Saint-Petersburg State University

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Developing methods for analysing the reliability of technical systems

Scientific specialisation 2.3.1.
System analysis, control and information processing, statistics

Dissertation is submitted for the degree of
candidate of physical and mathematical sciences

Translation from Russian

Scientific supervisor:
Doctor of Physical and Mathematical Sciences, Professor
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Saint-Petersburg — 2023

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INTRODUCTION

Relevance of the research topic. Nowadays, each of us depends on a wide range of technical devices in our daily lives. And the demands on the reliability of these products increase every year, from mobile devices and household appliances to large production lines. The expectations of today's world are that electricity, networks, transport systems must function without failures or delays. It is also not uncommon for failure of any equipment to have catastrophic consequences. More common are cases where defects and malfunctions in products cause consumer dissatisfaction and consequently the manufacturer's costs for remedying non-compliance under the warranty. In addition, due to the influence of competition and external circumstances, for many suppliers the reliability of their products has become a matter of survival in the market.

Reliability improvement methods have long evolved as a natural consequence of failure management. The «check-fix» approach was used long before formal data collection and analysis procedures were developed, since failures are an obvious phenomenon leading to design changes. Thus, for example, the development of systems closely related to safety (the railway industry as an example) was to some extent driven by innovative technology, but more so by bad experience. Nevertheless, even with this approach, the designed systems showed some degree of growth in reliability. In contrast to modern designs, 19th and early 20th century products were not subject to such stringent cost and lead times. For these reasons, relatively high levels of reliability were achieved by over-designing and over-reserving components. Quantitative assessment, as well as development of methods for its realisation, was not carried out. At the end of the 20th century the problem of reliability quickly came to the fore. At that time this task was regarded as the most important aspect in the creation of systems for military equipment, aviation, astronautics and the nuclear industry. Nowadays, reliability encompasses virtually all areas of industry and the development of technical and information systems.

Methods of reliability analysis based on both qualitative methods and quantitative approaches have been actively developed over the last 30 years. Despite the fact that the main discoveries in this field were made in the last century [1], the scaling of the fields of reliability theory applications necessitates the adaptation and development of methods with a thorough account of the characteristics of the application area [2].

This paper provides a reliability analysis for the rail transport sector, the attention to the components of which has been growing steadily in recent years.

The general statement of the problem is to develop methods and algorithms for analysis of operational reliability of technical systems in railway transport on the basis of mathematical statistics, differential calculus, methods of optimization, and control theory.

The aim and objectives of the work are to analyse and construct mathematical reliability models for components of rolling stock on the basis of application and modernisation of various scientific approaches to the specific topical tasks of modern industry. This main objective is realised on the basis of the Markov process apparatus in Chapter 2, statistical hypothesis testing methods in Chapter 3, using time series and survival analysis methods in Chapter 4 and several optimisation methods in Chapter 5.

In Chapter 2, realising this objective requires the following tasks:

1. Development of a structural diagram that most fully characterises the process of door operation within a wagon.
2. Construction of state transition diagram for the selected structural scheme of door car set reliability and subsequent calculation of reliability indicators by solving a system of Kolmogorov differential equations.

In Chapter 3, the aim is to find an analytical criterion for the treatment of reliability statistics:

1. Analysis of common statistical criteria in reliability theory and industrial analytics.
2. Finding the most appropriate criterion for application in the current industrial analytics tasks under consideration.
3. Application of the criteria to real-world problems in the form of the given examples and justification of the choice of method on the basis of the results of the criteria.

In Chapter 4, the objective of forecasting is achieved by addressing the following objectives:

1. Selection of a method for predicting the failure rate parameter.
2. Identification of time series adaptation coefficients based on survival functions.
3. Calculation and verification of the constructed model.

In Chapter 5, the aim is to build and examine a model for optimising maintenance costs, which is achieved by:

1. Introduction of technical measures to maintain the human influence factor in the model.
2. Construction of probabilistic reliability functions for both the system itself and the reliability due to human intervention.
3. Inclusion of reliability functions in the unit cost of maintenance model.
4. Analytical investigation of the constructed preventive cost functions and finding the condition for the existence of an optimum maintenance period.
5. Construction of an operational replacement cost function, taking into account the developments in the previous chapters of the paper and the application of dynamic programming to find the optimum replacement time for the system components.

