Review of the dissertation

"GENERAL APPROACH TO THEORY AND METHODOLOGY OF SINGULAR SPECTRUM ANALYSIS"

by Golyandina Nina Eduardovna

submitted for the Doctor of Physical and Mathematical Sciences degree in the scientific speciality 1.2.2. *Mathematical modelling, numerical methods and software complexes*

The singular spectrum analysis (SSA) method has been developed, first of all, for the analysis of one-dimensional time series. The name of the method only partially represents its nature. It rather reflects one of the main steps, which involves the singular value decomposition of a certain matrix (referred to as a trajectory matrix) related to the time series. The singular values in this singular value decomposition compose the singular spectrum of the operator determined by the matrix.

The SSA method should be placed, essentially, in between the adaptive method for the exploratory data analysis and the method of the best approximations using the least squares in the class of the signals representable in the form of a sum of products of polynomials, exponential and sine series. This class of signals is determined by the solutions of homogeneous linear differential equations. For signal extraction, one can describe the method using successive projections on the set of matrices with prescribed maximal rank and on the set of Hankel matrices. Iterating this sequence, we get in the limit a time series satisfying the signal model. This approach has been referred to as Cadzow iterations. The SSA method involves only one iteration; consequently, it is better adjustable for a particular problem. Therefore, the range of problems solvable by the SSA-like methods, such as subspace-based methods, is vast. It includes, for instance, the non-parametric decomposition of the series into trend, seasonality and noise, signal extraction, smoothing, noise reduction, forecast, gap filling, and estimation of parameters, such as the frequency of periodic components. Given such a wide range of problems, developing a general view of SSA is an important and timely task.

The dissertation consists of seven chapters. The first chapter introduces the research topic and describes a general approach to the SSA method, its modifications, and generalisations. The author discusses in a systematic way the general concepts and problems relevant to the SSA method. A comparison with other methods is also included. Furthermore, applications of SSA to the one-dimensional times series are discussed in detail, including the signal extraction via projections, parameter estimation and forecasting. As a result, Chapter 1 plays an essential and vital role in the dissertation.

РК № 33-06-1042 от 07.09.2023

The second chapter considers modifications of the decomposition step of the Basic SSA method The proposed modification called SSA with projections is a generalisation of the known versions of SSA, with single and double centering, developed to extract the constant and linear trends, respectively. The SSA with projections is a step away from non-parametric methods. Indeed, the projection includes information about the type of a certain component of the time series. The remainder is then decomposed with the use of an adaptive non-parametric approach. In particular, the work numerically demonstrates that using the SSA with projections prior to the linear regression significantly improves the precision of the linear trend extraction. Three other proposed modifications for improving the precision of the signal component extraction do not possess suitable approximation properties. For this reason, they can be applied only after signal extraction with a primary method. The proposed strategies for improving the separability of the signal components significantly extend the family of solvable problems. For instance, before the development of the proposed modifications, the SSA method failed to separate two sinusoidal series with identical amplitudes and two sinusoidal series with similar frequencies.

Chapter 3 contains the results that use, in various ways, a signal model. For the purposes of this review, we will consider one of them in more detail. Given a prescribed model with autoregressive noise, the signal extraction problem is standard in the sense that an optimal solution is obtained using the least squares method with weights. However, the problem is nonlinear, with a large number of extreme points. Solving it with an iterative approach requires, in particular, finding a sufficiently good initial approximation. In this setting, the author considers the Cadzow iterations with weights. Such iteration would provide a more straightforward solution; however, Theorems 7–9 in the dissertation demonstrate that the appropriate weights matrices do not exist. This is an elegant and deep result. Throughout its proof, the author develops an interesting and powerful technique. Furthermore, on a high level, this result justifies the importance of developing numerical methods for the approximate search of weights.

Chapters 4 and 5 contain results on multidimensional generalisations of the SSA method. The main achievement of Chapter 5 consists in developing the Shaped SSA method. This method allows reduction to a trajectory matrix for the objects of arbitrary shapes, for example, circular or triangular, with the use of the sliding window of arbitrary shape. One should point out here that, in fact, only the approach for building the trajectory matrix is changed, and, consequently, in most cases, the obtained generalisations can be combined with the ideas of Chapter 3, which are used to improve the separability.

Chapters 6 and 7 are devoted to the applications of the obtained results. Chapter 6 describes an approach to implementing the developed methods collected as an R software package. Chapter 7 discusses the practical applications of the SSA method. In the practical applications, the author utilises several important properties of the SSA, such as the ability to smooth data, construct an accurate prediction, estimate signal parameters, the ability to construct adaptive expansions for data of a given dimension and shape.

The dissertation is independently written by the author. It has internal unity; it contains new and significant scientific results and testifies to the personal scientific contribution of the author. The obtained scientific results can be qualified as a scientific achievement in the field of the analysis of data and digital images.

Minor shortcomings of the work include that some of the results, for example, on improving separability and gap filling, are described in detail only for one-dimensional time series. The dissertation only mentions that the same ideas apply to the general case of multidimensional objects. Indeed, from the description of the results, the method of generalisation to the multidimensional case is clear, but including the algorithms would be helpful. On the other hand, the absence of such algorithms in the work is understandable due to its large volume.

The dissertation of Golyandina Nina Eduardovna entitled "General approach to the theory and methodology of the singular spectrum analysis method" meets all basic requirements established by Order No. 11181/1 of November 19th, 2021 "On the procedure for awarding academic degrees at St. Petersburg State University". The applicant, Golyandina Nina Eduardovna, deserves to be awarded the Doctor of Physical and Mathematical Sciences degree in the scientific speciality 1.2.2. Mathematical modelling, numerical methods and software complexes. No violations of paragraphs 9 and 11 of the specified Procedure are found in the dissertation.

Member of the dissertation council
Full professor
School of Mathematics
Cardiff University
United Kingdom

Aliev, Iskander

August 7th, 2023