in the range from 0.1 to 35 MPa, similar to static strength. It is obtained in [19] that during mode II fracture of marble, an increase in the critical value of the dynamic stress intensity factor is also observed with increasing confining pressure. In [105], the failure of sandstone samples with different numbers of prefabricated cuts was experimentally studied using a split Hopkinson pressure bar system. It is shown that both hydrostatic pressure and the presence of defects (cuts) play an important role in the rock failure under the hydrostatic compressive pressure influence. Dynamic strength, elastic modulus and peak strain tend to increase with increasing hydrostatic pressure. In addition, with an increase in the number of defects in the samples, the dynamic strength and elasticity modulus decrease, and the nature of the failure passes from shear failure to a mixed tensile mode (shear along with tension). In [106], the influence of hydrostatic pressure on the dynamic fracture toughness of granite is considered. It was revealed that the higher the loading rate, the more significant the effect of hydrostatic pressure on the fracture toughness of granite. With increasing hydrostatic pressure, the limiting value of the stress intensity factor increases. To describe the behavior of the material in the case of dynamic loads, a formula is proposed for the dependence of the stress intensity factor on hydrostatic pressure and loading rate, which contains four parameters determined experimentally.

To describe the external factors influence on the dynamic strength and fracture toughness of materials, various parameters are introduced that characterize the damage of the material and the dynamic response to external influences. Often, works introduce a large number of parameters that are properties of loading processes, not the material, and also require a large number of experimental studies to find the values of these parameters for each case of external influences. The presence of additional external factors such as temperature and hydrostatic pressure complicates the determination of strength characteristics. Thus, to determine the dynamic strength and fracture toughness of materials in the case of the additional external factors influence, it is necessary to use a universal approach, a failure criterion that is valid for both static and dynamic loads. In this work, a structural-temporal approach is used, since it allows us to solve the above problems. In this approach, to describe fracture, parameters that are properties of the material are used: static strength (or static stress intensity factor limit for the case of a medium with a crack) and incubation time. This approach allows us to reduce the number of experiments required to determine the dynamic strength of materials.

Chapter 2. Effects of dynamic deformation and fracture in the Klein – Gordon wave field

In the modern world, each object does not exist separately, all elements are interconnected, so we have to consider the interaction of elements and model the contact between bodies. When making calculations, it is important to predict the behavior of interacting bodies and to study the results of the influence of one body on another. A special place is occupied by tasks related to the interaction of solids with various types of the environment. Often the environment has a significant impact on the material properties in contact with this environment.

There are many models that describe the interaction of silids with the environment, but the calculation of deformations and stresses under dynamic loading takes a special place, since many materials exhibit non-classical behavior under rapid dynamic loads. It is also significant to calculate critical stresses to prevent unplanned failure of structures under dynamic operating conditions.

This chapter is devoted to the study of the effects resulting from the dynamic deformation of a rod in contact with an elastic environment. A one-dimensional model is used to calculate the stresses and strains occurring in the rod.

This chapter considers an elastic model of longitudinal vibrations of a rod located in an elastic environment under dynamic loading. Numerical and analytical analysis of deformations and stresses in a rod in an elastic environment was carried out, and deformation profiles in specific sections of the rod were calculated. The waves propagation in a rod of finite length and the waves reflection from the free boundary of the rod are considered. Using the incubation time criterion, the rod fracture of finite length surrounded by an elastic medium was studied, and the possibility of spall fracture was shown. A time dependence curve of strength was constructed, i.e. threshold amplitude dependence on the initial pulse duration. One of the important and unexpected results is the non-monotonic nature of this dependence, which suggests the presence of optimal characteristics of the breaking impulse.

The results presented in this chapter were published in [79], [107], [108], [109].

2.1 Longitudinal vibrations of a rod in an elastic medium

Let us investigate the effects associated with dynamic deformation and fracture of a rod located in the elastic environment. Let's consider a one-dimensional model of a rod (Figure 2.1), which performs longitudinal vibrations (oscillations along the axis of the rod). It is assumed that the forces acting on the rod from the environment are directly proportional to the displacement. In addition, the cross-section of the rod is assumed to be constant along its entire length.

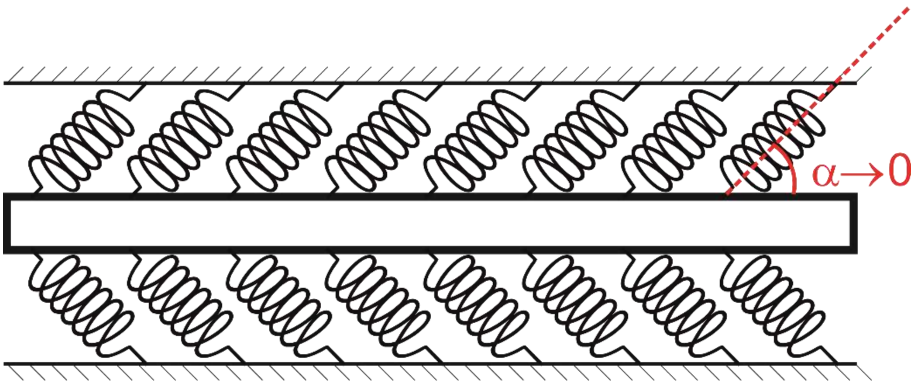


Figure 2.1. Rod element in an elastic medium

Derivation of the longitudinal vibration equation

Let us compose an equation for longitudinal vibrations of a rod surrounded by an elastic medium. To derive the equation, let carry out argumentation similar to that described in the works of Y. N. Rabotnov [110]. This work [110] considers the classic case of oscillations of a rod, when no additional resistance forces act on the rod. Let's carry out a similar analysis, but taking into account the resistance forces acting from the environment.

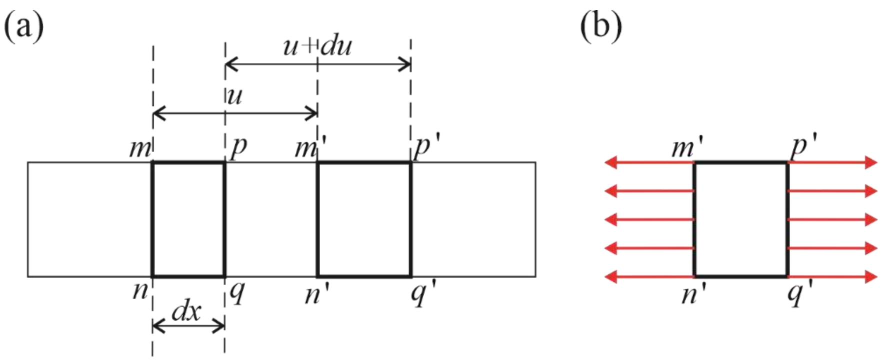


Figure 2.2. Rod element (a), rod element after deformation (b)

Figure 2.2, a shows a rod element, which in a non-deformable state was enclosed between sections mn and pq and had coordinates x and $x+dx$, respectively. Fixing the moment of time t when the section mn occupies position $m'n'$, the section pq takes position $p'q'$, let us denote the displacement of the left section, the initial coordinate of which was x, by u. The displacement u is a function of two variables – time t and coordinates in the undeformed state x, therefore the displacement of the section with the coordinate $x+dx$ will be $u+\frac{\partial u}{\partial x}dx$. Figure 2.2, b shows the element $m'n'p'q'$ separately. Let us denote the stress acting in the section $m'n'$ by σ, then the stress acting in the section $p'q'$ will be $\sigma+\frac{\partial \sigma}{\partial x}dx$. Along the entire element there will be a resistance force equal to $-kuP_0dx$, where k is the proportionality coefficient of the resistance forces and P_0 is the rod perimeter. It is assumed that the rod is surrounded by an elastic environment, so resistance forces act on the surface of the rod. Thus, when calculating the resistance force in each section of the rod, it is necessary to take into account the perimeter of this section. Since this work considers a one-dimensional model, it is assumed that the rod has the same cross-section along its entire length, and that the shape of the section does not change over time.

Let us write the equation of motion for the element $m'n'p'q'$

$$\rho Sdx\frac{\partial^2 u}{\partial t^2} = S\frac{\partial \sigma}{\partial x}dx - kP_0 udx, \tag{2.1}$$

where S is the cross-sectional area of the rod.
где S – площадь поперечного сечения стержня.

Substituting Hooke's law $\sigma = E\frac{\partial u}{\partial x}$ into equation (2.1), the following relation is obtained

$$\rho Sdx\frac{\partial^2 u}{\partial t^2} = SE\frac{\partial^2 u}{\partial x^2}dx - kP_0 udx. \tag{2.2}$$

Let us divide equation (2.2) by the product ρSdx, then the equation is derived

$$\frac{\partial^2 u}{\partial t^2} - a^2\frac{\partial^2 u}{\partial x^2} + b^2 u = 0, \tag{2.3}$$

where $a = \sqrt{\frac{E}{\rho}}$, $b = \sqrt{\frac{kP_0}{\rho S}}$.

The differential equation (2.3), which describes the longitudinal vibrations of a rod located in an elastic environment, turns out to be the well-known Klein – Gordon equation (generalized wave equation). The Klein – Gordon equation is a relativistic version of the Schrödinger equation and is used to describe fast moving particles that have mass. At the same time, this equation finds its application in the deformable solids mechanics.

Solution of the Klein – Gordon equation

As noted earlier, the purpose of this chapter is to study the wave propagation in a rod of finite length placed in an elastic environment. But to solve this problem, let firstly consider the auxiliary problem of longitudinal vibrations of a semi-infinite rod in an elastic environment.

According to [111], the Klein – Gordon equation belongs to equations of hyperbolic type, therefore, in order for the problem to be complete, it is necessary that two initial conditions be set. It is assumed that at the initial moment of time the displacement and the time derivative of the displacement are equal to zero. Thus, before the onset of the external force, there were no vibrations in the rod. In addition, we will assume that the load applied to the rod can be described by the Dirac delta function. Since the rod in the auxiliary problem is considered semi-infinite, after the impact the waves will propagate along the rod and will not return from infinity.

Mathematically, this problem can be described as follows (assuming $\rho = 1$kg/m^3):

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} - a^2 \dfrac{\partial^2 u}{\partial x^2} + b^2 u = 0 \\ u|_{t=0} = 0 \\ \left.\dfrac{\partial u}{\partial t}\right|_{t=0} = 0 \\ \left.\dfrac{\partial u}{\partial x}\right|_{x=0} = -\dfrac{1}{a^2}\delta(t). \end{cases} \tag{2.4}$$

The solution to problem (2.4) according to [112] is the following function

$$u_0 = \frac{1}{a} H\left(t - \frac{x}{a}\right) J_0\left(b\sqrt{t^2 - \frac{x^2}{a^2}}\right), \tag{2.5}$$

where $H(z)$ is the Heaviside unit function and $J_0(z)$ is the zeroth order Bessel function. Expression (2.5) is the fundamental solution to the Klein – Gordon equation.

Using Hooke's law, knowing the displacements, the corresponding stresses arising in a semi-infinite rod can be calculated:

$$\begin{aligned} \sigma_0(x,t) &= E\frac{\partial u}{\partial x} = \\ &= -\frac{E}{a^2}\delta\left(t - \frac{x}{a}\right) + \frac{E}{a} H\left(t - \frac{x}{a}\right) J_1\left(b\sqrt{t^2 - \frac{x^2}{a^2}}\right) \frac{bx}{a^2\sqrt{t^2 - \dfrac{x^2}{a^2}}} = \\ &= g_0(x,t) + h_0(x,t), \end{aligned} \tag{2.6}$$

where $g_0(x,t) = -\dfrac{E}{a^2}\delta\left(t - \dfrac{x}{a}\right)$, $h_0(x,t) = \dfrac{E}{a} H\left(t - \dfrac{x}{a}\right) J_1\left(b\sqrt{t^2 - \dfrac{x^2}{a^2}}\right) \dfrac{bx}{a^2\sqrt{t^2 - \dfrac{x^2}{a^2}}}$,

$J_1(z)$ – first order Bessel function.

Since the fundamental solution of the Klein–Gordon equation is known, using Duhamel's formula a solution to the equation with other boundary conditions is founded. Let solve the problem when an arbitrary force is applied to the end of a semi-infinite rod. In this case, the system of equations takes the following form

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} - a^2 \dfrac{\partial^2 u}{\partial x^2} + b^2 u = 0 \\ u\big|_{t=0} = 0 \\ \left.\dfrac{\partial u}{\partial t}\right|_{t=0} = 0 \\ \left.\dfrac{\partial u}{\partial x}\right|_{x=0} = -\dfrac{1}{a^2} f(t) \end{cases}, \tag{2.7}$$

where $f(t)$ is an arbitrary function. The solution to problem (2.7) according to Duhamel's formula is the following expression for displacements

$$u(x,t) = \int_0^t u_0(x, t-s) f(s) ds. \tag{2.8}$$

It is assumed that the deformations in the rod are elastic; similarly to the previous case, Hooke's law is applied and the corresponding stresses arising in the rod under the influence of an arbitrary force $f(t)$ is found

$$\sigma(x,t) = -\frac{E}{a^2} f\left(t - \frac{x}{a}\right) + \int_0^t h_0(x, t-s) f(s) ds. \tag{2.9}$$

The auxiliary problems discussed above make it possible to solve the problem of finding deformations and stresses in a rod of finite length, surrounded by an elastic environment, under the action of impulse loads. Figure 2.3 shows a model of a finite rod. As before, it is assumed that all cross sections of the rod have the same shape and are subject to the same influences from the environment, so let us consider the one-dimensional problem of longitudinal vibrations.

Let the length of the rod be l. Let direct the axis Ox along the axis of the rod; take the left end of the rod as the origin of coordinates. Let us assume that an arbitrary dynamic force $f(t)$ is applied to the left end of the rod, and the second end of the rod is stress-free.

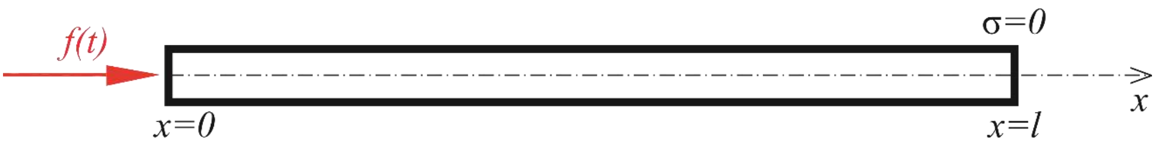


Figure 2.3. Finite length rod model

Mathematically, this displacement problem is written in the following form

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} - a^2 \dfrac{\partial^2 u}{\partial x^2} + b^2 u = 0 \\ u|_{t=0} = 0 \\ \left.\dfrac{\partial u}{\partial t}\right|_{t=0} = 0 \\ \left.\dfrac{\partial u}{\partial x}\right|_{x=0} = -\dfrac{1}{a^2} f(t) \\ \left.\dfrac{\partial u}{\partial x}\right|_{x=l} = 0 \end{cases} . \qquad (2.10)$$

The last condition is the free end condition. The problem of wave reflection from the free boundary of a rod is considered.

When solving problem (2.10) and subsequent calculation of stresses according to Hooke's law, it turns out that the stresses arising in the rod are equal to the sum of two waves, direct and reflected, and are expressed by the formula

$$\sigma(x,t) = \sigma_-(x,t) + \sigma_+(x,t). \qquad (2.11)$$

When a compressive force is applied, a direct compression wave will first propagate along the rod, expressed by the following formula

$$\sigma_-(x,t) = -\frac{E}{a^2} f\left(t - \frac{x}{a}\right) + \int_0^t h_0(x, t-s) f(s) ds, \qquad (2.12)$$

where $h_0(x,t) = \dfrac{E}{a^3} H\left(t - \dfrac{x}{a}\right) J_1\left(b\sqrt{t^2 - \dfrac{x^2}{a^2}}\right) \dfrac{bx}{\sqrt{t^2 - \dfrac{x^2}{a^2}}}$.

As soon as the direct wave reaches the free edge of the rod, it will be reflected from it and change its sign to the opposite. Initially a compression wave after reflection will become a tension wave

$$\sigma_{+}(x,t)=+\frac{E}{a^2}f\left(t-\frac{2l-x}{a}\right)-\int_0^t h_1(x,t-s)f(s)ds, \qquad (2.13)$$

where $h_1(x,t)=\frac{E}{a^3}H\left(t-\frac{2l-x}{a}\right)J_1\left(b\sqrt{t^2-\frac{(2l-x)^2}{a^2}}\right)\frac{b(2l-x)}{\sqrt{t^2-\frac{(2l-x)^2}{a^2}}}$.

From the obtained solution it is clear that when the wave is reflected from the free edge of the rod, the initial compressive stresses become tensile, since the initial impulse changes its sign from negative to positive. Many materials have high compressive strength, i.e. withstand high compressive stresses, but are easily failure under tensile loads. Therefore, when a wave is reflected from the free boundary of the rod, spall fracture is possible. One example of a material that has these properties is concrete.

2.2 Examples of the impact force action on a rod

Let us consider examples of wave propagation along the entire length of a rod, when a force $f(t)$ is applied to one of the rod ends. The rod length equals l.

$$f(t)=\begin{cases} P\,\mathrm{Sin}\left(\frac{\pi t}{t_0}\right), 0\le t\le t_0 \\ 0, \quad t\ge t_0 \\ 0, \quad t\le 0 \end{cases} \qquad (2.14)$$

where P is the amplitude of the applied load. The force graph $f(t)$ is shown in Figure 2.4.