Main statements to be defended.

1. Method for determining standardised reliability indicators based on structural and Markov analyses.

2. Justification of the choice of statistical hypothesis testing criteria suitable for reliability and industrial analytics tasks.
3. Short-term prediction model based on time series and survival theory.
4. Model for calculating the optimum period for preventive maintenance.
5. Algorithm for finding the replacement period of complex system components based on a constructed cost function incorporating the main results of the whole performance.
6. Design of a reliability assurance system for the product life cycle based on the models and algorithms presented in the paper.

Methodology and methods of the research. Various areas of mathematical apparatus are used: Markov analysis, application of statistical criteria for hypothesis testing, time series analysis and optimisation methods.

The theoretical and practical significance of the work consists in proposed theoretical methods for solving applied urgent problems of modern enterprises on the basis of existing industrial needs. Approaches not previously used in the field of reliability analysis of technical systems are presented. A system for analysing and ensuring reliability at various stages of the technical product life cycle is introduced. The results obtained are applied for solving real problems and have high efficiency in rapidly changing conditions of the production market.

Scientific novelty

1. In Chapter 2, a method of setting standardised reliability indicators based on integrated application of structural reliability analysis and Markov approach is proposed. This approach to the definition of indicators for the coordination of technical documentation at the stage of system design (system components as part of an aggregated item) between the manufacturer and the customer will allow to correctly set standardised indicators, reasonable values of which in further operation will be used for reasoned control of reliability by the supplier and the customer.

2. Chapter 3 analyses common statistical criteria for reliability analysis and describes real-world problems in the field of industry, which can be solved by correctly chosen statistical criteria. A criterion not yet widely used, especially in the field of reliability and industrial analytics, is proposed, and its advantage over a number of other tests is shown.
3. A method for predicting reliability indicators based on time series is proposed, the selection of adaptation parameters of which is based on the apparatus of the survival theory. Short-term forecasting is an under-reported area of reliability due to the prevalence of a probabilistic approach. The model developed in the Chapter 4 makes it possible to obtain accurate predictions of reliability, the application of which is necessary for planning technical measures for the maintenance of products in operation.
4. A new method for finding the optimum preventive maintenance period has been developed. The human operator influence and the internal reliability of the system are included and the unit cost function is investigated on this basis. The lemmas and theorems about the conditions of existence of the optimal solution are proved.
5. A component replacement cost function for a complex system is constructed, incorporating the results of the entire operation. Based on the apparatus of dynamic programming, the optimum replacement period is found, which is the result of a multi-stage approach to the scheduling of planned component replacements.

The validity and credibility of the results are ensured by the correctness of problem statements obtained from the literature, as well as real requests of manufacturing firms and teaching experience for employees of enterprises in different fields of industry. The results have been presented in a number of publications in Russian and international journals, have been presented at conferences and successfully applied to industrial problems on an ongoing basis.

Personal contribution. The dissertation is an independent work of the author. All results presented in the work have been achieved by the author of the work, except where explicitly stated with a reference to a primary source or joint work with other researchers.

Approbation of results. The results presented in this dissertation have been presented at conferences:

1. XIV International Conference on Electromechanics and Robotics «Zavalishin's Readings-2019» April 17–20, 2019, Kursk, Russia.
2. XV International Conference on Electromechanics and Robotics «Zavalishin's Readings-2020» April 15–18, 2020, Ufa, Russia.
3. LIII International Scientific Conference on Control Processes and Stability (CPS) for students and graduate students (CPS'22) April 4–8, 2022, St. Petersburg, Russia.

Publications. The results of the dissertation are reported in [3–12], of which five articles have been included in the list of Higher Attestation Commission of the Russian Federation [3], [5], [7], [9], [10], one in a journal indexed in the Scopus database [12].

