CENTRO DE INVESTIGACION Y DE ESTUDIOS AVANZADOS DEL IPN

DEPARTAMENTO DE CONTROL AUTOMATICO

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on dissertation presented by

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Titled

“Mathematical methods of analysis and synthesis of time-delay systems”

Submitted for the Candidate of Physical and Mathematical Science degree in Scientific specialty 2.3.1 “System analysis, information control and processing,statistics”
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The doctoral work of **Aleksei Aliseiko**
was developed at the Department of Control Theory, St Petersburg state university, under the supervision of **Professor A.P. Zhabko**, Doctor of Physical and Mathematical Sciences.

The general framework of this thesis work is the stability analysis of linear systems with delays. More precisely, the development of results in the Lyapunov-Krasovskii framework. The manuscript is written Both in Russian and English languages. The present report refers to the English version of the manuscript, which consists of 234 pages organized into an introduction, six chapters, a conclusion, and two appendices.

The **introduction** starts with a detailed historical review of the results on the stability theory of delay systems focused at results in the Lyapunov Krasovskii framework, more precisely the approach of so-called complete type functionals and matrices for linear systems. It is followed by the exposition of the aims of this work “Finding new classes of time delay systems to which it is possible to construct the Lyapunov matrix analytically and develop new approximate algorithms to obtain the Lyapunov matrix. The description of the manuscript follows. The introduction ends with an outline of the main contributions and corresponding publications in international journals (3), national journals (1), and international conferences (1).

Chapter 1 presents a reminder of the basic concepts and results of the general theory for linear systems, Lyapunov functional of complete type, delay Lyapunov matrix, and its construction, including the basic properties of Kronecker products.
The information in this chapter, which is clear and concise gives the reader the appropriate background and preliminaries for the main results of the manuscript.

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In **Chapter 2,** Aliseiko revisits the case of systems with distributed delay with exponential kernel studied in [48]. In this case, the delay Lyapunov matrix, obtained as the solution of a two points boundary problem, was known to suffer singularities. By a judicious choice of new boundary conditions, Aliseiko solved these issues. Moreover, he was able to prove the uniqueness of the solution, an open problem up to now, whenever the Lyapunov condition holds. *I consider that this chapter is a very solid contribution as it clarifies and completes pending theoretical issues of Chapter 4 in the monography by Kharitonov [51]. Of course, the provided effective construction of the delay Lyapunov matrix for this case is crucial for applying existing results such as stability tests, optimal control, etc...*

Chapter 3 is devoted to an alternative method [38] for the construction of the Lyapunov matrix for systems with distributed delays. By introducing a new variable, the system with distributed delay with exponential kernel is transformed into a retarded type delay system of higher dimension. Here, Aliseiko establishes the connections between the distributed delay system and the extended system, between solutions, fundamental matrices, and finally, Lyapunov matrices. The analysis of the Lyapunov condition and uniqueness of solutions for the respective two-point boundary value problems shows that the transformation must be handled with care. Two examples clearly illustrate the findings.
It is worthy of mention that the authors in [38] outlined that the connection between Lyapunov matrices was not evident. Thus, this chapter completely clarifies this point. A remark with a discussion of the advantages/limitations/complexity of using the extended systems would be useful.

In **Chapter 4**, Aliseiko addresses the construction of the delay Lyapunov matrix of the interconnection of input delay systems with the filtered predictor introduced in [59] in the framework of the "finite spectrum assignment" theory. This closed loop is a distributed delay system of a special form. Thus, the results of the previous two chapters can be employed. A block partition of the delay Lyapunov matrix allows considering the special form of the problem's initial functions for the state and delay.
The result allows the determination of exponential estimates of the closed-loop solutions.
The problem addressed here is an interesting application of the theoretical results of the two previous chapters. Some additional comments on why it is better to use the block partitioning formulation. Also, one wonders if the construction method employed is that of Chapter 2 or 3?

In **Chapter 5**, the case of piecewise constant kernels is addressed. As in Chapter 2, the definition of appropriate boundary conditions allows the construction as well as the uniqueness proof of the solution of the boundary value problem. An academic illustrative example is given.
This Chapter remedies pending issues outlined in Chapter 2. Moreover, the case of piecewise constant is of interest in several applications. Thus, the presented results are significant and useful.

A comment concerning chapters 3, 4, and 5 is that all presented examples are academic. Although I am fully aware of the theoretical nature and value of this research work, I consider that some practical examples in engineering and biology, such as connected cruise control or dehydration process, would enhance the thesis. Particularly, I have in mind examples where the Lyapunov matrix was constructed for distributed delays with exponential or Gama-distribution kernels, despite the issues the author is indeed now solving, with the method introduced in [51] in the context of several problems, robustness, stability analysis, input delay compensation. The author may look at some of the references at the end of this report.

Chapter 6 addresses an important problem: the continuous dependence of the Lyapunov matrix with respect to small perturbations on the right-hand side. By representing the right-hand side by a Stieltjes integral and under mild assumptions, the convergence of the kernels of the right-hand side implies the convergence of the Lyapunov matrices. The result generalizes previous results for exponentially stable systems with delay perturbation to a general class of perturbed systems, not necessarily exponentially stable.
The results of this chapter, published as a single author in a high-impact journal of the web of science Category "Applied Mathematics", show clearly that Aleksei Aliseiko, can produce significant results based on tools of higher sophistication than those at the beginning of his thesis.

The paper ends with a brief conclusion.
At this point of the manuscript, some final comments on possible future research or ongoing work on the topic may be appreciated by readers.

Summarizing, *the research presented in this manuscript shows a high mastery of the topic of delay Lyapunov matrices as well as a solid knowledge of the existing results in the literature.*
The thesis addresses significant pending and novel problems in the study of the delay Lyapunov matrix, ranging from construction issues and uniqueness of solutions to continuous dependence with respect to perturbations. The construction of the Lyapunov matrix of the filtered finite spectrum assignment problem for input delay systems is of interest from a control engineering perspective. Although I am well aware of the spirit of the "Results speak by themselves" style employed in the thesis, I consider that some explanatory comments and non-strictly academic examples along the manuscript would help enhance the value of the contributions and make the manuscript more reader-friendly.
The results, all proven with detail, mathematical rigor, and clarity, are published in 3 top indexed international journals and 1 prestigious Russian Journal. Unfortunately, the COVID-19 epidemic only allowed a single international conference.
It is worthy of mention that the single authorship of the 4 journal publications shows the scientific independence of the defendant, and his capacity to produce novel results, prove them accurately, and communicate them successfully to the scientific community.

In view of the above, **I give a positive evaluation of the thesis authored by Aleksei Aliseiko, titled "Mathematical methods of analysis and synthesis of time-delay systems"**, which meets the highest standard of international scientific research. Thus, **I consider that Aleksei Aliseiko deserves to be granted the Candidate of Physical and Mathematical Science degree** in Scientific Specialty 2.3.1 "System analysis, information control and processing, statistics" of the Saint Petersburg State University, Russia.

Mexico City, Mexico, July 28th

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