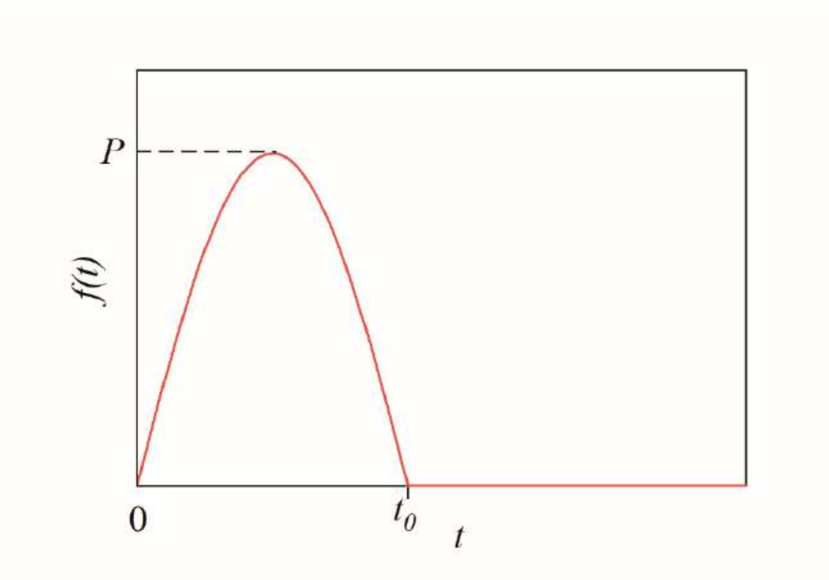


Figure 2.4. Graph of the force applied to one rod end

The classic case of wave propagation (a triangular pulse) along a semi-infinite rod and wave reflection from the free boundary of the rod in the absence of an additional environment were considered in [55], [60]. When no resistance forces act on the rod, the resistance coefficient of the medium is $b = 0$. The equation describing the longitudinal vibrations of the rod is transformed from the Klein – Gordon equation (2.3) into the wave equation.

In this case, when the wave propagates along the rod, the pulse does not change its shape; when the pulse is reflected from the free boundary of the rod, the shape of the pulse does not change, but the sign changes, the initially compression wave turns into a tension wave.

Let us consider waves in a rod of finite length located in an elastic environment. Let the same force $f(t)$ act on the rod as in the previous case, expressed by formula (2.14). Depending on the duration of the external pulse and the resistance coefficient of the medium b, the longitudinal vibrations occurring in the rod exhibit different characters. Three options for wave propagation can be distinguished. For definiteness, let assume that the speed of wave propagation in the rod is $a = 1$ m/s, Young's modulus is $E = 1$ Pa, and the length of the rod is $l = 50$ m.

1. For sufficiently large duration values t_0 and a small value of the medium resistance coefficient b (for example, at $t_0 = 10$ s, $b = 0.1$ s^{-1}), then the wave in the rod will propagate as shown in Figure 2.5. From the results of numerical experiments it is clear that after applying an external load, a compression wave begins to propagate along the rod (Figure 2.5, a). Even when the wave passes directly behind, a stretching «tail» begins to form (Figure 2.5, b).

Then the wave reaches the free boundary of the rod and is reflected from it (Figure 2.5, c, d); upon reflection, the original compression wave turns into a tension wave and is combined with a stretching «tail». As a result, the amplitude modulus of the total tensile stress is greater than the amplitude modulus of the initial compressive pulse (Figure 2.5, e). It is in this situation that the probability of spall fracture is high, since by acting on the rod with a load initially less than the tensile strength of the material, after the combination of the direct and reflected waves, tensile stresses may exceed the tensile strength.

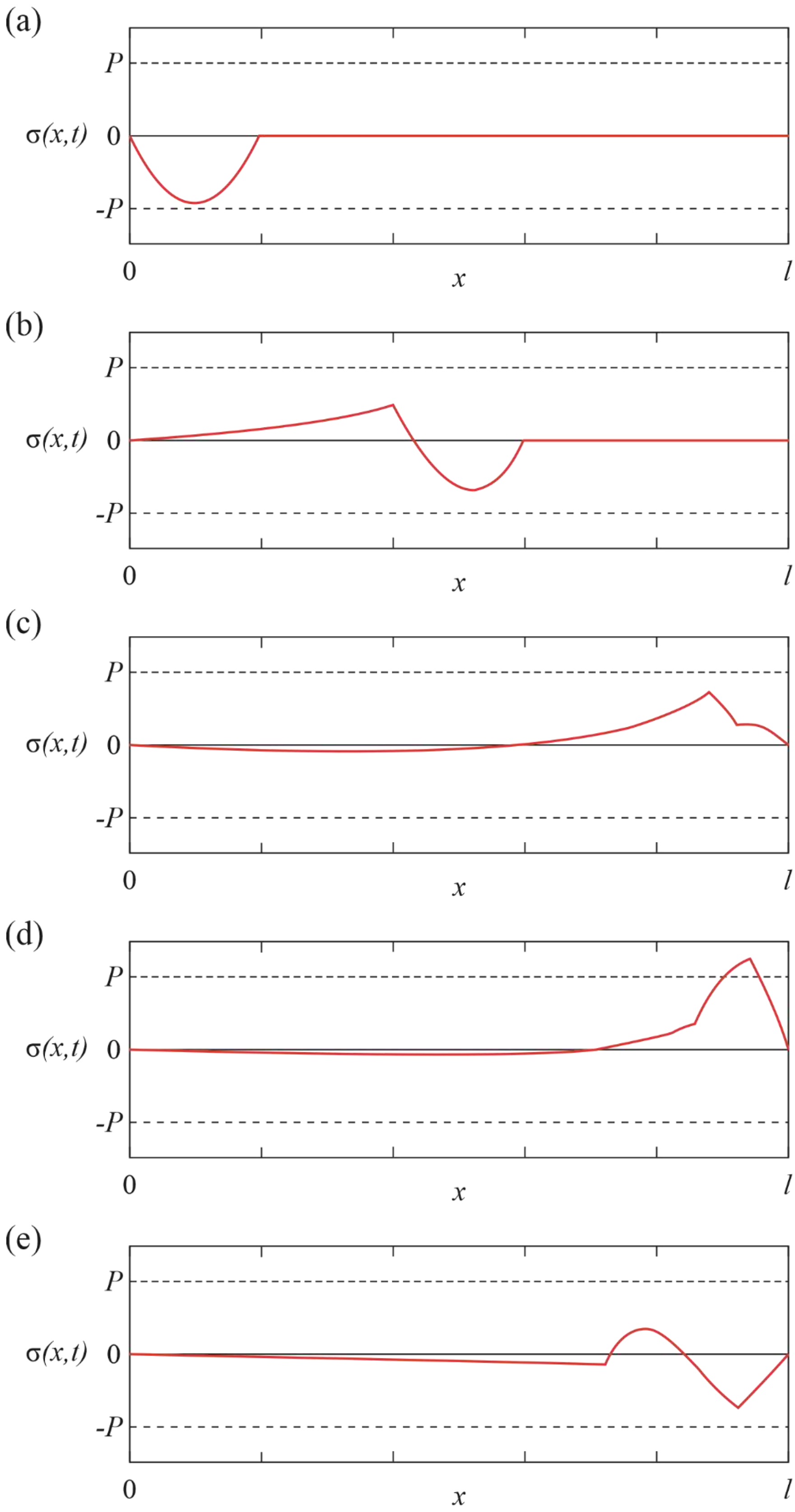


Figure 2.5. Wave propagation in the rod at $t_0 = 10$ s, $b = 0.1$ s^{-1}

2. Consider the case when the duration of the initial pulse is $t_0 = 38$ s, and the resistance coefficient of the medium is $b = 0.5$ s^{-1}.

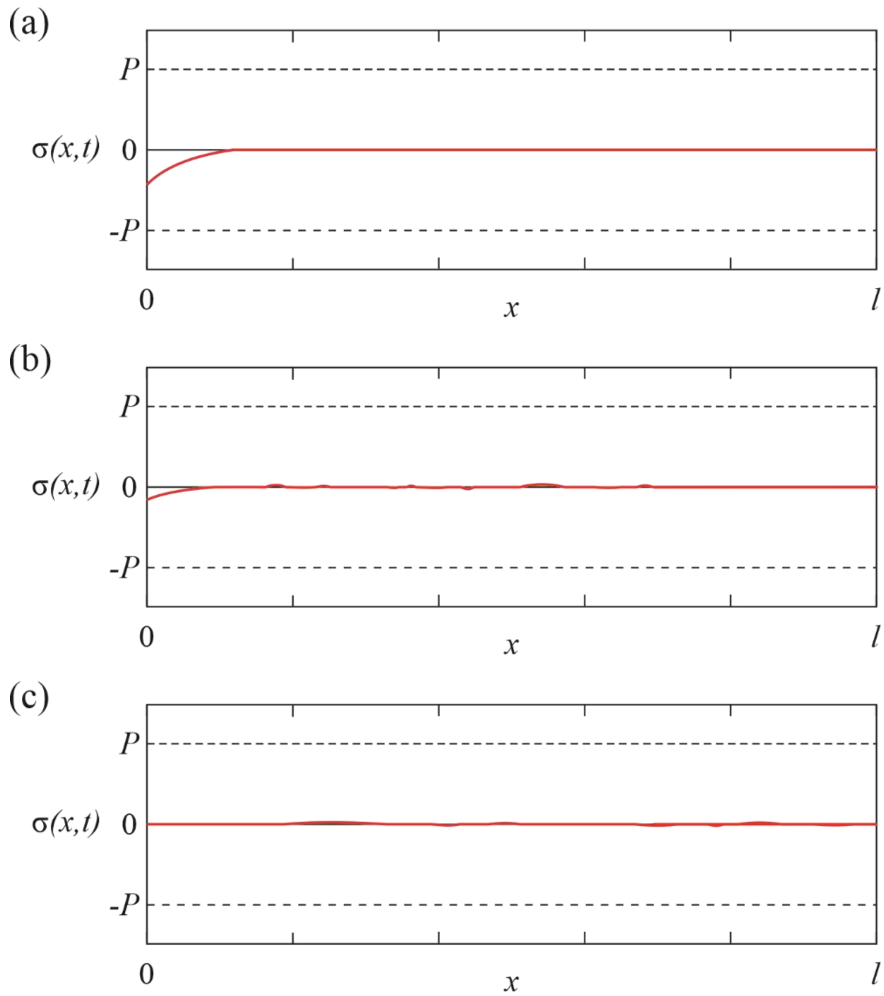


Figure 2.6. Wave propagation in the rod at $t_0 = 38$ s, $b = 0.5$ s^{-1}

In this case, the compression wave begins to propagate along the rod (Figure 2.6, a, b), but then the initial pulse is almost completely damped by the medium (Figure 2.6, c). Thus, under loads with a sufficiently long duration of impact, the environment has a damping effect. Therefore, a rod immersed in a medium with a resistance coefficient $b \geq 0.5$ s^{-1} can withstand high loads, since the medium dampens the resulting vibrations.

3. Let the pulse duration is $t_0 = 3$ s, and the resistance coefficient of the medium is $b = 0.5$ s^{-1}.

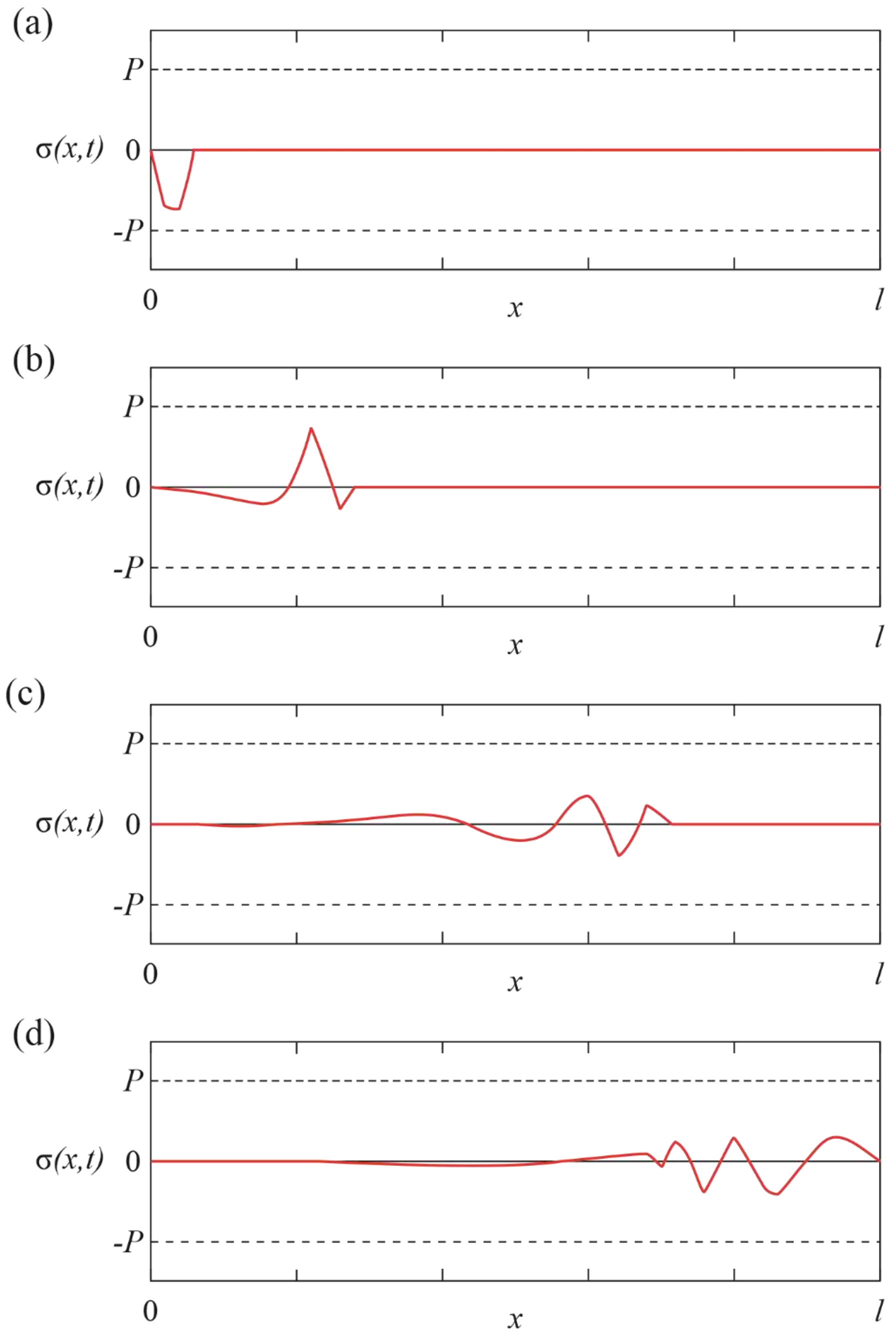


Figure 2.7. Wave propagation in the rod at $t_0 = 3$ s, $b = 0.5$ s^{-1}

A force similar to the previous cases is applied to the end of the rod, but with a shorter duration (Figure 2.7, a). Due to the fact that the resistance coefficient of the medium b is greater compared to the first case, the medium has a more significant influence on the wave propagation in the rod. The initial pulse is divided into many compression and tension waves, so in this case we observe the dispersion phenomenon (Figure 2.7, b, c, d). Dispersion is the dependence of velocity on wavelength and, as a consequence, distortion of the original pulse shape [110].

Calculations show that, depending on the medium resistance coefficient value b and the pulse duration, different patterns of wave propagation are observed. Thus, the environment can act both as a damper and dampen vibrations, or, conversely, enhance the resulting disturbances. These effects demonstrate that the medium has a significant influence on the wave processes in the rod. Unlike the case when there is no environment, in the presence of resistance forces acting on the rod from the environment, when waves propagate, the initial impulse does not retain its shape.

These unique properties can be explained by considering the dispersion relation of the Klein – Gordon equation (2.3). As is known, the dispersion relation in wave theory is the relationship between the frequency and the wave vector. To derive this relationship, let substitute $u = e^{ikx}e^{i\omega t}$, where k is the wave number and ω is the frequency, into equation (2.3). Then the following expression is obtained

$$\omega(k) = \pm\sqrt{a^2k^2 + b^2} \qquad (2.15)$$

A graph of frequency versus wave number is shown below in Figure 2.8.

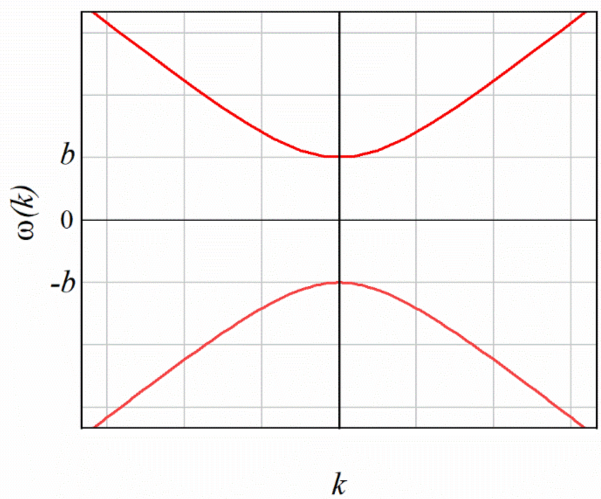


Figure 2.8. Frequency versus wave number graph

It turns out that the ratio of the oscillation frequency ω to the wave number k is not a constant value, it follows that wave dispersion occurs. In addition, equality (2.15) follows that the wave number is related to frequency according to the formula

$$k = \pm \frac{1}{a}\sqrt{\omega^2 - b^2}. \tag{2.16}$$

When performing the inverse Fourier transform, the following integral is obtained

$$u(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} u^F(k,t)e^{ikx}e^{i\omega t}dk. \tag{2.17}$$

From formula (2.16) it follows that for $\omega < b$, that is, for small values ω (i.e., for large values of the acting force duration t_0), k is a complex number, which means that according to formula (2.17), displacements, and therefore stresses, will rapidly decrease. This is observed in the considered case 2. At large values $\omega > b$, i.e. at small values of duration t_0, according to formula (2.17), oscillations is observed, as in case 3.