Support. This research was supported by the Russian Foundation for Basic Research under scientific project №20-38-90218.

Scope and structure of the work. The dissertation consists of an introduction, a literature review, five chapters, a conclusion, a list of references and appendices. The thesis consists of 135 pages, including 23 figures, and 5 tables. The reference list contains 99 titles.

Summary of work. The introduction contains the relevance of the issues dealt with in the paper, the formulation of the research objectives and the description of the research tasks.

The first chapter reviews the literature sources used for this research topic, describing the general approaches used in the work to address the formulated objectives.

The second chapter describes the problem of supplier-customer interaction in comparing reliability performance of rolling stock components. There are a number of reliability structural diagrams which characterise the operational functioning of doors in a wagon. Based on the chosen structural scheme, a state transition diagram is constructed and Kolmogorov differential equation system which corresponds to the given graph is solved. As a result of applying Markov processes apparatus the calculation of failure rate parameter for carriage door set is done.

The third chapter analyses existing variants of statistical criteria in the field of reliability and industrial analytics on the basis of a large body of literature and practical experience in different industrial sectors. A comparison is made on applied reliability problems.

In chapter four, the rationale for short-term reliability forecasting is given, then a prediction is made using the Holt – Winters method on the basis of operational data. Time series adaptation parameters are selected using survival functions. Examples of forecasting on real data are given.

The fifth chapter pays much attention to the reliability component due to the human factor. On this basis, a model for minimising the unit cost of system maintenance is developed and investigated. The final part of the study presents a method for constructing and minimising a replacement cost function for components of a complex system using the main results of the entire study.

The conclusion summarises the work, notes the relevance of the results for today's manufacturing sector, and reflects the prospects for the development of this research.

CHAPTER 1. Overview of Methods

The fundamental sources on reliability are well-known works [1], [13], [14], [15], [16]. The books describe the basic apparatus of reliability theory necessary for any task in engineering practice.

Relevant sources on many areas of reliability theory and its applications are [17], [18]. Publications presented in these journals allow reliability specialists in any vector of its application to take into account new trends and in-demand areas of development in their fields.

A study of complex systems with ageing elements to derive reliability estimates for cases with limited information as well as optimal redundancy issues is given in the fundamental work [19]. The authors have also made a major contribution to the field of optimum preventive maintenance timing; the models developed in [16] have formed the basis for many upgraded later methodologies.

A wide range of methods for the correct statistical treatment of experimental data are given in [20]. The options for describing statistical criteria make it possible to choose a method for any purpose. It is worth noting that this book describes in detail applied statistical analysis specifically for engineering fields by means of examples.

Monograph [2] forms a modern multidisciplinary approach to reliability analysis. This edition discusses important methodologies in systems reliability research, such as Fault Tree Analysis, Failure Mode and Effects Analysis, Root Cause Analysis, and many other methods necessary for a full-fledged calculation of reliability metrics. One of the authors of this paper has also published a book on Risk Assessment [21], which not only provides an extensive theoretical basis for application in the field of especially hazardous failure analysis, but also allows to dive into the subject in terms of considering historical incidents.

In [22] considerable attention is given to the role of modern applied mathematics in the field of engineering. A large part of the work is devoted to logic and probabilistic methods, which are not often and extensively discussed in the literature. The author emphasises the clarity of the logic and probabilistic calculus required in the field of security, and outlines the problem of technical service professionals' mastery of these methods.

In order to determine reliability performance, manufacturers carry out reliability tests. This is often long-term and expensive, especially with rapidly changing technical specification requirements. To reduce the cost and time of testing, accelerated reliability assessment is used by means of a probabilistic-physical approach to solving it. These a priori and a posteriori reliability calculation methods are given in [23]. Also, basic tools for correct interpretation of test results are proposed in [24].

Details of sensitivity analysis (methods of estimation of influence of system parameters tolerances on its characteristics), construction of graphs on various electronic schemes are considered in [25]. The book contains also the extensive description of Boolean models of reliability, that allows to conduct popular in modern analysis of risks Fault Tree Analysis with use of method of the minimum sections and minimum paths.

