REVIEW

by Joaquim Ortega Cerdà, member of the dissertation council for the dissertation of Pavel Alexandrovich Mozolyako on the topic: "Discrete models of the boundary behaviour of harmonic functions", submitted for the degree of doctor of Mathematics in the scientific specialty 1.1.1. Real, Complex and Functional Analysis.

Review:

The dissertation is a very notable recollection of the original work done by Pavel A. Mozolyako in a classical field of function theory, with an emphasis on the study of harmonic functions. Most of the results have been published in prestigious international journals, like the International Mathematical Research Notices, Mathematische Annalen, Revista Matemática Iberoamericana, C. R. Math. Acad. Sci. and the Journal of the London Mathematical Society among others.

The thesis nevertheless is presented in a coherent way with a nice long Introduction where all results are collected followed by eight chapters, with detailed proofs.

The first three chapters are devoted to the problem of studying function theory in the polydisk. The main tool is to have a discrete model the d-tree. The first chapter delves with the definitions and discrete models and the second properly with the analysis in the discrete setting, more concretely the study of Carleson measures and Hardy embeddings. A deep study of potential theory is realized. Finally in the third chapter the results are transferred to the polydisk, where Carleson measures are analyzed. More precise results are obtained in the 2 and 3 dimensional trees.

Then, there is a change of topic and the thesis turn on the study of the study of harmonic functions with a prescribed growth. On the fourth chapter is devoted to the characterization of harmonic functions with a prescribed growth at infinity in the upper half space in terms of its wavelet decomposition. In the fifth chapter he proves an iterated logarithmic growth of a weighted average of an harmonic function with slow growth along a ray that finishes in the boundary of a Lipschitz domain. Of course the techniques of the previous chapter do not apply since we are working on a Lipschitz domain. Instead an approximation by harmonic Bloch functions and an ingenious use of martingales are in order. This follows a well established expertise of Mathematicians in St. Petersburg, like prof. Makarov who was one of the pioneers on that technique. The next chapter looks a bit different, but the same techniques of discretizing the problem and stating in in terms of a dyadic martingale apply. It is the description of the size of the set, in terms of the Haussdorff measure, where the fractional divided differences of a Holder function can be big. In the seventh and last chapter of this topic, he presents a generalization of Cartwright result to several variables. The main result is that under some regularity conditions on the weight that prescribes the growth, from above, of an harmonic function in the ball as we approach the boundary, any such function has a bound from below of the same order, thus upper growth implies two-sided growth. This is a very non-trivial extension to several variables of a classical result.

Finally the last and eighth chapter is in a different topic. It is a generalization of a theorem of Bourgain on the variation along rays of a bounded holomorphic function. Bourgain proved that the image of radius has finite length on a set of angles with a big Haussdorff dimension. Here he presents an extension to positive harmonic functions in smooth domains in Euclidean space. In the thesis the proof is presented for the ease of presentation in dimension 2.

Finally some conclusions and reflections on future work are presented.

Altogether I find that the thesis is very complete. I value specially the first topic where, although the techniques of transfering problems from the polydisk to the trees are not new, here they are developed to a superior level, obtaining non-trivial results that could not have been obtained otherwise.

It is also remarkable that he has established a net of connections and collaborations with top researchers in Analysis, as Volberg, Malinnikova, Logunov and Nicolau. This continued collaborations are the hallmarks of a successful academic career. I have no reservations whatsoever with the thesis. I think it is very fine job in Mathematics.

Summary:

The dissertation on the topic: "Discrete models of the boundary behaviour of harmonic functions" **meets** the basic requirements established by Order No.11181/1 dd. 19.11.2021 "On the procedure for awarding academic degrees at St. Petersburg State University". The applicant Pavel Alexandrovich Mozolyako **deserves** to be awarded the academic degree of doctor of Mathematics in the scientific specialty 1.1.1 Real, Complex and Functional Analysis. No violations of paragraphs 9 and 11 of the specified Order have been detected.

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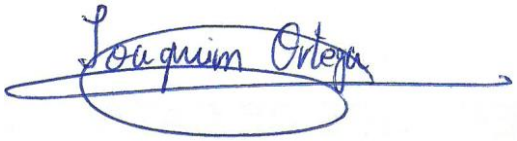

Joaquim Ortega Cerdà

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