2.3 Fracture in the Klein – Gordon wave field

Before putting any structure into operation, it is necessary to examine its strength characteristics and study possible fracture options to ensure safety. Let us consider possible scenarios for the fracture of finite length rod located in an elastic environment.

As was described earlier, under the action of a compressive force of the form (2.14), a direct compression wave first travels along the rod; when it reaches the free edge of the rod, it is reflected from it and becomes a tension wave. The initially compressive wave becomes a tensile wave after reflection. The resulting stress that arises in the rod is equal to the sum of two waves, direct and reflected.

Usually, when the wave is reflected from the stress-free edge of the rod, and the initial compressive stresses become tensile, spall failure in the rod occurs [113]. On the other hand, according to calculations, in the presence of other material around the rod, even when the wave passes directly through the rod, tensile stresses arise, which can lead to the rod fracture. Therefore, rods in various media can be fractured not only as a result of the wave reflection from the free boundary (spall fracture), but also as a result of the direct initially compressive wave propagation.

Let us calculate the threshold pulses, namely the threshold force amplitudes that disturbs the rod, at which the rod is fractured. Threshold pulses are those pulses that, at a fixed duration, have the smallest failure amplitude [114].

To study the threshold amplitude dependence on the duration of the impact, we use the incubation time criterion to predict the conditions for the brittle fracture initiation of the rod under the action of an applied dynamic load [56], [57], [58]. It is assumed that the rod under study does not have initial imperfections, therefore, according to [56], it is possible to use the criterion of incubation time of destruction.

In this case, the criterion is written in the following form

$$\int_{t-\tau}^{t} \sigma(s)ds \le \sigma_c \tau, \tag{2.18}$$

where τ – the incubation time, σ_c – the critical stress value in the sample (these are specified values determined experimentally for each sample).

To estimate the threshold value of the force amplitude, let use the incubation time fracture criterion (2.18). Let us introduce dimensionless quantities: $T = \frac{t}{\tau}, T_0 = \frac{t_0}{\tau}, X = \frac{x}{\tau a}, L = \frac{l}{\tau a}$, then the for the incubation time criterion takes the form: $\int_{T-1}^{T} \sigma(s)ds \le \sigma_c$.

The results of calculating the threshold fracture amplitudes P^* on T_0 (the loading pulse duration) under the force action of the form (2.14) for the cases of $b =$ 0 s^{-1}, $b = 0.1$ s^{-1}, $b = 0.5$ s^{-1} are presented in Figure 2.9. The $b = 0$ s^{-1} case corresponds to the classical rod fracture case in the absence of additional resistance forces from the medium; in this case, a monotonic threshold amplitude dependence on duration is observed, which is typical for dynamic material fracture.

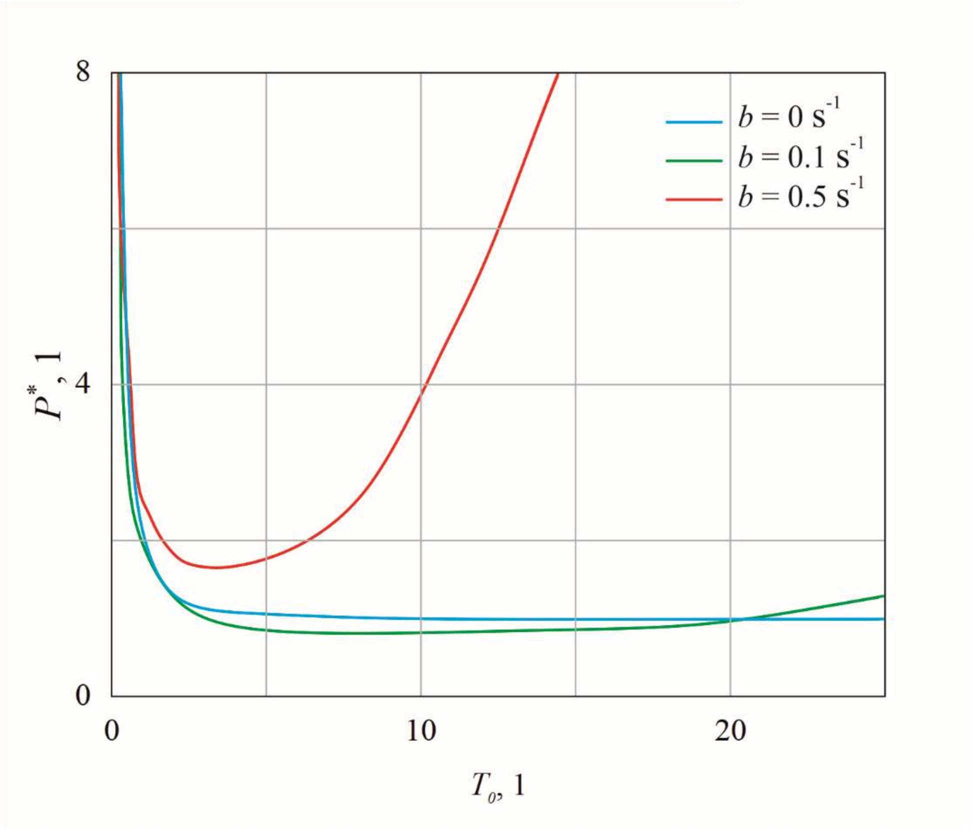


Figure 2.9. Threshold amplitude dependence on impact duration

The highest stress in the rod at $b = 0.1$ s^{-1} occurs after the composition of direct and reflected waves, i.e., spall fracture occurs. Figure 2.9 shows that the curve of the threshold amplitude dependence on the duration of the rod at a small resistance coefficient ($b =$ 0.1 s^{-1}) is similar to the curve at $b = 0$ s^{-1}, when no resistance forces act on the rod (classical case). The classic case of a triangular symmetrical wave propagation in a rod in the absence of additional influences from the medium was considered by Y. V. Petrov [115]. However, unlike the classical case, at $b = 0.1$ s^{-1} the graph P^* versus T_0 non-monotonic, has a minimum point, and, therefore, optimal pulse mode is possible, in which the rod fracture occurs under the action with the smallest amplitude. Thus, pulses with amplitudes that would not be breaking in the absence of additional forces acting on the rod from the medium can cause the rod fracture in the presence of an elastic medium

When the resistance coefficient of the medium increases to $b = 0.5$ s^{-1}, the maximum tensile stress in the rod occurs even during the passage of a direct wave, before reflection from the free boundary of the rod. Similar to the previous case, the strength curve (Figure

2.9) has a minimum point, therefore, again there is an optimal loading mode. The non-monotonic nature of the curve is due to the fact that fracture occurs at different times. It turns out that with short pulses, fracture in the rod occurs even when the wave passes directly through the rod, and with increasing duration of load, rod spall fracture is observed.

It is worth noting that, as shown earlier, the medium can either have a damping effect on the wave propagation in the rod and make the structure stronger, or cause rod fracture, enhancing the initial impulse. Therefore, the study of strength characteristics and the calculation of stresses and deformations under dynamic loads in rods located in various environments have significant practical applications.

2.4 Comparison with experimental data

To prevent fracture occurring in the rod, it is necessary to conduct numerical and experimental studies to study the strength characteristics of materials. A large number of results were obtained from the study of materials that are not affected by an additional environment. But at the moment, not much experimental data is known that describes the deformations that occur in rods immersed in any environment.

For example, a number of experiments were carried out to measure the deformation profile in a rod under dynamic influence (Smirnov I. V., 2020). The rod is made of PMMA (plexiglass). Two cases are considered. The first is when the rod is not affected by resistance forces from the environment (Figure 2.10, a). The second is when there is silicone around the rod (Figure 2.10, b). The size of the plexiglass rod in which the deformation profile was measured was 10x10x400 mm.

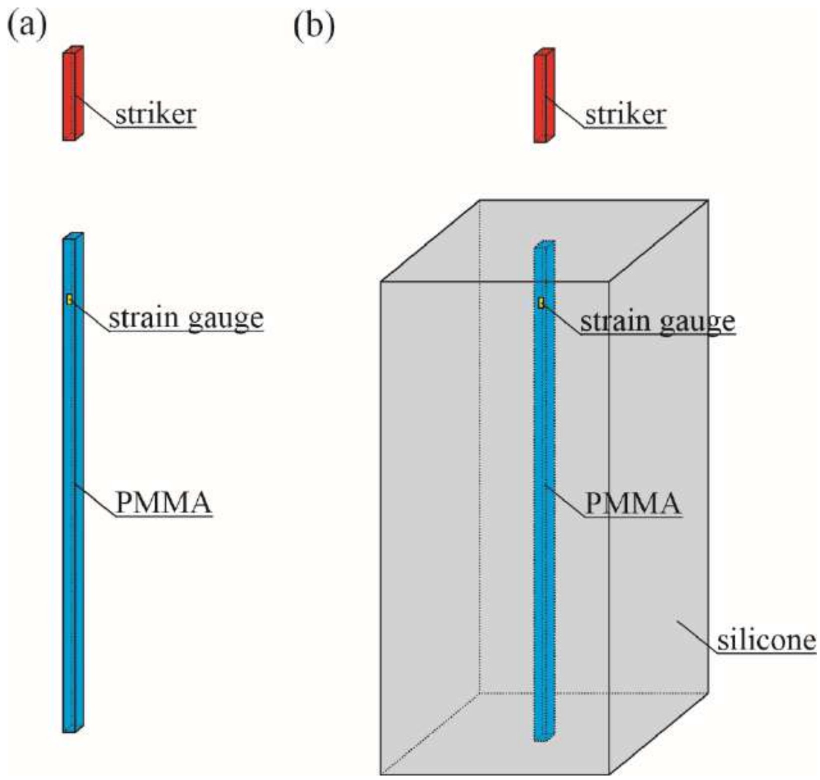


Figure 2.10. Scheme of the experiment (Smirnov I. V., 2020)

During the experiment, the striker, made, like the rod, from PMMA, fell onto the rod under the influence of gravity, causing waves to propagate in the rod. A strain gauge was located on the rod at a distance of 5 cm from the edge to which the force was applied, using which the strain profile in the rod was measured.

The approximation graph of the initial impulse taken for calculations is shown in Figure 2.11. The applied pulse has a «tail»; this form of initial action was chosen in order to take into account the reflection of the wave packet from the side surfaces of the rod in a real experiment. Since, according to [116], in order to use a one-dimensional model to describe an actually three-dimensional object under dynamic loading, it is often necessary to consider the influence of the rod boundaries on the wave processes.

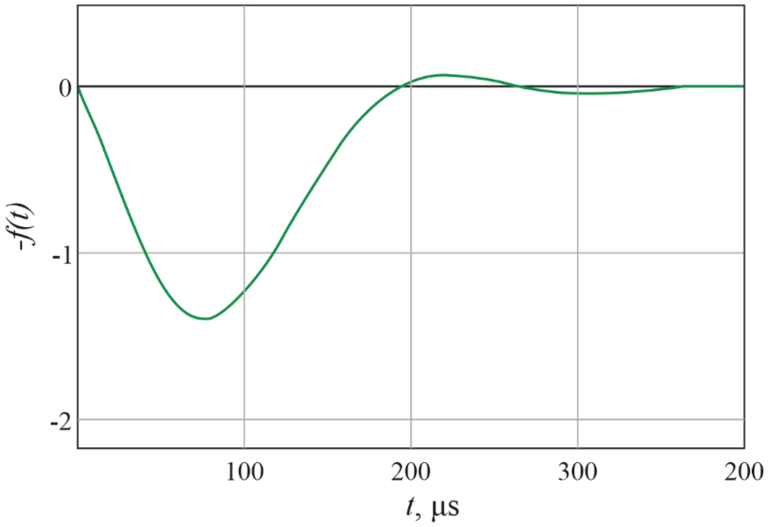


Figure 2.11. Impulse acting on the rod

A comparison of the calculation results with experimental data obtained in the absence of an additional environment is shown in Figure 2.12. The theoretical and experimental strain profiles (green and black lines, respectively) occurring in the rod at a distance of 5 cm from the rod edge to which the force is applied are shown.

According to the experimental results, longitudinal vibrations in the rod are clearly observed; the initial pulse is slightly damped after several reflections from the free ends of the rod. When comparing experimental data with the analytical solution obtained earlier, it turns out that in the absence of an environment there is almost complete agreement between calculations and experimental data. But the model constructed during the work does not take into account the attenuation that occurs in the rod over time, but the periods of oscillation completely matches.

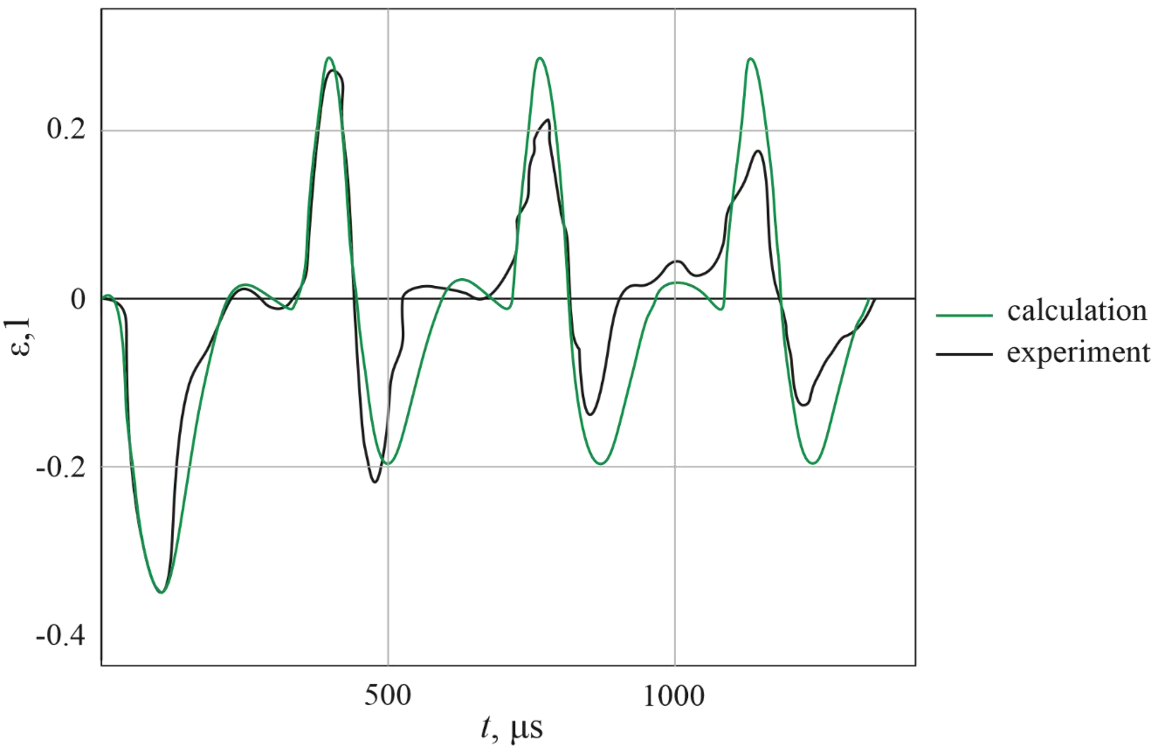
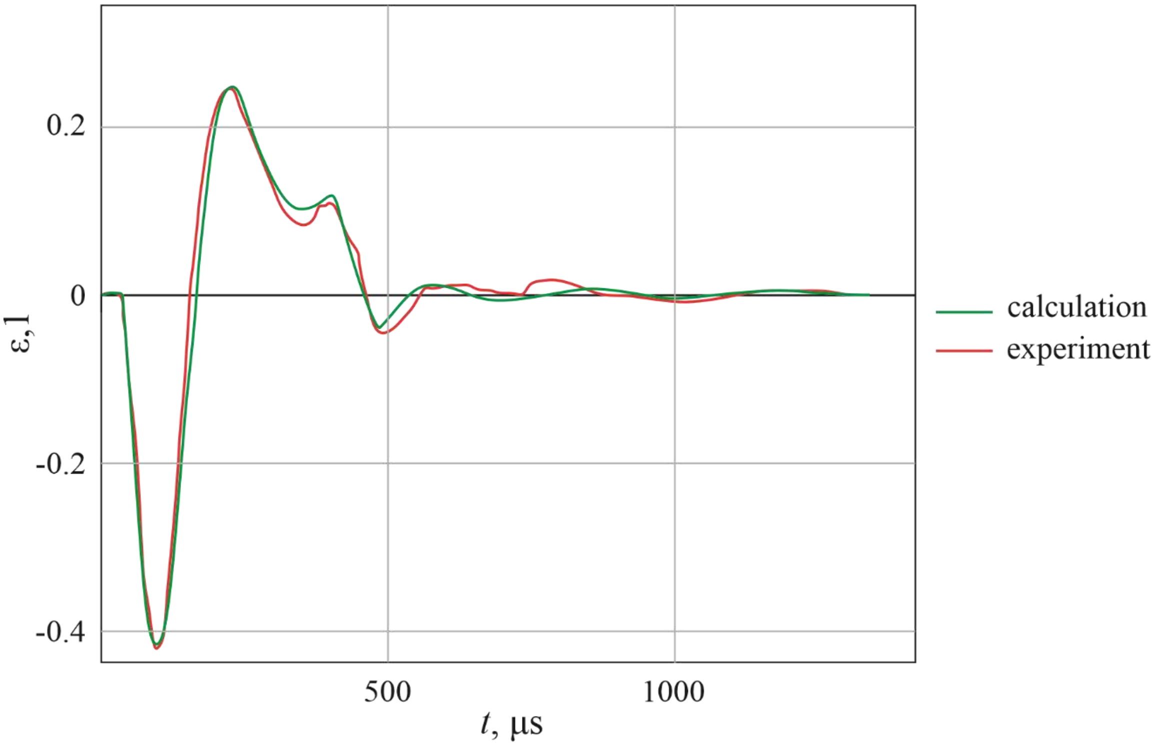


Figure 2.12. Deformation profiles in the rod near (5 cm) to the impact site in the absence of a medium (experimental data (Smirnov I. V., 2020) and theoretical calculation)

Figure 2.13 shows the analytical calculation and experimental results of deformations in the cross section of a rod immersed in silicone, also located at a distance of 5 cm from the edge to which the external force is applied. The wave formed in the rod, after several oscillations, is rapidly extinguished by the medium, which confirms the strong influence of the environment on the deformations occurring in the rods.