A variety of practical methods for determining reliability of technical systems are outlined in [26]. Most importantly, the book pays special attention to such aspects as determination of reliability parameters at different stages of the system life cycle: design, construction stage, series production, as well as verification of reliability requirements during operation.

The full cycle of reliability analysis steps is given in [27]. The author explains how to correctly approach the stage of collecting data on failures as well as of design review, types of redundancy, and also describes the stage of repair and maintenance works evaluation. Of particular value is the detailed description of the importance of studying the dynamics of changes in the failure rate parameters and the specifics of their behaviour at different stages of a technical object's life cycle.

Of great practical relevance to the application of theoretical reliability distribution functions is work [28]. The author details a large variety of statistical hypotheses with different types of samples, as well as methods for assessing process suitability and decision-making based on control charts.

The peculiarities of the application of theoretical distribution laws to estimate the reliability of technical systems with different types of failure processes are described in [29]. Also in this paper, fundamental deductions on the transformation of recovery functions are given.

Design methodology with regard to the reliability of mechanical objects is elaborated in [30]. The author describes the field of failures caused by fatigue failure due to cyclic loading, breaking down catastrophes and mass failures that have occurred in the history of global industry. Numerous schematic illustrations are provided to complement the methodology and identification of failure mechanisms. A special emphasis is placed on accelerated reliability testing, which is an effective method of determining the lifetime of products at the lowest cost, up to the production phase.

The theoretical foundations of technical systems maintenance as well as aspects of mathematical models implementation for estimation of serviceability indicators are presented in [31]. The work contains both description of basic concepts of reliability theory and optimal models of object operation according to durability indicators.

Human Reliability Assessment (HRA) has gained a significant place in the field of reliability and safety in recent years. This section describes the assessment of the human factor impact using quantitative and qualitative methods. A systematic approach covering many aspects of human factors influence is presented in [32]. Because of the increased hazards and environmental impact of the nuclear industry, this topic is dealt with in detail, both with meaningful examples of nuclear accidents on the International Nuclear and Radiological Event Scale (INES) and in terms of the basic tools of the logic and probabilistic approach of reliability theory. The work of the same author [33] is of great value for applications in the field of transport systems. The role of personnel and passenger errors in the operation of various types of transportation facilities is of great importance, so the methods and experiences described in this monograph have potential for development in the field of reliability assessment of rolling-stock components.

1.1 Basic Concepts

Here are the basic concepts and terms of reliability theory used in this paper of this paper. The list is based on standard [34].

Dependability is a property of an object to retain over time, within prescribed limits, the values of all parameters that characterise the ability of the object to perform the required functions in the specified modes, conditions of use, maintenance strategies, storage and transportation.

Remark. Dependability is a complex property which, depending on the purpose of the object and its conditions of use, may include reliability, durability, maintainability and storability, or certain combinations of these properties.

Reliability is a property of an object to remain in serviceable condition continuously for a certain time or runtime under specified modes and conditions of use.

Maintainability is a property of an object which consists in its adaptability to maintain and restore the serviceability of the object by means of maintenance and repair.

Recoverability is a property of an object which consists in its ability to recover from failure without repair.

Durability is a property of an object to remain in serviceable condition until a limit state is reached under an established system of maintenance and repair.

Storability is a property of an object to retain, within specified limits, the values of parameters that characterise the ability of the object to perform the required functions during and after storage and/or transportation.

Perfect (flawless) state is a state of an object in which all the parameters of the object meet all the requirements set out in the documentation for that object.

Imperfect state (flaw) is a state of an object in which at least one parameter of the object does not comply with at least one of the requirements set out in the documentation for that object.

Up state condition is a state of an object in which the values of all the parameters that characterise its ability to perform a given function meet the requirements of regulatory and technical documentation.

Remark. Lack of necessary external resources may hinder the operation of a facility, but this does not affect the facility's stay in an operable state.

It is also worth noting that a technical object may be defective but still operational. For instance, the presence of a defect (deformation, corrosion) indicates non-compliance with certain requirements of the technical documentation, but does not affect the performance of the given functions.