Drawing. 2.13. Deformation profiles in the rod near (5 cm) to the impact site in the presence of the environment (experimental data (Smirnov I. V., 2020) and theoretical calculation)

After analyzing the environment influence on the waves propagating in the rod, it was hypothesized that the environment has such a strong effect on the deformations in the rod that the wave is extinguished even before it reaches the free rod boundary, which is exactly what the experimental data indicate. Therefore, if it is assumed that the wave is not reflected from the free edge of the rod, the best agreement between experiment and calculations for a medium resistance coefficient of $b = 0.02$ s^{-1} can be obtained.

The results obtained demonstrate the applicability of the constructed model of longitudinal vibrations in a rod immersed in an elastic environment. Since there is a qualitative agreement between the theoretical calculation and the experimental data.

2.5 Conclusions to the chapter 2

The effects of dynamic deformation and fracture of a rod located in an elastic environment have been studied. The deformations and stresses that arise in a rod undergoing longitudinal vibrations under dynamic loading are found. The propagation and reflection of waves in a rod having a finite length is studied. The influence of the environment on the rod fracture is considered, and the possibility of spall fracture is shown.

The mathematical model is constructed to describe these processes. The resulting equation for the longitudinal vibrations of the rod turned out to be the well-known Klein – Gordon equation. An analytical solution to this problem is found. Graphs of wave propagation in the rod were constructed for different values of the medium resistance coefficient. At small values of the resistance coefficient of the medium $b = 0.1$ s^{-1}, after the wave is reflected from the free boundary of the rod, the stress in the rod becomes greater in amplitude compared to the initial load, thus, in this case, rod spall fracture is possible as a result of the interference of direct and reflected waves. When the characteristics of the environment change, leading to an increase in the resistance coefficient of the environment ($b \approx 0.5$ s^{-1}), the nature of wave propagation depends on the load duration. With short pulses, oscillations are observed in the rod, and the wave packet breaks up into many compressive and tensile waves, thus, wave dispersion appears. With longer exposures, the waves are completely damped by the environment, i.e. the environment acts as a damper.

The rod fracture under dynamic influence was studied using the incubation time criterion. It was revealed that the rod fracture can occur both during the direct passage of a wave along the rod, and as a result of the spall. Curves of the strength time dependence were constructed, it was shown that they can have a non-monotonic character, which allows us to speak about the presence of optimal characteristics of the shock-pulse effect causing spall fracture.

The results of theoretical calculations are compared with experimental data. It is shown that the model of wave propagation in a rod constructed in this work makes it possible to qualitatively describe the wave processes of longitudinal vibrations both in the presence of an elastic environment and in the absence of an additional external environment.

Many structures consisting of rods are located in different environments, so the construction of models that describe the deformations and stresses arising in rods under dynamic loads is of particular interest, both from a scientific and practical point of view. By correctly selecting the materials from which the rods are composed, and taking into account the influence of the environment in which these rods would experience loads, it is possible to achieve either effective rod fracture, or, conversely, strengthening of the structure.

Chapter 3. Influence of external factors on the dynamic material fracture toughness

Known experimental data on the study of the additional external factors influence on the dynamic fracture toughness of materials are analyzed based on the incubation time criterion. The dependence of fracture toughness on the stress intensity rate for hydrostatically compressed and pre-heat-treated granite samples, as well as for thermally treated barite and standard cement mortars, was studied. Based on experimental data, the values of incubation times were estimated. The effect of fracture toughness inversion is discussed, which consists in the fact that when comparing two material samples processed at different temperatures, one sample may have lower fracture toughness under quasi-static loads, but will have higher values of the same characteristic compared to the second sample in dynamics, i.e. under high speed impact.

The results presented in this chapter were published in [117], [118].

3.1 Dynamic fracture toughness calculation

The structure-time approach makes it possible to describe fracture processes in a wide range of dynamic problems, including problems of crack propagation [119].

The incubation time criterion for determining the dynamic fracture toughness takes the following form [66]

$$\frac{1}{\tau_K}\int_{t-\tau_K}^{t} K_I(t')dt' \le K_{Ic}, \tag{3.1}$$

where K_{Ic} is the static limit of stress intensity factor of fracture mode I, and τ_K is the fracture incubation time corresponding to the limiting condition for fracture toughness.

The stress intensity factor under static external influence K_{Ic} is a characteristic of the material; this value determines the amount of energy required for the emergence of new surfaces in the material. In the static case, if in the fracture region the stress intensity factor reaches the value K_{Ic}, then a crack appears and propagates in the material until the value K_I becomes less K_{Ic} [120]. The fulfillment of equality in expression (3.1) determines the condition for initiating crack propagation in a medium with a crack or both static and dynamic loads.

One of the possible tests to determine the dynamic material fracture toughness is an experiment on the dynamic fracture of a semicircular granite sample with a notch (Figure 3.1, a) using a system of split Hopkinson rods [121]. For a given sample shape, fracture occurs in mode I: the crack edges move perpendicular to the plane of crack propagation in the sample.

In the experimental results considered in this work, the dynamic stress intensity factor is defined as the maximum value of the stress intensity factor in the sample over the entire loading history. When considering the external force dependence on time in these experiments, there is a time range in which the external force changes according to a linear law. In addition, it is believed that the material fracture also occurs at the stage of increasing force. Therefore, for the calculations in this work, it is assumed that the external force can be determined by the following formula:

$$P(t) = \dot{P}tH(t), \tag{3.2}$$

where $\dot{P}$ is the rate of force increase and $H(t)$ is the Heaviside function.

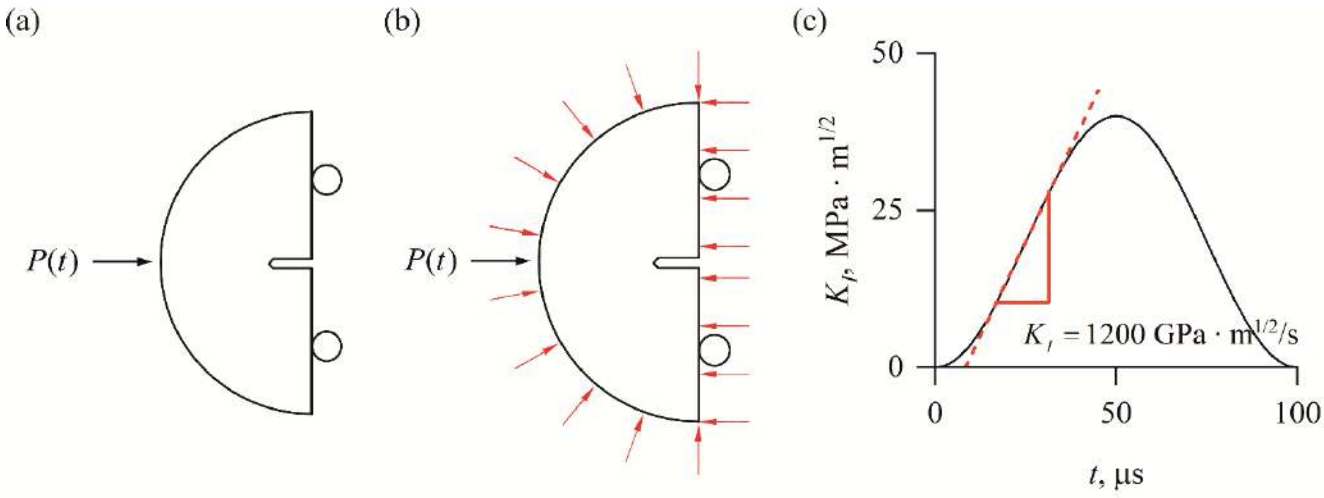


Figure 3.1. Experimental scheme for determining fracture toughness (a), experimental scheme for determining fracture toughness in the presence of hydrostatic pressure (b), the stress intensity rate determination (c)

In experimental studies on the dynamic fracture of a semicircular specimen with a notch, the stress intensity factor according to [121] is expressed by the following formula

$$K_I(t) = \frac{P(t)S}{BR^{3/2}} Y\left(\frac{a}{R}\right), \tag{3.3}$$

where $P(t)$ is the external force, R is the sample radius, B is the sample thickness, S is the distance between support pins, $Y(a/R)$ is a dimensionless function depending on the geometric parameters, which can be obtained numerically, for example, in the ANSYS application package [121], [122] (for example, when $a/R = 0.5$ $Y(a/R) = 0.5037 + 3.4409a/R - 8.0792(a/R)^2 + 16.489(a/R)^3$). It turns out that in these experiments the relationship between the stress intensity factor and the external dynamic force acting on the sample is linear, the proportionality coefficient depends on the sample geometry and the distance between the support pins. Accordingly, for calculations, taking into account the dynamic force expressions (3.2), the stress intensity factor is calculated using the formula:

$$K_I(t) = \dot{K}_I t H(t) \tag{3.4}$$

where $\dot{K}_I$ is the stress intensity rate.

The stress intensity factor change rate in experimental studies is equal to the slope of the linear part of the graph of the change in stress intensity factor over time (Figure 3.1, c).

Applying the fracture incubation time criterion (3.1) to find the dynamic stress intensity factor K_{Id} dependence on the rate of change in fracture toughness $\dot{K}_I$, we obtain the following expression:

$$K_{Id}(\dot{K}_I) = \begin{cases} K_{Ic} + \frac{\tau_K}{2}\dot{K}_I, \frac{K_{Ic}}{\dot{K}_I} \geq \frac{\tau_K}{2}, \\ \sqrt{2K_{Ic}\tau_K\dot{K}_I}, \frac{K_{Ic}}{\dot{K}_I} < \frac{\tau_K}{2}. \end{cases} \quad (3.5)$$

To construct theoretical dependencies (3.5), it is necessary to know the incubation time and static fracture toughness. The incubation time is calculated by the least squares method based on the results of an experiment on the dynamic fracture of samples [123]; the value of static fracture toughness is considered a material property and is also determined from known experimental data.

3.2 Dynamic fracture toughness of thermally treated granite

Nowadays, the mining industry is developing rapidly, so mining and mineral extraction work is carried out at great depths. Rocks in natural conditions are under the influence of static hydrostatic pressure. As the depth of the rock increases, the level of hydrostatic pressure grows and has an increasing impact on the rock strength characteristics. In addition, rocks are subject to dynamic fracture caused by impulse loads such as explosions and seismic activity.

The static fracture toughness and strength of rocks is well studied. These characteristics tend to grow with increasing hydrostatic pressure [101], [102]. However, the dynamic response of rocks under different hydrostatic pressures is of particular interest, since understanding the mechanisms of rock failure is a key aspect for solving many problems arising in modern rock mechanics and geoengineering. Dynamic loads that cause failure and crushing of rocks are the essence of many industrial processes in the extraction and further processing of minerals. In this regard, it is important to be able to predict the material fracture caused by dynamic impacts during mining and blasting [120].

Rock fracture occurs as a result of the combined effect of accumulated deformation energy and external dynamic disturbance [124]. Also, the nature of fracture is influenced by many factors, such as temperature [16], pressure [106], water saturation [20], material anisotropy [125] and internal stresses [126]. With increasing depth of underground mining, it became clear that the mechanical properties of rock masses depend not only on dynamic load and hydrostatic pressure, but also on temperature [97], [98]. In addition, with the development of geothermal energetics, the amount of geothermal energy produced increases, since geothermal energy is an environmentally friendly, renewable source of energy. With the development of the nuclear industry, radioactive waste is being buried. These processes require an understanding of the mechanisms of dynamic strength and fracture toughness of rocks previously subjected to heat treatment [97]. In this regard, it is necessary to assess the possibility of the occurrence and spread of dynamically unstable fracture, as well as to identify explosive areas. It is necessary to take into account the temperature and speed characteristics of the rock fracture mechanism.

According to experimental studies, the classical strength theory is not applicable to determine the dynamic material fracture toughness, since the material fracture under high-speed loads differs from the case of static impact [127], [128], [129], [130], [131]. In addition, the influence of various external factors, such as hydrostatic pressure [106], water saturation [1], [61], temperature [16], has a significant impact on the dynamic fracture toughness of rocks. The value of dynamic fracture toughness depends on the loading history, while static fracture toughness is considered a constant value and characteristic of the material. When the load duration, the loading pulse profile shape, the sample geometry, and the applying the load method change the rate dependences of the material dynamic fracture toughness.

Using the incubation time criterion in [68], [119], based on experimental results, it is shown that dynamic fracture toughness is not a material property. In addition, it is revealed that the structural-temporal approach makes it possible to describe the effects inherent in dynamic fracture processes, for example, the fracture toughness dependence on fracture time, as well as the unstable behavior of strength rate dependences. The use of the incubation time criterion is a promising method for solving problems of fracture in a wide range of loading rates.

Let consider the experimental results [16] on the dynamic fracture of a semicircular granite sample with a notch (Figure 3.1, a). In [16], the change in the dependence of dynamic fracture toughness on loading rate during tension of granite as a result of preliminary exposure to temperature was studied. Granite fracture occurs in mode I using the split Hopkinson pressure bar system.

In [16], to prepare for dynamic tests, the samples were first gradually heated to a fixed temperature, then the temperature was maintained constant for some time to uniformly heat the samples. After this, the samples were cooled naturally at room temperature (25 °C) to room temperature 25 °C. Next, these samples were subjected to dynamic influences at different loading rates.

The static granite fracture toughness values after pretreatment at various temperatures were studied in [4]. It was shown that static fracture toughness decreases with increasing temperature. Based on experimental data [4], the following static fracture toughness values were obtained for calculation (Table 3.1). Based on the results of [16], the incubation times for granite were calculated (Table 3.1).

T (°C)	25	100	250	450	600	850
K_{Ic} (MPa · m$^{1/2}$)	0.482	0.434	0.3376	0.2272	0.1488	0.1036
τ_K (μs)	340	253	376	379	457	605

Table 3.1. Values of static fracture toughness and incubation time of granite, corresponding to different temperature levels

The dynamic fracture toughness dependences on the stress intensity rate (3.5), constructed based on the results of the experiment [16], are shown in Figure 3.2. In [16], granite samples were tested under impact loads (Figure 3.1, a) by preheating the samples to temperatures of 25 °C, 100 °C, 250 °C, 450 °C, 600 °C, and 850 °C.

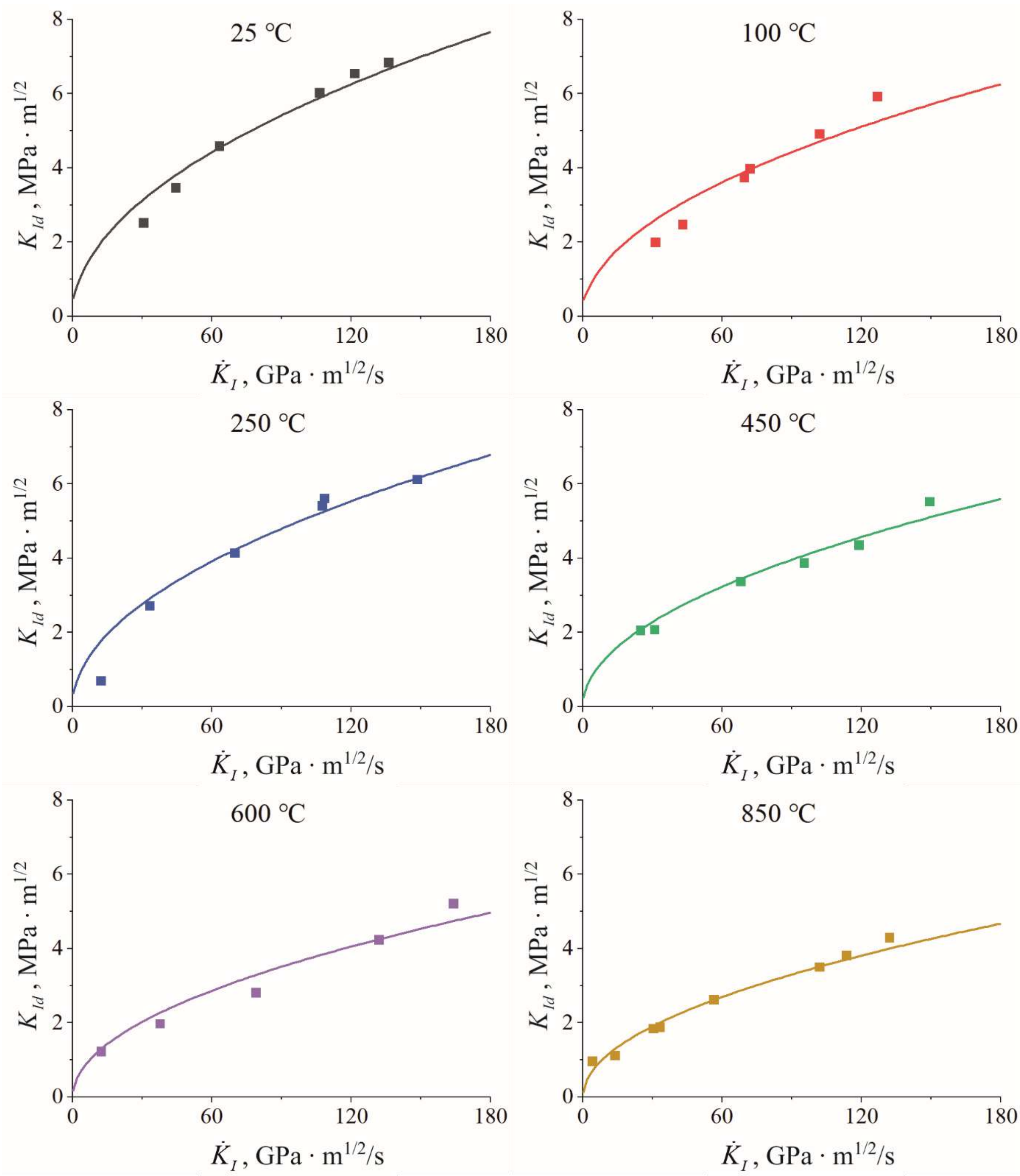


Figure 3.2. Rate dependences of the granite dynamic fracture toughness, calculated at different temperatures, experimental data [16]

There is a qualitative compliance between the experimental data and the calculated curves (3.5). With an increase in the stress intensity rate for all temperature values, the dynamic fracture toughness increases, and the higher the temperature, the smaller the grow in dynamic fracture toughness with an increase in the stress intensity rate becomes. With increasing heating temperature, a slight decrease in dynamic fracture toughness is observed, and the higher the loading rate, the more noticeable the effect of temperature becomes.

3.3 Effect of granite fracture toughness inversion

According to theoretical calculations (3.5), based on experimental data [16], it turns out that the fracture toughness rate dependence curves of the granite pre-treated at temperatures of 100 °C and 250 °C intersect at a loading rate equal to 1.46449 GPa · $m^{1/2}s^{-1}$ (Figure 3.5). Therefore, it can be concluded that under slow loads, granite processed at a temperature of 100 °C has the highest fracture toughness, but under high-speed loads, the highest fracture toughness corresponds to granite processed at a temperature of 250 °C, thus, an inversion of fracture toughness is observed.

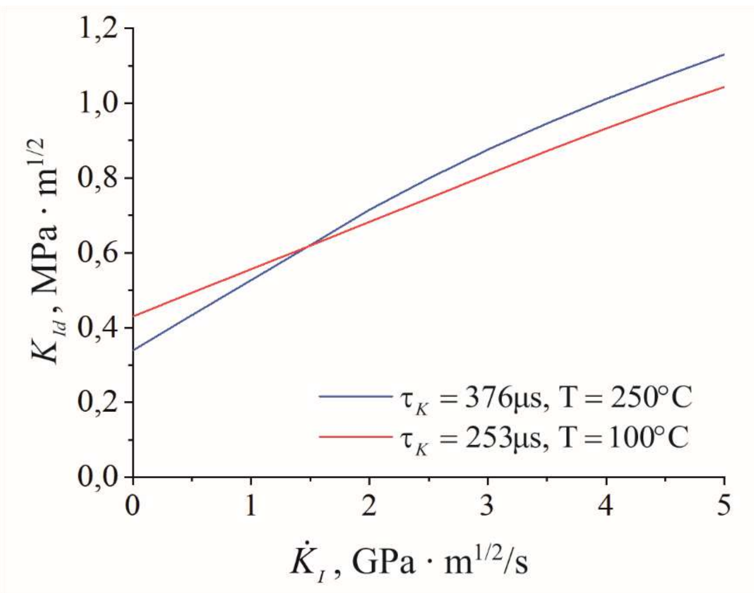


Figure 3.3. Granite dynamic fracture toughness inversion

This example shows that when calculating the crack resistance of materials and choosing on this basis a material with greater load-bearing capacity, one cannot focus only on the values of static fracture toughness; it is necessary to take into account the reaction of the material to dynamic loads. Thus, the incubation time criterion (3.1) can be used for these purposes, since it allows us to give a unified description of the dynamic material fracture for slow and fast speed ranges.

3.4 The influence of hydrostatic pressure on the granite fracture toughness

In this section, the dynamic fracture of rock in the presence of external hydrostatic pressure is analyzed using a structural-temporal approach. The rate dependences of the granite fracture toughness under the action of various levels of hydrostatic pressure is calculated.

Let us consider the experimental results presented in [106], which examines the dynamic fracture of rock, namely Laurentian granite, under the influence of static hydrostatic pressure. In this work, a series of experiments are carried out on the dynamic fracture of semicircular notched granite samples (Figure 3.1, b) using the split Hopkinson pressure bar system to measure the dynamic fracture toughness of rock at five different levels of hydrostatic pressure (0 MPa, 5 MPa, 10 MPa, 15 MPa, and 20 MPa). In the experiment, the material fracture according to mode I is realized. To implement hydrostatic pressure, two chambers were added to the split Hopkinson pressure bar system, the first exerts lateral limiting pressure on the sample, the second provides axial pressure on the rods and the sample.

The limiting value of the stress intensity factor for the case of static load according to [106] is equal to 1.5 MPa $\cdot$ $m^{1/2}$. The incubation time values are calculated based on the experimental results [106].

The rate dependences of the dynamic fracture toughness (3.5) of granite at hydrostatic pressures equal to 0 MPa, 5 MPa, 10 MPa, 15 MPa, and 20 MPa, constructed using the experimental results [106], are shown in Figure 3.4. A qualitative compliance between the theoretical calculation and experiment was obtained. The results show that with increasing hydrostatic pressure level, the dynamic fracture toughness value increases for all loading rates. This is explained by the closure of microcracks in rocks. In addition, the dynamic fracture toughness for all levels of hydrostatic pressure grows with increasing loading rate. The higher the loading rate, the greater the impact hydrostatic pressure has on the granite fracture toughness. With increasing hydrostatic pressure, an increase in the incubation time of fracture is observed.

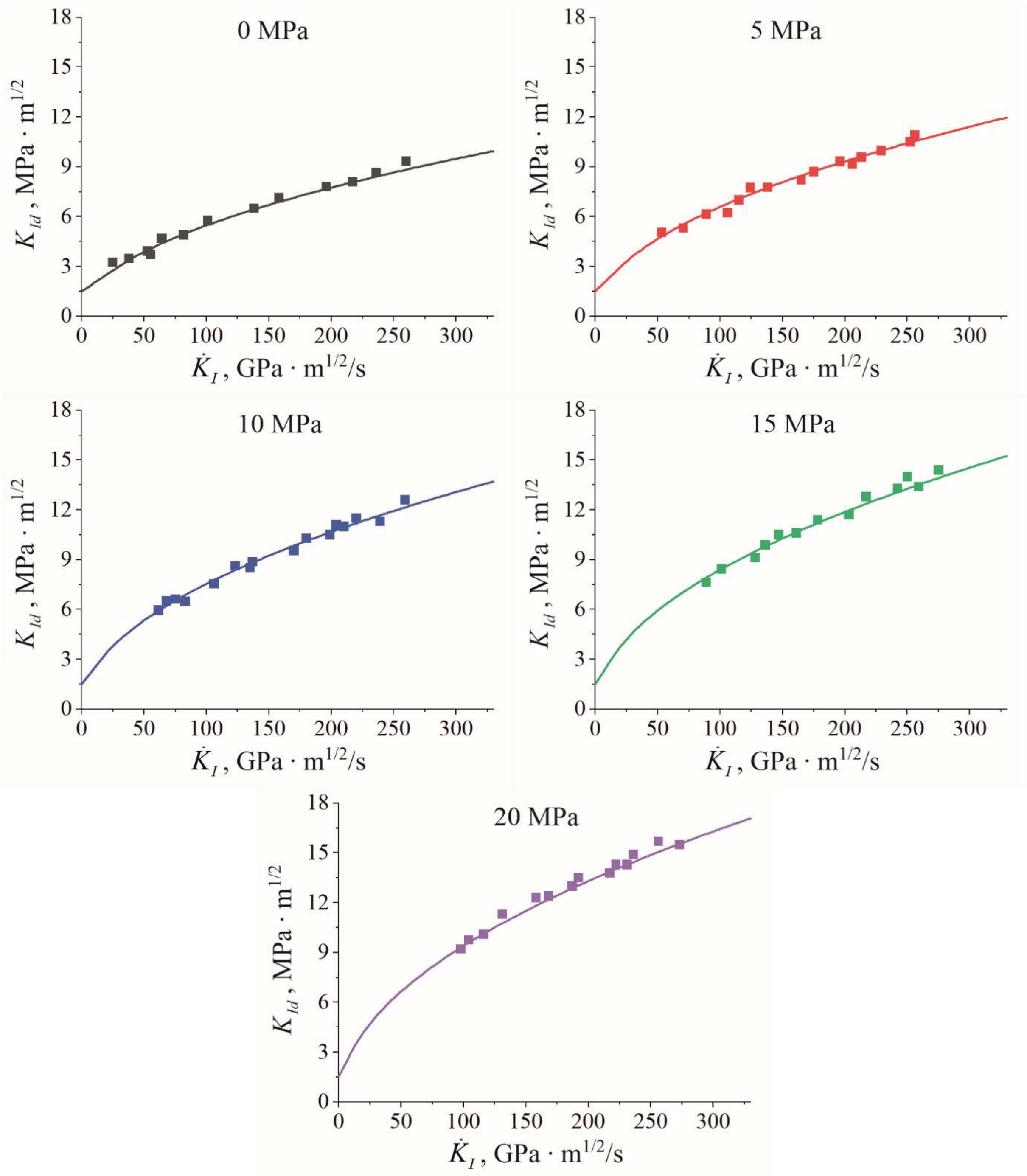


Figure 3.4. Rate dependences of granite fracture toughness, calculated for different levels of hydrostatic pressure, experimental data [106]

3.5 Dynamic fracture toughness of cement mortars

Cement is one of the main constituent materials used in construction. Concrete, masonry mortar, plaster, and screed are made using cement. Today, cement and concrete are the most used materials for construction throughout the world. Cement, together with reinforced concrete, makes up more than half of all materials produced by mankind [132]. The safety use of structures, buildings and structures directly depends on the physical and mechanical properties of cement mortar, therefore the study of the cement mortars strength characteristics is the crucial task.

As a result of various external influences, both man-made and natural, buildings and structures are exposed to high temperatures. Heat treatment occurs during fires, as well as during geothermal energy extraction and radioactive waste disposal. In addition, critical structures are subject to high-velocity loads, explosions and impacts caused by accidents, terrorist attacks, earthquakes, and mining operations. Thus, buildings and structures (tunnels, mines, nuclear reactors) are subject to the combined action of temperature and shock loads. In this regard, the development of mathematical models that make it possible to calculate the values of dynamic strength and dynamic fracture toughness of building materials is necessary to determine the bearing capacity of structures, as well as to restore structures damaged under the high temperature influence.

Let us consider the effect of preliminary temperature treatment on the dynamic fracture toughness of two cement mortars (with barium sulfate admixture and standart), using experimental data [99]. The first mortar consists of sand, cement, water and barium sulfate with mass fractions of 0.62, 0.25, 0.07 and 0.06, respectively, the second consists only of sand, cement and water with mass fractions of 0.48, 0.37 and 0.15. In [99], using the split Hopkinson pressure bar system, the dynamic fracture tests were carried out on semicircular samples with a cutout (Figure 3.1, a) to determine dynamic fracture toughness. The samples are preheated to temperatures of 150°C, 250°C, 350°C, 450°C and 600°C, and then tested at 25°C.

Figure 3.5 shows a comparison of the theoretical calculation of the dynamic fracture toughness of barite cement mortar and cement mortar without admixtures (3.5) with the experimental results presented in [99]. The incubation time is calculated based on the experimental results; the static limit of the stress intensity factor is determined in [99] (Table 3.2).

The obtained calculated data qualitatively describe the trends in changes in fracture toughness with varying loading rates, and also make it possible to take into account the influence of preliminary temperature treatment. Both solutions show a similar response to increasing loading rate and increasing pre-temperature treatment. Dynamic fracture toughness increases with increasing loading rate at all values of temperature treatment. The dynamic fracture toughness of barite cement mortar is higher compared to cement mortar without admixtures at the same temperature treatment values for all loading rates, and the higher the loading rate, the more significant the difference between the values of dynamic fracture toughness becomes. As the preheating temperature increases, a decrease in fracture toughness is observed for both mortars.

	T, °C	25	150	250	350	450	600
m1	τ_K, μs	94	93	116	126	117	128
	K_{Ic}, MPa · m$^{1/2}$	1.11	0.94	0.78	0.67	0.48	0.36
m2	τ_K, μs	95	106	103	125	134	153
	K_{Ic}, MPa · m$^{1/2}$	0.92	0.7	0.6	0.51	0.4	0.23

Table 3.2. Summary table of the values of the stress intensity factor static limit and incubation time for cement mortars (m1 – barite cement mortar, m2 – standard cement mortar)

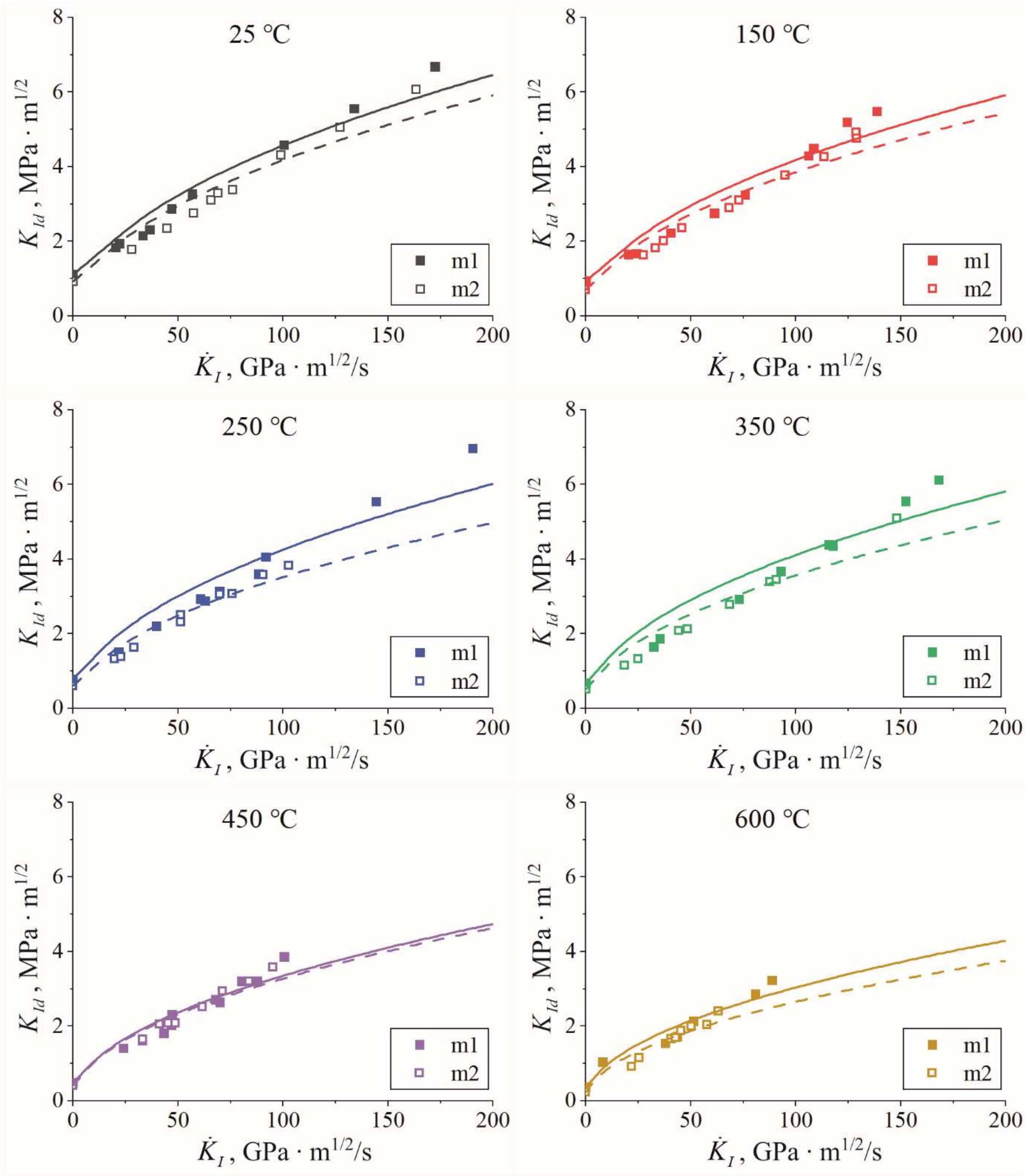


Figure 3.5. Rate dependences of the cement mortars dynamic fracture toughness at different temperatures (solid line – calculation for barite cement mortar, dashed lines – calculation for standard cement mortar), experimental data [99] (m1 – barite cement mortar, m2 – standard cement mortar)

The fracture toughness inversion effect is also observed (Figure 3.6) when comparing the characteristics of mortars preheated to different temperatures. The fracture toughness of one mortar treated at one temperature is higher than that of a second mortar treated at a different temperature under slow loads, while at the same time, for dynamic loads, the second mortar exhibits greater fracture toughness. The intersection point of the fracture toughness rate dependences is the fracture toughness inversion point. A summary table comparing the dynamic and static fracture toughness of mortars treated at different temperatures is given in Table 3.3.