Down state condition is a state of an object in which the value of at least one of the parameters that characterise the ability of an object to perform a given function does not comply with the requirements of the documentation for that object.

Remarks.

1. For complex objects it is possible to divide their down states. In this case, partially down states, in which the object is able to partially perform the required functions, are separated from the set of down states.

2. A flawless object is always up state, a flaw object can be either up state or down state. An up state object may be up state or flaw, an down state object is always flaw.

Failure is an event consisting in the disturbance of the operable state of an object.

Remarks.

1. Failure can be complete or partial.
2. Complete failure is characterised by the object moving into an inoperable state.
3. Partial failure is characterised by an object moving into a partially inoperable state.

In [1] a failure is formulated as an event, after the occurrence of which, the output characteristics of the hardware are out of acceptable limits.

Sudden failure is a failure characterised by a sudden transition of an object from an operable state to an inoperable state.

Degradation failure is a failure due to the natural processes of ageing, wear and tear, corrosion and fatigue, provided that all established rules and/or standards of design, manufacture and operation of the object are respected.

Operating time (runtime) is the duration or workload of the facility.

Remark. Runtime can be a continuous value (operating time in hours, kilometres travelled, etc.) or a discrete value (number of operating cycles, starts, etc.).

Restoration time is the time spent directly on the operation of the facility's recovery.

Restoration is a process and event of moving an object from an inoperable state to an operable state.

Remarks.

1. Restoration as a process is characterised by the operations and duration of time from the moment a failure occurs to the moment the object is restored to an operable condition.

2. Restoration as an event is characterised by the moment at which an object is restored to an operable state after a failure.

(Instantaneous) failure intensity is a limit of the ratio of the probability of failure of a recoverable object in a sufficiently short time interval to the duration of that interval, tending towards zero.

Restoration rate is the conditional probability density of recovery of an object's operable state, determined for the time point in question, assuming that the recovery has not been completed before that point.

Redundancy is a way of ensuring the reliability of a facility by using additional means and/or capabilities beyond the minimum required to fulfil the required functions.

Dependability specification is the establishment of quantitative and qualitative requirements for the reliability of a facility in the normative and technical documentation.

Remark. Dependability specification includes selection of a nomenclature of standardised reliability indicators; establishment and feasibility study of reliability parameters of an object and its components; setting requirements for accuracy and reliability of initial data; determination of criteria of failures, damages and limiting states; setting requirements for reliability control methods at all stages of the object life cycle.

Specified dependability measure (indicator) is a reliability index, the value of which is regulated in the normative and technical documentation of a facility.

1.2 Organisation of Reliability Program in Enterprises

Virtually every manufacturer struggles to expand and maintain market share in its industry. The quality and reliability of a product are competitive features for this purpose. At the same time, achieving a certain level of reliability must be optimally matched to the costs involved. One of the ways of realising this is competent planning of reliability work at all stages of the technical facility life cycle [35]. The cumulative approach to the reliability assurance program at different stages of project creation is both rapidly being implemented at modern production [36] and mentioned by authors of theoretical works in the early years of the development of this field [37]. Reliability measures are not independent of each other: they are integrated into the engineering design in stages and contribute to the optimal completion of each stage of the life cycle. The multifaceted approach to enterprise project implementation in an applied and theoretical form is described in [38]. The following tasks and stages are not exhaustive, it is one possible way of organising the work on the reliability of a technical system.

This section presents a system for building and ensuring reliability. It takes into account all stages of the product life cycle. Its functional structure is shown in Figure 1.1. Each stage/block has its own meaning, and as a whole the scheme gives an idea of the general algorithm for ensuring reliability and optimisation of production processes, taking into account this most important factor. Detailed description and filling of the stages is given in the following sections of the thesis.

The reliability building system developed in this study can both be applied partially at different stages of project creation in the product life cycle direction (see Fig. 1.1) and be used to model functional relationships between the fundamental areas of reliability assessment when forming a complex algorithm.