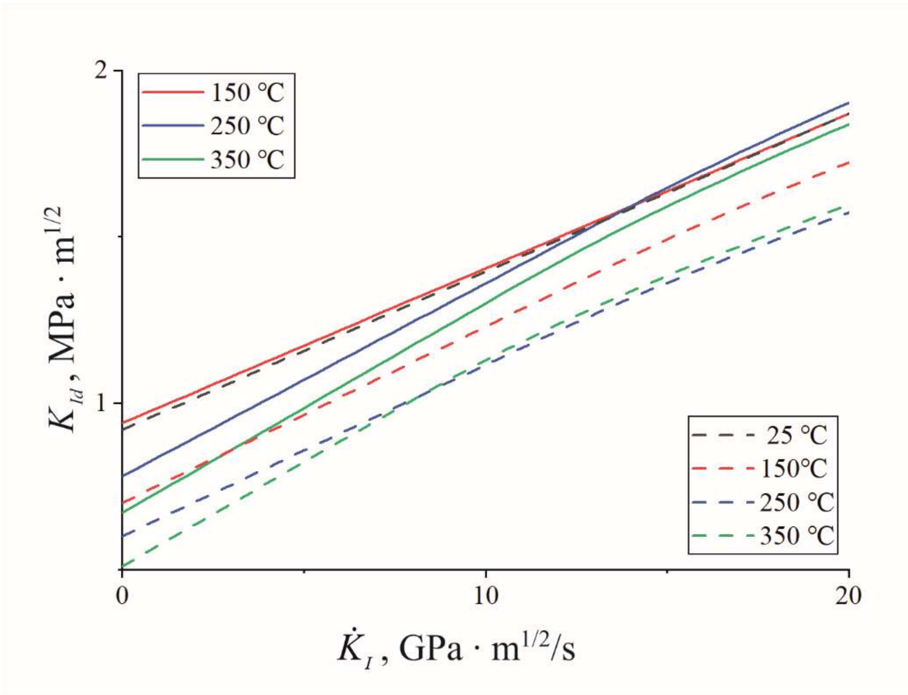


Figure 3.6. Dynamic fracture toughness curves inversion for cement mortars (solid lines – barite cement mortar, dashed lines – standard cement mortar)

It is worth noting that the same cement mortar treated at different temperatures exhibits the fracture toughness inversion effect. For example, at a stress intensity rate less than 14.8 GPa · $m^{1/2}$/s, barite cement mortar treated at a temperature of 150 °C has greater fracture toughness compared to barite cement mortar treated at a temperature of 250 °C. When the rate of change of stress intensity factor becomes higher than 14.8 GPa · $m^{1/2}$/s, higher fracture toughness corresponds to barite cement mortar processed at a temperature of 250 °C.

Cement mortar with greater fracture toughness under slow loads, processing temperature	Cement mortar with greater fracture toughness under high-speed loads, processing temperature	Fracture toughness inversion point
m1, 150 °C	m1, 250 °C	14.8 GPa · $m^{1/2}$/s
m2, 25 °C	m1, 250 °C	13.3 GPa · $m^{1/2}$/s
m2, 150 °C	m1, 350 °C	3.1 GPa · $m^{1/2}$/s
m2, 250 °C	m2, 350 °C	8.4 GPa · $m^{1/2}$/s

Table 3.3. Comparative characteristics of cement mortars fracture toughness (m1 – barite cement mortar, m2 – standard cement mortar)

3.6 Hydrostatic pressure and incubation time

As a result of the above calculations of the granite dynamic fracture toughness at different hydrostatic pressure levels, it turns out that the incubation time grows with increasing hydrostatic pressure. Figure 3.7 shows a graph of incubation time versus hydrostatic pressure. The increase in incubation time with increasing hydrostatic pressure is linear and can be expressed by the following formula

$$\tau_K = \tau_{Kp0} + kp, \tag{3.6}$$

where τ_{Kp0} is the incubation time value in the absence of hydrostatic pressure (for granite $\tau_{Kp0} =$ 97 μs), k is the proportionality coefficient (for granite $k = 9.6$ μs/MPa), p is the value of hydrostatic pressure.

Table 3.4 shows the incubation time values for granite with increasing hydrostatic pressure.

Granite	p , MPa	0	5	10	15	20
	τ_K , μs	100	145	190	235	295

Table 3.4. Summary table of hydrostatic pressure and incubation time values for granite

The results obtained confirm the hypothesis proposed in [61] that the difference in incubation times for rocks with different degrees of saturation is caused by different levels of hydrostatic pressure arising from rapid impact. Hydrostatic pressure slows down the incubation process of microcracking, therefore, the characteristic relaxation time increases. Water-saturated rocks have a higher density compared to dry rocks, so they have a higher strength and a longer incubation time of fracture.

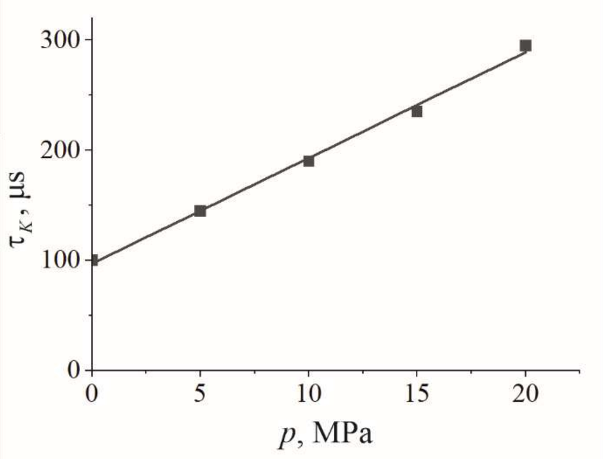


Figure 3.7. Incubation time dependence on hydrostatic pressure

3.7 Temperature and incubation time

According to the calculation results, it was found that with increasing temperature, the granite incubation time value increases (Figure 3.8, a). To a first approximation, we can assume that the dependence of incubation time on temperature is linear and can be written as:

$$\tau_K = \tau_{KT0} + kT, \tag{3.7}$$

where τ_{KT0} is the value of incubation time at zero temperature (for granite τ_{KT0} = 269 μs), k is the proportionality coefficient (for granite $k = 0.35$ μs/°C), T is the treatment temperature.

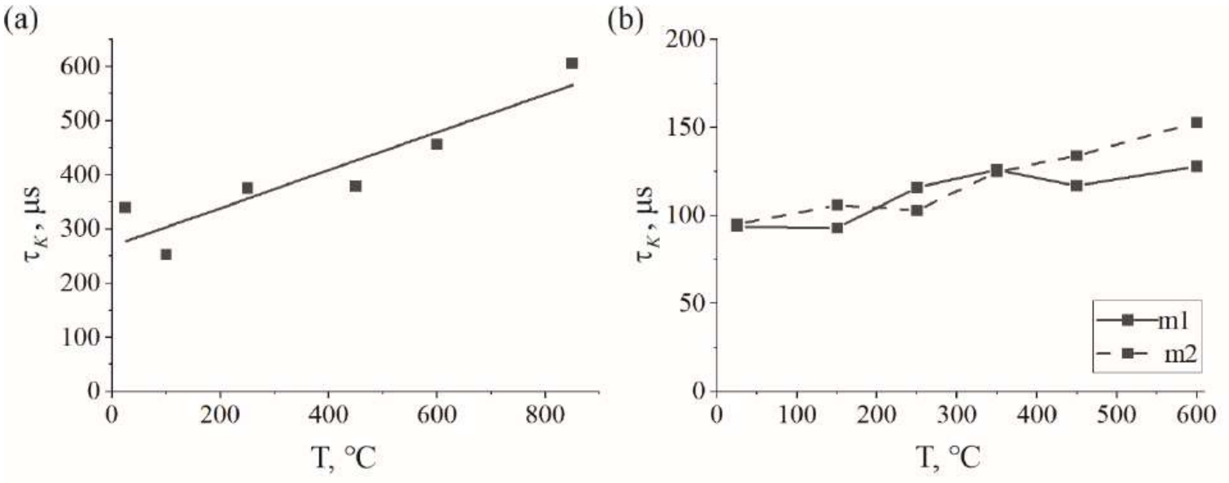


Figure 3.8. Dependence of incubation time on temperature for granite (a) and for cement mortars (b) (m1 – barite cement mortar, m2 – standard cement mortar)

Figure 3.8, b shows the fracture incubation time dependence on the pre-treatment temperature for two cement mortars (standard and barite). The incubation time τ_K, corresponding to the condition for determining dynamic crack resistance, for cement mortar with an admixture of barium sulfate is on average lower than the incubation time of destruction for cement mortar without impurities at the same processing temperatures. The dependences of the incubation time on the preheating temperature for cement mortar with an admixture of barium sulfate and for ordinary cement mortar are similar. The incubation time, corresponding to the condition for determining dynamic fracture toughness τ_K, on average increases with increasing temperature.

3.8 Conclusions to the chapter 3

Using a structural-temporal approach based on the concept of fracture incubation time, theoretical rate dependences of the dynamic fracture toughness of granite and cement mortars (barite and standard) were constructed, taking into account the temperature treatment influence. As the loading rate increases, the values of the dynamic fracture toughness of thermally treated cement mortars and granite increase.

The dynamic fracture toughness of the materials considered decreases with increasing pretreatment temperature. The dynamic fracture toughness of barite cement mortar is higher than that of a mortar without admixtures for all values of temperature treatment. Incubation time increases linearly with increasing temperature. The effect of the material fracture toughness inversion upon transition from slow to high-speed loading is discussed (depending on the loading rate, a material sample can be stronger under quasi-static loads and at the same time less strong under impact loads compared to the second sample).

The dynamic fracture toughness dependence on the stress intensity rate for granite at different levels of hydrostatic pressure was found. With increasing rate of external influence, the strength characteristics of granite increase for all values of hydrostatic pressure. With increasing hydrostatic pressure, an increase in the granite dynamic fracture toughness is observed. A linear increasing relationship is established between incubation time and external hydrostatic pressure.

Due to the good correspondence of the theoretical calculation to the experimental data, it can be conclude that it is possible to use the incubation time criterion when predicting the material fracture under the influence of dynamic loads. Thus, to describe the material fracture toughness rate dependences at different pretreatment temperatures and under the action of hydrostatic pressure, two material constants proposed in the structure-time approach are sufficient: static fracture toughness and incubation time. The results obtained show the effectiveness of the structural-temporal approach for predicting the rate effects of dynamic fracture in the presence of additional external influences.

Chapter 4. The influence of external factors on the material dynamic compressive strength

Known experimental data on studying the additional external factors influence on the dynamic fracture of sandstone and cement mortars are analyzed based on the incubation time criterion. Compressive strength dependences on loading rate were obtained for hydrostatically compressed and pre-heat-treated sandstone samples, as well as for standard cement mortar and mortar with an admixture of barium sulfate subjected to pre-heat treatment. A relationship is established between incubation time and external hydrostatic pressure. The preliminary heat treatment influence on the dynamic compressive strength of materials is assessed, and the incubation time values are calculated for each pretreatment temperature. The effect of compressive strength inversion is discussed.

The results presented in this chapter were published in [118], [133].

4.1 Dynamic compressive strength calculation

The incubation time fracture criterion for finding the material dynamic compressive strength is expressed by the following formula:

$$\frac{1}{\tau_\sigma}\int_{t-\tau_\sigma}^{t}\sigma(s)ds \le \sigma_c\,, \qquad (4.1)$$

where σ_c is the static compressive strength, and τ_σ is the incubation time of fracture corresponding to the compression condition. The fracture occurs at the point in time at which the equal sign in the condition (4.1) is reached.

The compressive strength under static loading σ_c is the material constant; this value determines the stress value in the sample, upon reaching which the fracture begins in the case of static loads. The fulfillment of equality in expression (4.1) determines the condition for the onset of fracture propagation for the entire range of speed impacts.

This chapter considers the known experimental data on determining the dynamic compressive strength of materials [100], [134]. Experiments are carried out using the split Hopkinson pressure bar system to fracture cylindrical samples (Figure 4.1, a) with different loading rates. In these experiments, the stresses in the sample depend linearly on time over a certain loading interval; fracture occurs at the stage of stress growth. Thus, the stresses dependence on time for calculations is expressed by the following formula $\sigma(t) = \dot{\sigma} t H(t)$, where $\dot{\sigma}$ is the loading rate (stress change rate) and $H(t)$ is the Heaviside function. The loading rate is equal to the slope of the tangent to the linear section of the stress versus time graph (Figure 4.1, c).

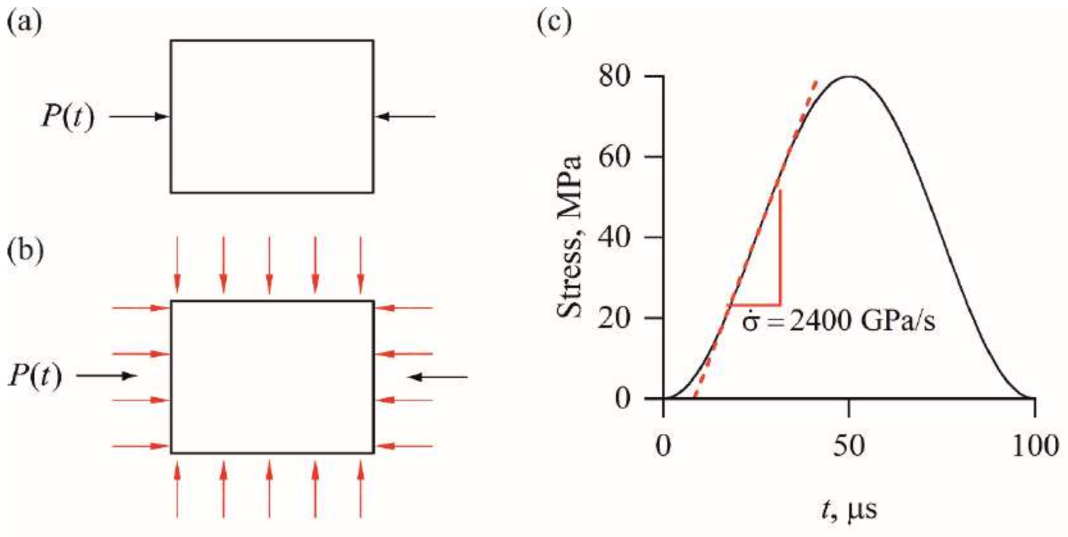


Figure 4.1. Experimental scheme for determining the dynamic compressive strength of a cylindrical sample (a), experimental scheme for determining the dynamic compressive strength of a cylindrical sample in the presence of hydrostatic pressure (b), the stress change rate determination (c)

Applying the incubation time fracture criterion (4.1), we obtain the dependence of dynamic compressive strength σ_d on the loading rate $\dot{\sigma}$:

$$\sigma_d(\dot{\sigma}) = \begin{cases} \sigma_c + \dfrac{\tau_\sigma}{2}\dot{\sigma}, \dfrac{\sigma_c}{\dot{\sigma}} \geq \dfrac{\tau_\sigma}{2}, \\ \sqrt{2\sigma_c \tau_\sigma \dot{\sigma}}, \dfrac{\sigma_c}{\dot{\sigma}} < \dfrac{\tau_\sigma}{2}. \end{cases} \quad (4.2)$$

To construct theoretical dependencies (4.2), the incubation time is calculated by the least squares method based on experimental results; static fracture toughness is the material property and is also taken from known experimental data.

4.2 Compressive strength of thermally treated sandstone

Let us consider the experimental results [134] on the dynamic fracture of preheated sandstone samples. Work [134] considers the change in the rate dependences of dynamic strength during sandstone compression as a result of preliminary heat treatment. In [134], to prepare for dynamic tests, the samples were first gradually heated to a fixed temperature (250 °C, 450 °C and 600 °C), then the temperature was maintained constant for some time to uniformly heat the samples. After this, they were cooled under natural conditions, and tests were carried out at room temperature using the split Hopkinson pressure bar system.

The static strength characteristics of materials after preliminary heat treatment usually decrease with increasing processing temperature [17], [96], [135]. Based on experimental data [134], the following static sandstone compressive strength values were taken for the calculation (Table 4.1). The incubation times calculated from the data in [134] are also presented in Table 4.4.

T, °C	25	250	450	600
σ_c, MPa	30.5	30	29	28.5
τ_σ, μs	45	37	46	36

Table 4.1. Static compressive strength and incubation time of sandstone corresponding to different temperature levels, calculated from the experimental data [134]

Graphs of dynamic compressive strength rate dependences on loading rate, calculated from experimental data [134], are shown in Figure 4.2. The results obtained according to the calculation formula (4.2) qualitatively coincide with the experimental results. As the loading rate increases for all temperature values, the dynamic compressive strength increases, and the higher the temperature, the smaller the increase in dynamic compressive strength with increasing loading rate becomes. The strength properties of a heat-treated sandstone sample are lower compared to a sample that was not exposed to temperature. The sample thermally treated at a temperature of 600 °C has the lowest compressive strength for the entire range of speed impacts.

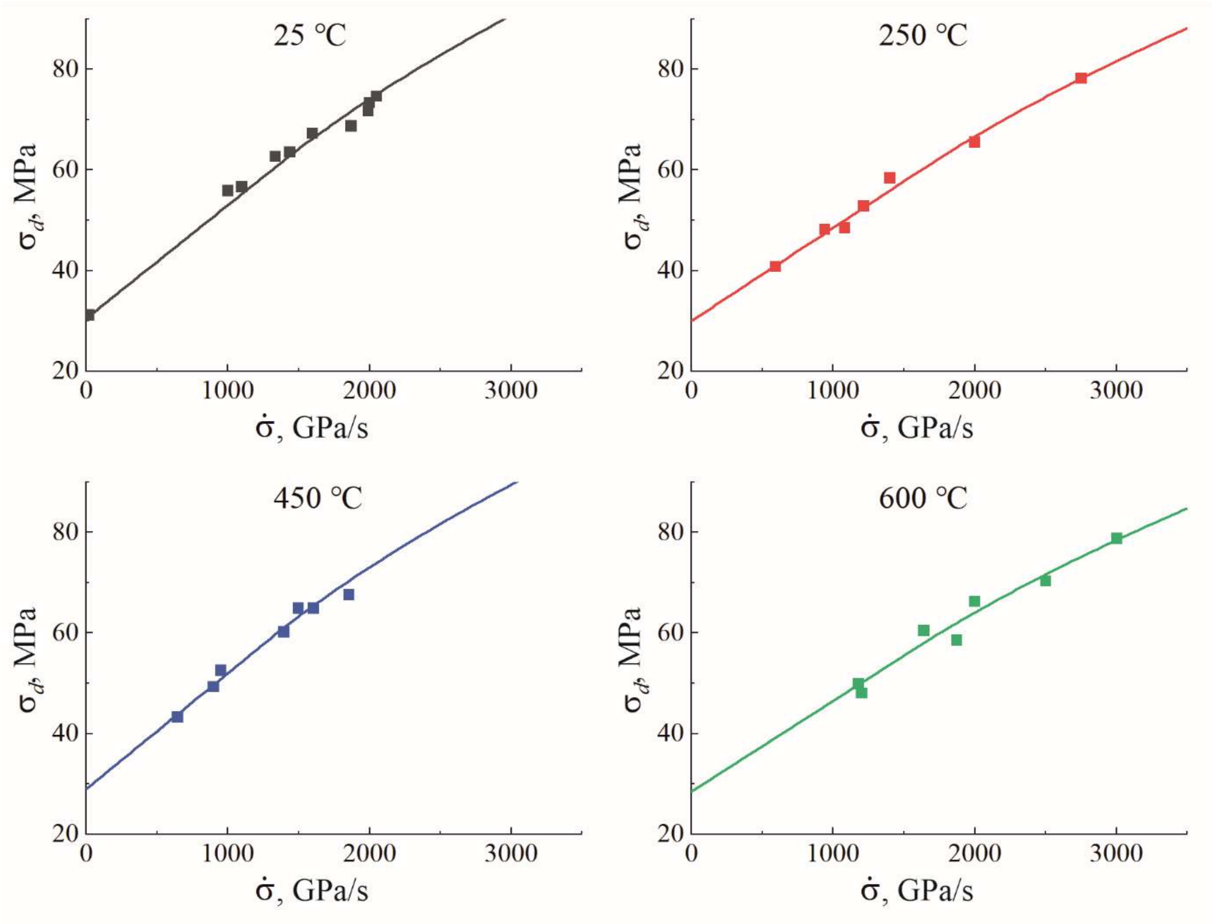


Figure 4.2. Dynamic compressive strength dependencies on loading rate for sandstone, calculated at different temperatures, experimental data [134]

According to theoretical calculations (4.2), based on the experimental data [134], it turns out that the rate dependence curves of the compressive strength of sandstone pre-treated at temperatures of 250°C and 450°C intersect at a loading rate of 222.22 GPa/s (Figure 4.3). At low loading rates, sandstone processed at a temperature of 250°C has the greatest compressive strength, and under dynamic loads, sandstone processed at a temperature of 450°C is more durable, i.e., the compressive strength inversion is observed.

The good agreement between the theoretical calculation (4.2) and the experimental data confirms the possibility of using the incubation time criterion when predicting dynamic fracture considering the heat treatment influence.

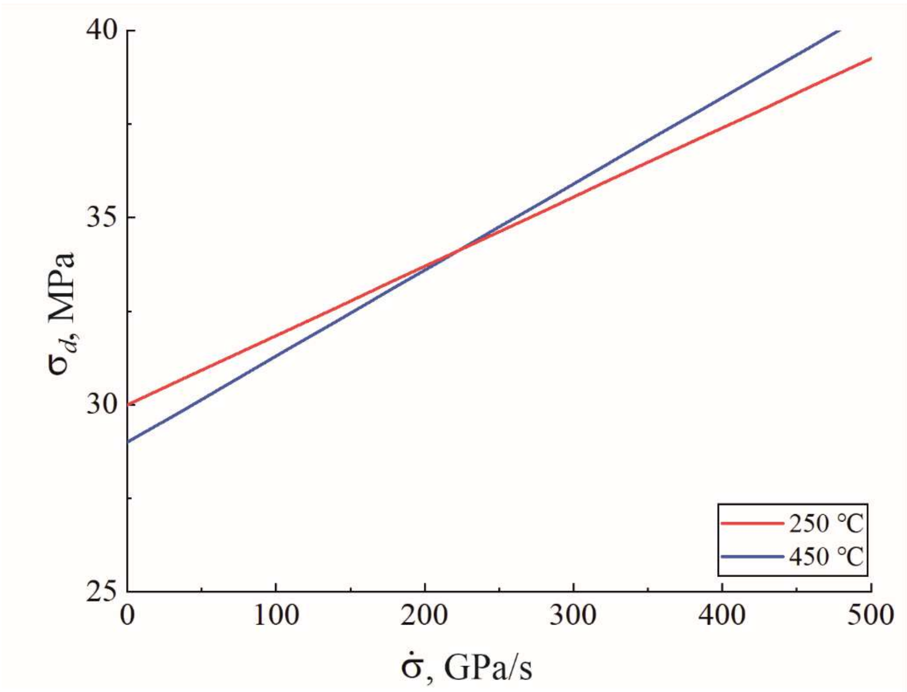


Figure 4.3. Inversion of sandstone compressive strength rate dependences

The results obtained show that the effect of strength inversion can also appear during compression of materials. In this regard, to determine the material strength characteristics, it is important to consider the possibility of material failure for both quasi-static and high-speed loads (for the entire range of loading speeds).

4.3 Dynamic compressive strength of cement mortars

Let us consider the dynamic compressive strength of two cement mortars that differ in composition (barite and standard) based on the experimental data presented in [100]. The compositions of the mortars are identical to those presented in section 3.5.

To determine the dynamic compressive strength, in [100] the split Hopkinson pressure bar system is used. Before testing, cylindrical cement mortar samples are preheated to temperatures of 150°C, 250°C, 350°C, 450°C, 600°C, and 850°C, then dynamic fracture experiments are carried out at room temperature 25°C. The experimental scheme for determining the dynamic compressive strength of a cement mortar cylindrical sample is presented in Figure 4.1, a. Experiments were carried out with different loading rates. Table 4.2 presents the values of σ_c and τ_σ determined from experimental data [100].

	T, °C	25	150	250	350	450	600	850
m1	τ_σ, μs	42	38	42	39	44	41	62
	σ_c, MPa	61.41	56.19	46.34	40.55	32.44	25.49	7.72
m2	τ_σ, μs	52	49	51	49	51	45	69
	σ_c, MPa	40.12	37.12	31.56	28.1	22.09	18.78	6.46

Table 4.2 Static compressive strength and incubation time values for cement mortars (m1 – barite cement mortar, m2 – standard cement mortar)

Figures 4.4 and 4.5 show the two cement mortars compressive strength dependence on the loading rate at different pretreatment temperatures. The theoretical curves qualitatively correspond to the experimental results described in [100]. The results show that with increasing pretreatment temperature, there is a decrease in compressive strength for both cement mortars. For all temperatures, compressive strength increases with increasing loading rate. Barite cement mortar has higher compressive strength than standard cement mortar at equal pre-treatment temperatures, and the higher the pretreatment temperature, the smaller the difference in compressive strength between the two cement mortars becomes.

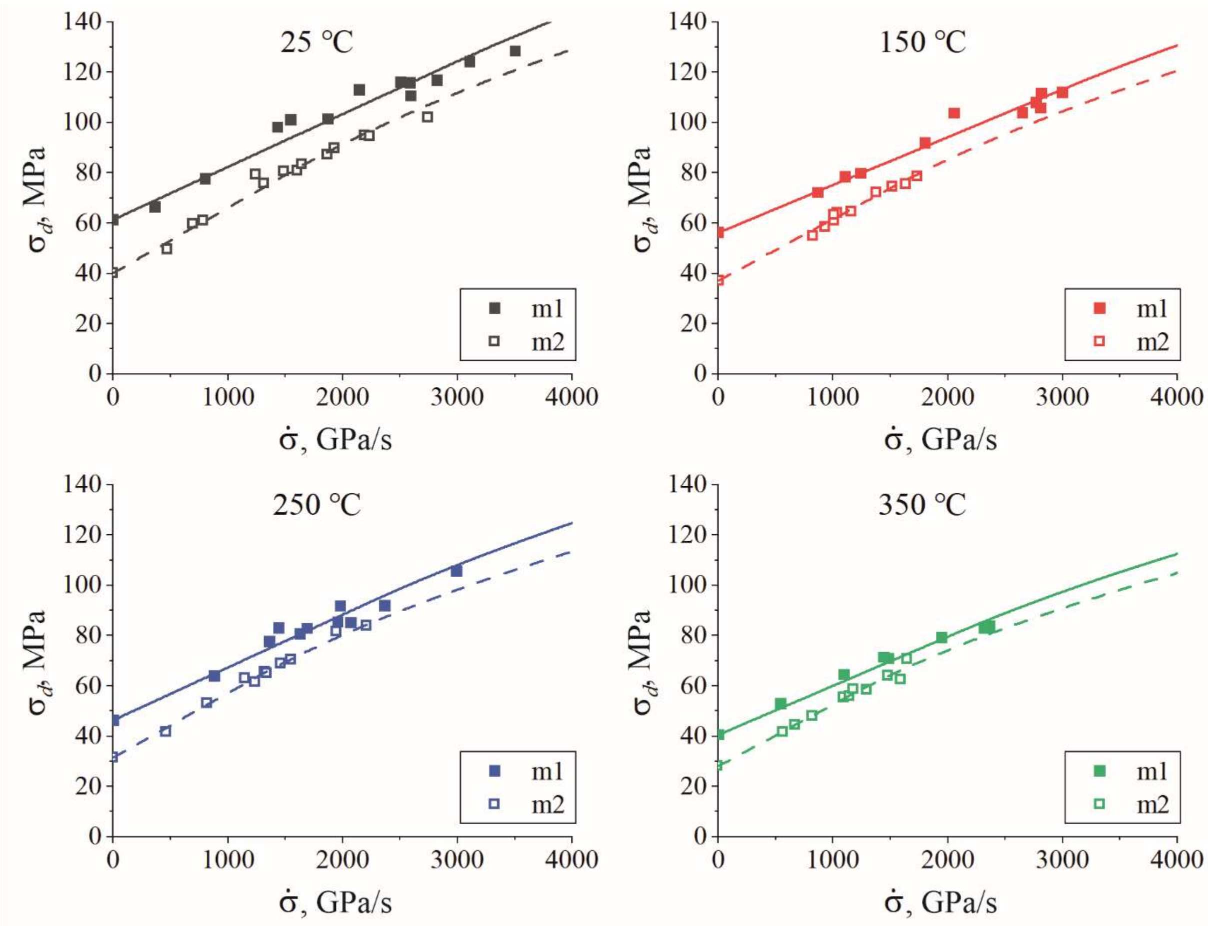


Figure 4.4. Cement mortars dynamic compressive strength rate dependences at temperatures of 25–350 °C (solid lines – calculation for barite cement mortar, dashed lines – calculation for ordinary cement mortar), experimental data [100] (m1 – barite cement mortar, m2 – standard cement mortar)

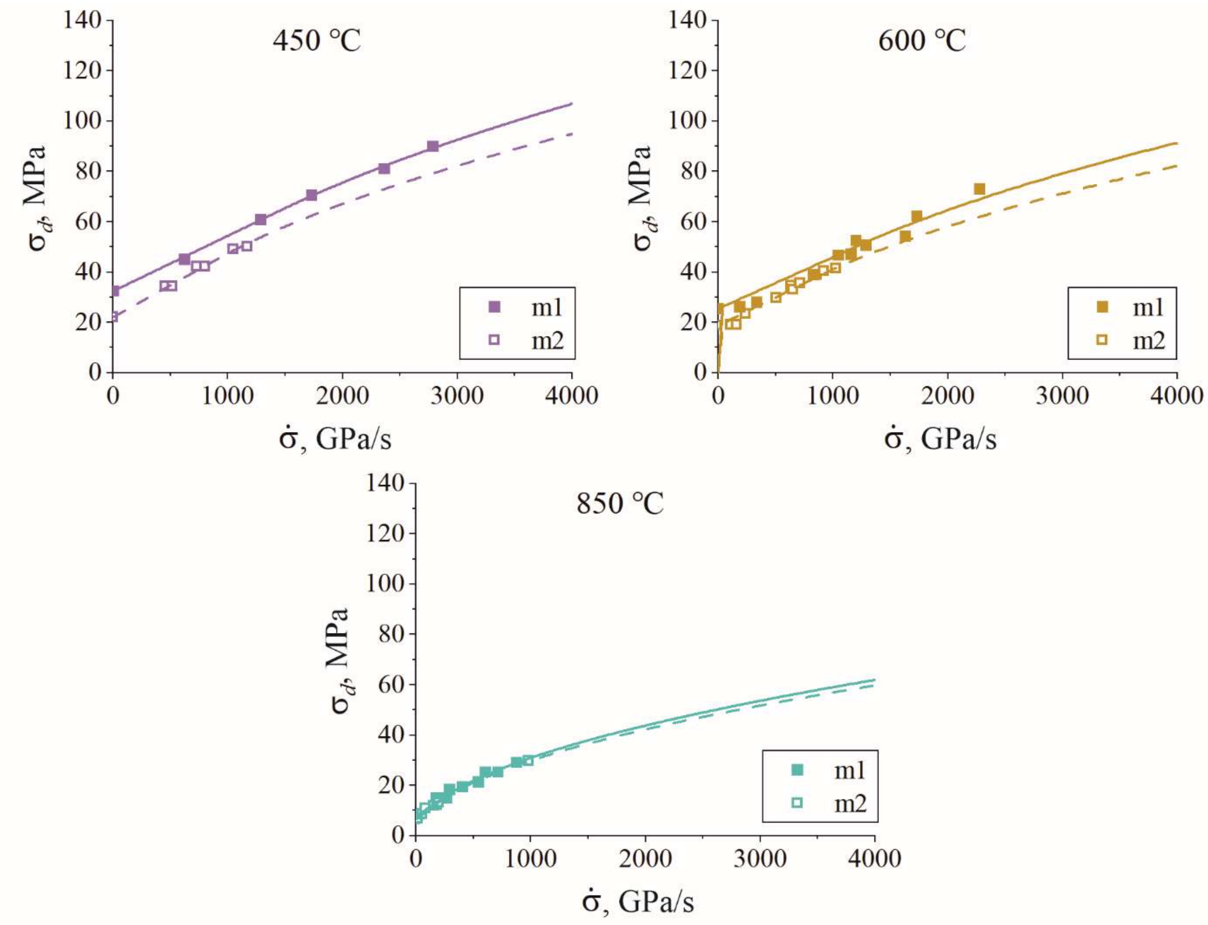


Figure 4.5. Cement mortars dynamic compressive strength rate dependences at temperatures of 450–850 °C (solid lines – calculation for barite cement mortar, dotted lines - calculation for ordinary cement mortar), experimental data [100] (m1 – barite cement mortar, m2 – standard cement mortar)

When comparing the two mortars, the compressive strength inversion effect is observed (Figure 4.6). At different pre-treatment temperatures, barite cement mortar has greater compressive strength under external influences at low loading rates, but becomes less strong under high-speed loads compared to standard cement mortar. The intersection point of the compressive strength velocity curves in Figure 4.6 corresponds to the transition loading rate at which inversion of the compressive strength occurs. Table 4.3 presents a comparative description of the strength properties of two cement mortars processed at different temperatures.

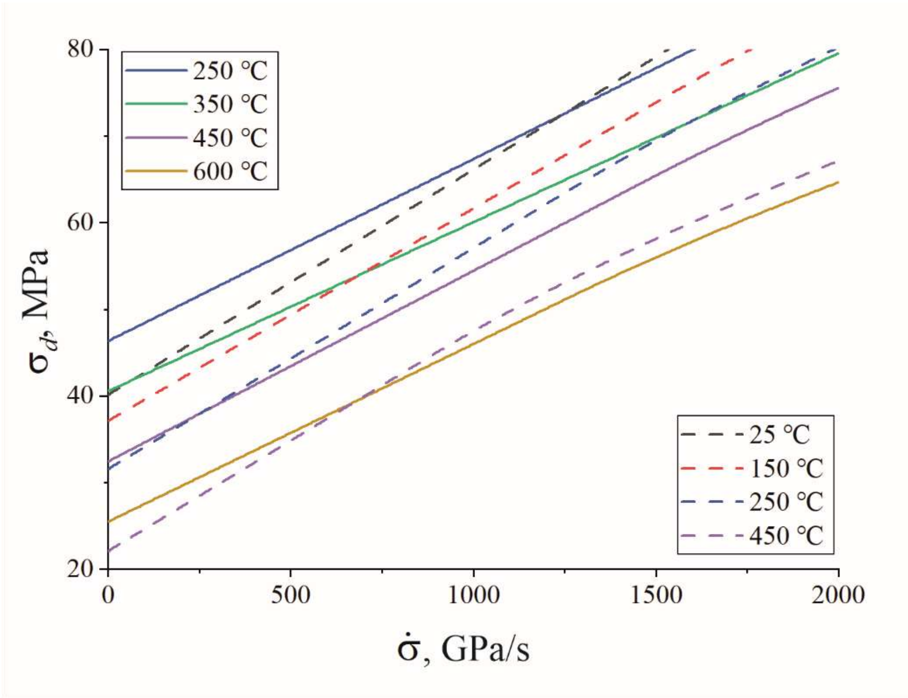


Figure 4.6. Dynamic compressive strength inversion for cement mortars (solid lines – barite cement mortar, dashed lines – standard cement mortar)

Cement mortar with greater compressive strength under slow loads, processing temperature	Cement mortar with greater compressive strength under high-speed loads, processing temperature	Loading rate corresponding compressive strength inversion
m1, 250 °C	m2, 25 °C	1216 GPa/s
m1, 350 °C	m2, 25 °C	67 GPa/s
m1, 350 °C	m2, 150 °C	714 GPa/s
m1, 350 °C	m2, 250 °C	1481 GPa/s
m1, 450 °C	m2, 250 °C	233 GPa/s
m1, 600 °C	m2, 450 °C	667 GPa/s

Table 4.3. Comparative characteristics of the compressive strength of two cement mortars (m1 – barite cement mortar, m2 – standard cement mortar)

4.4 Effect of hydrostatic pressure on dynamic compressive strength

In [136], sandstone samples were tested to determine dynamic compressive strength under the influence of hydrostatic pressure (Figure 4.1, b). Experiments are carried out using the split Hopkinson pressure bar system modified with two constraint systems (axial and radial). The axial system is used to provide preload in the axial direction, and the radial system is used to apply lateral restraining pressure on the sample. Different rates of dynamic loading are achieved by changing the pressure of the air gun and the striker location in the shot chamber.