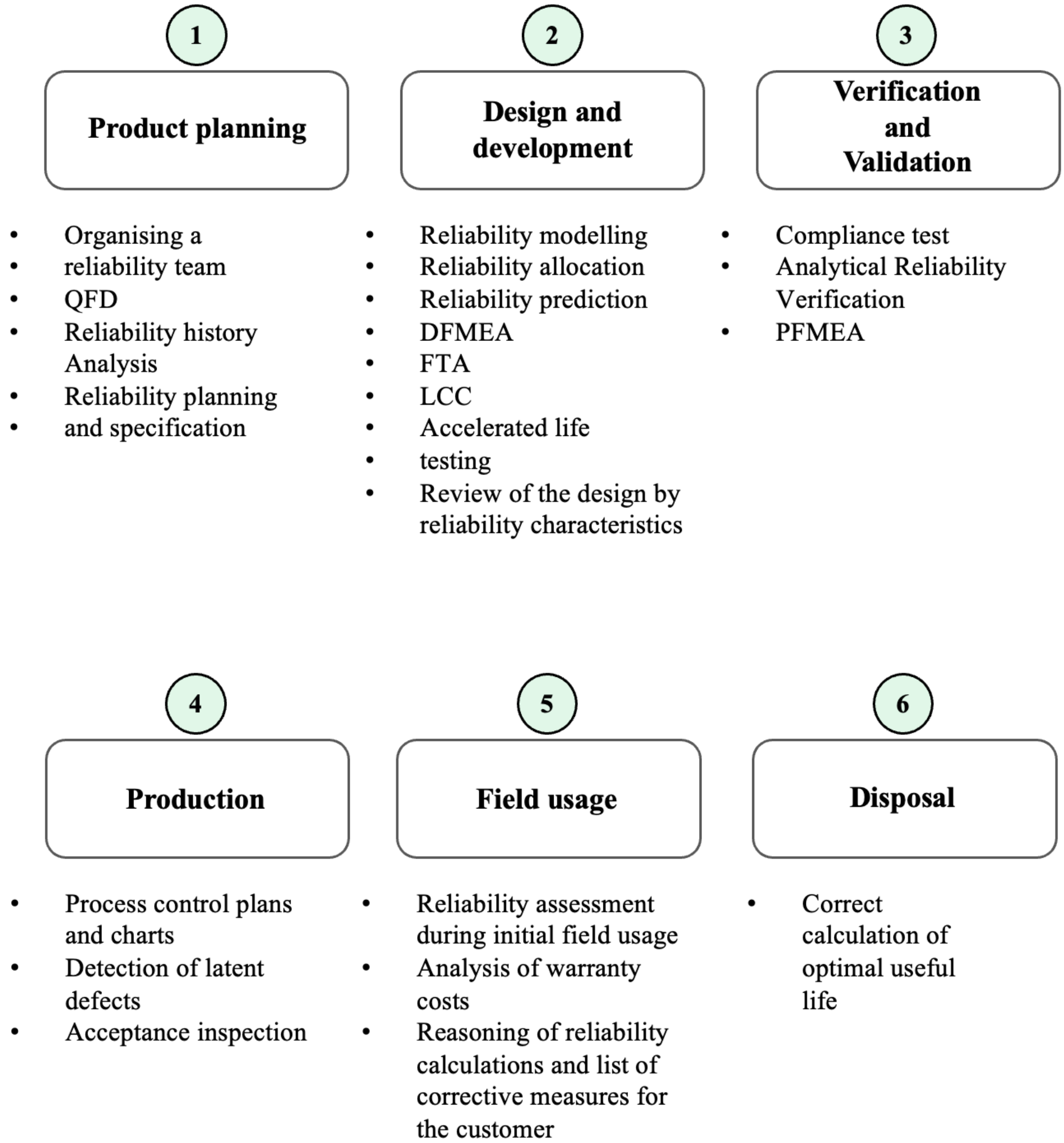


Figure 1.1 — Stages in applying the methods of this study to the various stages of a technical project

Separately, each of the sections can be applied to other reliability tasks at different stages of the project. All the reliability processes to which the methods developed in this study apply are shown in Figure 1.1 in the sequence of project phases.

Schematically, the main blocks of reliability management, which are described sequentially in the chapters of this study, are shown in Figure 1.2.

The initial stage is *Product planning*. In Figure 1.1 this stage is marked as Block №1:

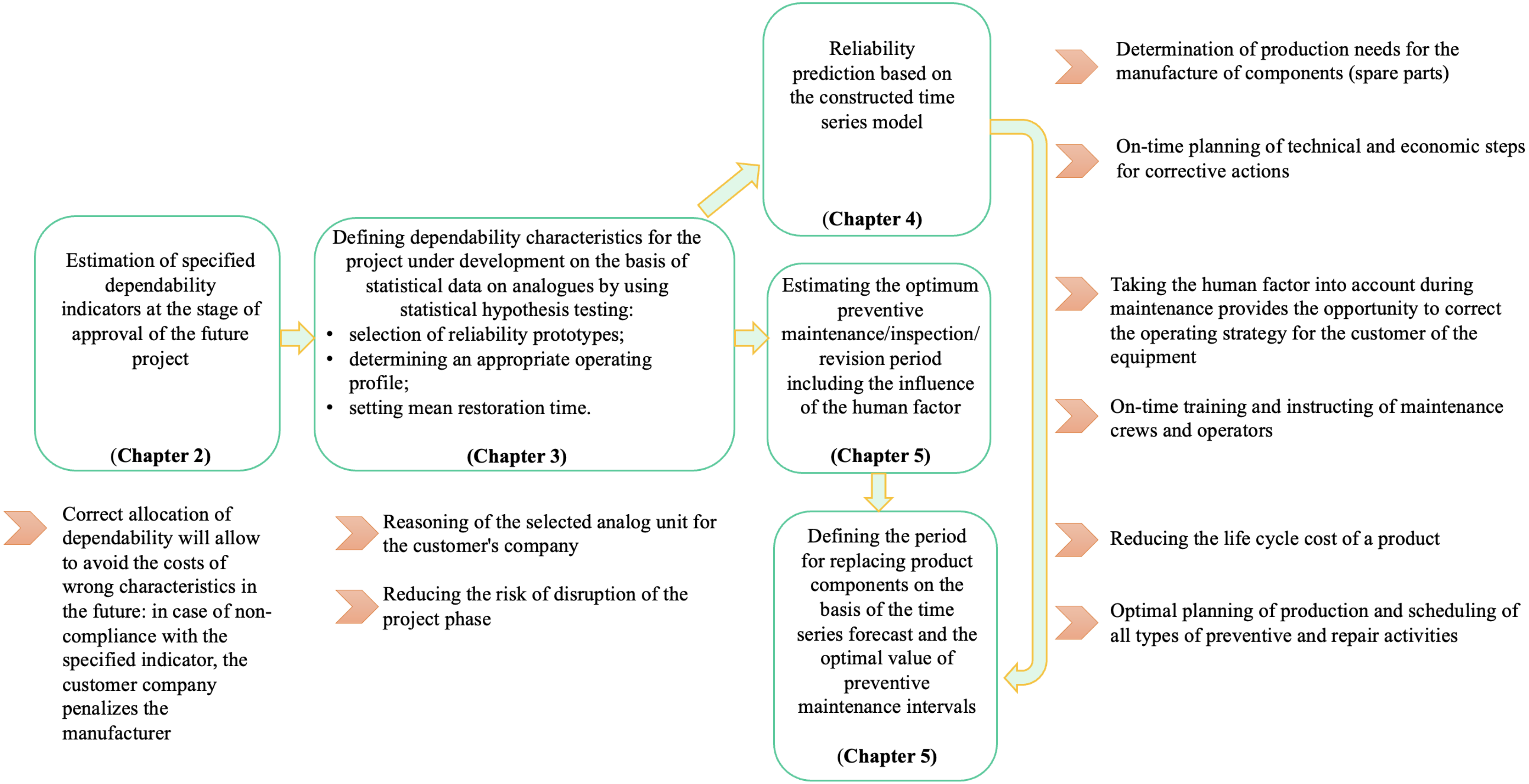


Figure 1.2 — Developed methods for technical product cycles

1. *Organising a reliability team:* the first step is to review the project with several units. This will help to build the reliability management processes of the project, taking into account both reliability analysis and design factors, manufacturing and maintenance technology, testing, operation, and market research.