Tests are carried out on sandstone samples at four hydrostatic pressures of 0 MPa, 7 MPa, 14 MPa, 21 MPa, and 28 MPa. After hydrostatic pressure is applied, a dynamic load begins to act on the sample; the loading rate is in the range of 1000–5000 GPa/s. The incident, transmitted and reflected strain signals are monitored by strain gauges mounted on the straight and the transmitting rods. It is believed that the experiment is carried out in a one-dimensional wave theory. When deriving the basic relationships of the Hopkinson split pressure bar method, it is assumed that due to the very short length of the sample compared to the loading pulse length, a uniaxial stress state with a uniform distribution of stresses and strains along its length is realized in the sample during the test.

Based on these experimental results [136] on studying the sandstone dynamic fracture under compressive impact at various hydrostatic pressures, theoretical calculations of the sandstone dynamic strength dependence on the loading rate using the incubation time fracture criterion (4.1) are made. The compressive strength for the case of static loads according to [136] is 70 MPa. The incubation time, similar to the previous example, is determined by the least squares method based on the experiment results [136].

A graph of dynamic compressive strength versus loading rate at different levels of hydrostatic pressure (0 MPa, 7 MPa, 14 MPa, 21 MPa, and 28 MPa) in comparison with experimental data is shown in Figure 4.7. A qualitative agreement between the theoretical calculation and experiment was obtained. At low values of hydrostatic pressure (0 MPa, 7 MPa), an exact agreement between the theoretical calculation and experimental data is observed.

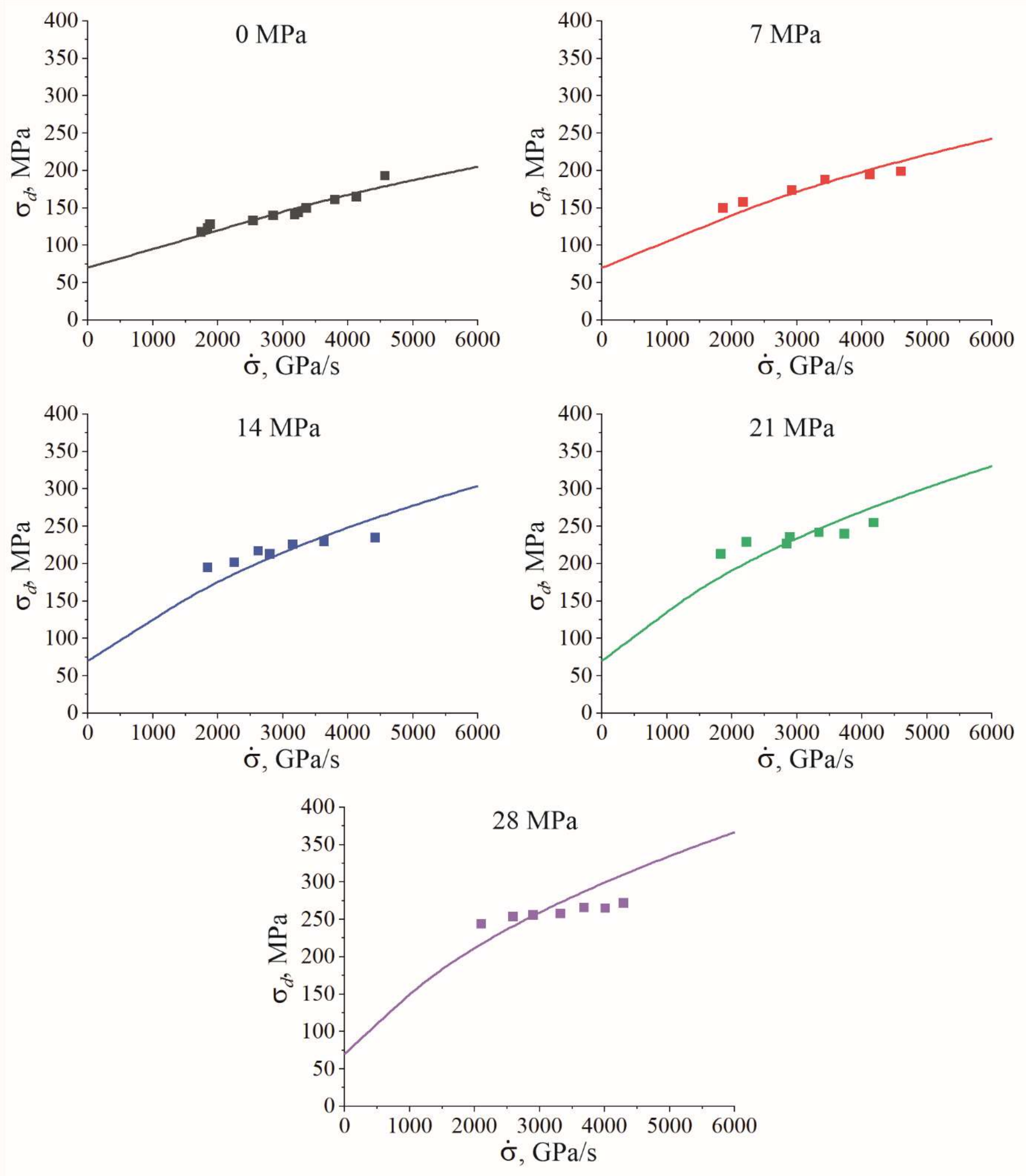


Figure 4.7. Dynamic compressive strength dependence on loading rate for sandstone at various hydrostatic pressures, experimental data [136]

With increasing hydrostatic pressure, the deviation of experimental data from the values obtained from theoretical calculations is no more than 20% (at a hydrostatic pressure of 28 MPa). The results of the theoretical calculation can be refined by carrying out additional tests to determine the static compressive strength of hydrostatically compressed sandstone samples. For all levels of hydrostatic pressure, an increase in compressive strength is observed with increasing loading rate. In addition, the higher the hydrostatic pressure, the higher the dynamic compressive strength and the higher the incubation time. This is explained by the microcracks closure in rocks.

The results obtained show that for a qualitative description of the rate dependences of compressive strength under additional external influences (hydrostatic pressure, temperature), two parameters are sufficient: incubation time and static compressive strength.

4.5 External factors influence on the incubation time

According to the calculation results, it was found that with increasing temperature, the value of incubation time decreases slightly (Figure 4.8, a). The incubation time of sandstone changes weakly under the influence of temperature treatment. To a first approximation, it can be assumed that the sandstone incubation time dependence on temperature is linear and can be written as:

$$\tau_\sigma = \tau_{\sigma T0} + k\mathrm{T}, \tag{4.3}$$

where $\tau_{\sigma T0}$ is the incubation time value at zero temperature (for sandstone $\tau_{\sigma T0}$ = 44 μs), k is the proportionality coefficient (for sandstone $k =$ -0.0088 μs/°C), T is the material processing temperature.

Figure 4.8, b shows the incubation time dependence on the pretreatment temperature for two cement mortars. The incubation time for compressive strength tests for cement mortar with an admixture of barium sulfate is on average lower than the incubation time for cement mortar without admixtures at the same processing temperatures. The fracture incubation time dependence on the preheating temperature for cement mortar with an admixture of barium sulfate and for standard cement mortar is of a similar nature. When both solutions are heat treated in the temperature range from 150 °C to 600 °C, the incubation time corresponding to the condition for determining compressive strength τ_σ

almost does not change, its value differs slightly from the incubation time value of thermally untreated cement mortar. When the samples are heated to a temperature of 850 °C, the incubation time value τ_σ rapidly increases for both mortars. When comparing the results obtained with the finding presented in Chapter 3, it turns out that the incubation time corresponding to the condition for determining fracture toughness τ_K is approximately two times higher than the incubation time corresponding to the condition for determining compressive strength τ_σ, both for barite cement mortar and for cement mortar without admixtures.

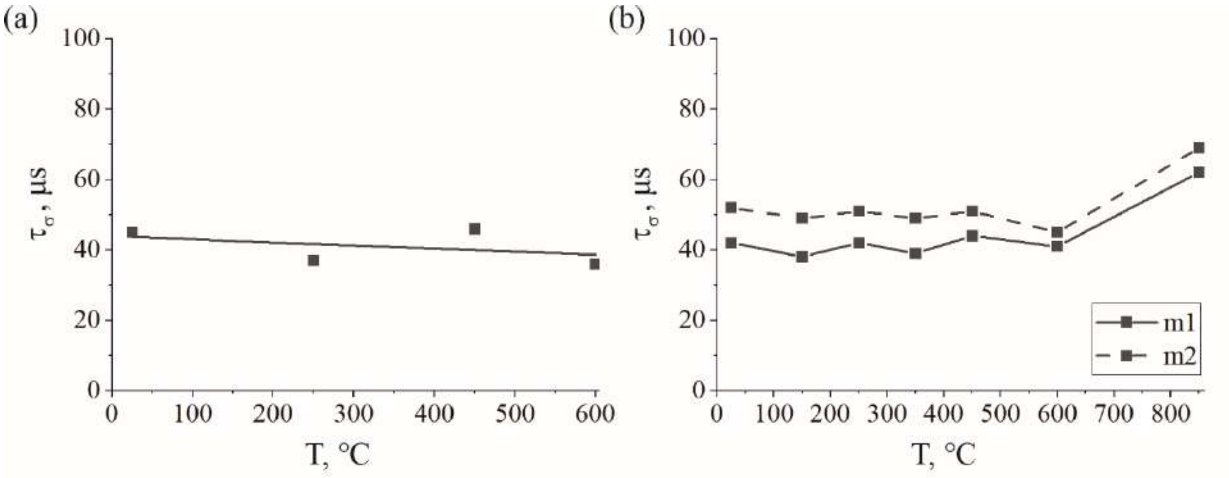


Figure 4.8. Incubation time dependence on temperature for sandstone (a) and for cement mortars (b) (m1 – barite cement mortar, m2 – standard cement mortar)

As a result of the above calculations of the sandstone dynamic compressive strength at various levels of hydrostatic pressure, it turns out that the incubation time increases with increasing hydrostatic pressure (Table 4.4). Hydrostatic pressure slows down the incubation process of microcracking, therefore, the sandstone characteristic relaxation time increases. Figure 4.9 shows a incubation time versus hydrostatic pressure graph. The increase in incubation time with increasing hydrostatic pressure is linear and can be expressed by the following formula

$$\tau_\sigma = \tau_{\sigma p0} + kp, \tag{4.4}$$

where $\tau_{\sigma p0}$ is the value of incubation time in the absence of hydrostatic pressure (for sandstone $\tau_{\sigma p0} = 48$ μs), k is the proportionality coefficient (for sandstone $k = 4$ μs/MPa),

p is the value of hydrostatic pressure. Table 4.4 shows the incubation time values with increasing hydrostatic pressure for sandstone.

p, MPa	0	7	14	21	28
τ_σ, μs	50	70	110	130	160

Table 4.4. Hydrostatic pressure and incubation time values for sandstone.

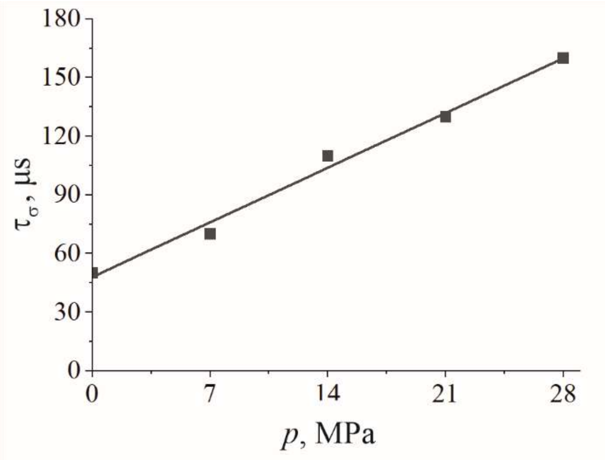


Figure 4.9. Incubation time dependence on hydrostatic pressure for sandstone

4.6 Conclusions to the chapter 4

The heat treatment and hydrostatic pressure influence on the dynamic strength properties of materials was studied using the example of sandstone and cement mortars. The rate dependences of the dynamic compressive strength were constructed taking into account the external factors influence. It was revealed that the material dynamic compressive strength decreases with preliminary exposure to high temperatures. The incubation time slightly changes when the pretreatment temperature changes in the temperature range (0 °C – 600 °C). However, with further sample heating, a rapid increase in the incubation time for cement mortars is observed. With increasing hydrostatic pressure, the strengthening effect for sandstone is shown for all values of external impact velocities. The incubation time in the studied range increases linearly with increasing hydrostatic pressure.

The dynamic compressive strength of barite cement mortar turns out to be higher than that of a mortar without admixtures for all temperature treatment values. With increasing temperature, the strength properties of both mortars decrease. As the loading rate increases, the dynamic strength of thermally treated cement mortars increases.

The compressive strength inversion effect is studied, according to which, when comparing two material samples processed at different temperatures, depending on the loading rate, one material can be stronger under quasi-static loads and at the same time less strong under impact loads.

The loading rate influence on the rock dynamic strength with five levels of hydrostatic pressure was studied. The results show that the rock dynamic strength exhibits a similar response to loading rate at different hydrostatic pressures, showing an increasing trend with growing loading rate.

It is shown that to describe the fracture processes taking into account the additional external factors influence, two material constants proposed in this work are sufficient, namely the incubation time and the static compressive strength. The results obtained, verified by experimental data, show the effectiveness of the applied approach for predicting the dynamic rock fracture.

Conclusions

The dissertation work investigated the influence of external temperature and force factors on the strength characteristics of materials under high-speed loads. It is shown that to predict the material fracture as a result of dynamic loads and additional external factors, a structural-temporal approach based on the concept of incubation time can be effectively used.

The main results of the work are as follows:

1. The wave propagation and reflection in a rod located in an elastic environment is modeled. The case is considered when the rod, like the medium, exhibits elastic behavior. Longitudinal vibrations of the rod are described by the well-known Klein – Gordon equation. It is shown that, depending on the resistance coefficient value of the elastic medium, the propagation of waves in the rod has a different character. When the resistance coefficient of the medium is low, after reflection from the free edge of the rod, the effect of increasing the tensile amplitude compared to the absolute value of the initial pulse can be observed, increasing the possibility of medium spall fracture. With an increase in the resistance coefficient of the medium, the external influence duration becomes a determining factor in the nature of wave propagation. With fast impacts, wave dispersion is observed; with slower impacts, the medium almost completely damps the vibrations.
2. Based on the structural-temporal approach and the incubation time criterion, the fracture threshold amplitudes for the rod interacting with the elastic medium were calculated. It was revealed that, depending on the characteristics of the external influence and the environment, brittle fracture can occur both as a result of spall

caused by wave reflection, and before the wave reaches the free edge of the rod. The presence of optimal durations of exposure leading to the rod fracture at threshold loading is shown. It was also revealed that there are duration values of external force that will not cause the rod fracture.

3. A comparison is made and a qualitative agreement is shown between the results of modeling wave propagation in a rod located in an elastic medium and experimental data on wave propagation in a plexiglass (PMMA) rod immersed in silicone.
4. Based on the structural-temporal approach, the change in the fracture toughness of granite and cement mortars (barite and standard) and the compressive strength of sandstone and cement mortars under dynamic loads as a result of preliminary heat treatment was studied. It is shown that with increasing processing temperature, dynamic fracture toughness and compressive strength tend to decrease. The crack resistance inversion effect for both granite and cement mortars and the compressive strength inversion effect for both sandstone and cement mortars were revealed during the transition from quasi-static loads to rapid impacts. A linear dependence of the incubation time on the treatment temperature is found in the considered range of preheating temperatures for sandstone and granite.
5. The hydrostatic pressure influence on the sandstone dynamic compressive strength and the granite dynamic fracture toughness was studied. With increasing hydrostatic pressure, an increase in the sandstone dynamic compressive strength and granite fracture toughness is observed. As the loading rate increases, the granite fracture toughness and the sandstone compressive strength increase for all hydrostatic pressure values. It is shown that in the studied range the incubation time dependence on hydrostatic pressure is linear, and the greater the hydrostatic pressure, the greater the incubation time value. The relationship between hydrostatic pressure and moisture saturation is discussed.
6. It is shown that the structure-temporal approach can be successfully applied to describe the dynamic material fracture in the presence of additional external factors. From the studies carried out it follows that when determining the material strength characteristics taking into account the influence of temperature treatment

and hydrostatic pressure, in many cases it is sufficient to know two parameters: the static tensile strength (static fracture toughness) and the incubation time. The resulting theoretical calculations were verified by known experimental data.

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