   The interrelationship of reliability bureaus with other structural units of manufacturing companies is discussed in [7].
2. *Quality function deployment (QFD).* Competition is forcing manufacturers to produce systems which meet customer requirements as closely as possible, also in terms of reliability. *QFD* enables important customer expectations to be highlighted and transformed into technical requirements. The design, production and verification of the project are then based on these expectations.
3. *Analysis of historical reliability data:* collection and analysis of reliability data of peer products. This procedure makes it possible to identify problems in the operation of previous generations of systems, whether the product to be developed will have similar features and to find opportunities for improvement.
4. *Reliability planning and specification:* establishing an equilibrium level of reliability. The reliability team determines the reliability objective value by comparing all factors, taking into account economic feasibility and the developed reliability program.

The next step in terms of reliability performance is *Design and development.* This block of tasks allows to take into account potential failures in the use of the developed object and to ensure its robustness against them. Let's review the main options for reliability analysis at this stage.

1. *Reliability modelling:* representation of the architecture of a technical object. Most often, the logical relationships are represented in the form of reliability block diagrams. An application of this approach is presented in Chapter 2.

2. *Reliability allocation:* reliability objective value set in the product planning phase must be correctly accounted for in the subsystems and elements. The reliability of a system is expressed as a function of the reliability of its constituent components, and this relationship must be correctly established according to several criteria: the individual technical characteristics of each element and their logical relationship in terms of functioning as part of a complex system.

   The problem of reliability distribution is addressed in the design of any complex system. In particular, when specifying standardised reliability parameters for large rolling-stock units (railway vehicles, wagons, etc.), it is necessary to take into account the inclusion of rolling-stock components in the overall structure of an enlarged object. An example of such a task is discussed in Chapter 2. An approach to this step based on a non-linear programming problem, where the end goal is a function minimum of some technical and economic indicator, is given in [39].
3. *Reliability prediction:* A project design calculation is usually performed to compare several design variants of a product under development in terms of reliability performance. This step is often referred to as «reliability forecasting» because of the fact that the values of reliability characteristics at the initial stage are carried out with a certain degree of uncertainty. This process is quite variable in methods. Some manufacturers use outdated but popular approaches in the form of manuals on failure rates (data have been collected for different types of components since the mid-twentieth century). In this case an exponential distribution of any type of component in a complex system is usually assumed and total failure rates are constructed by summing up the failure rates of its constituent components.

   At the same time, a large number of works are devoted to theoretical distribution laws for evaluating empirical failure data. The collection of operational data from many technical devices has made it possible to identify groups of systems that can be aggregated according to the change in failure behaviour over time [40]. Examples of such grading are shown in Figures 1.3 and 1.4.

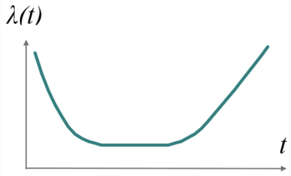
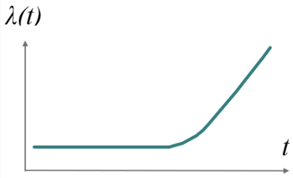
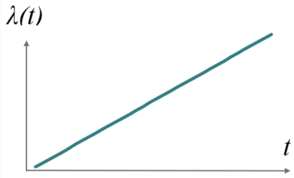
Failure behavior		General characteristics	Typical examples
A		**"bathtub curve"**	**old steam engine (late 18th to early 19th century)** **Various modern complex systems**
B		**simple devices** **complex machines with bad design (one single dominating type of failure)**	**car water pump** **shoelace, cords** **Chevrolet Vega engine (1974)**
C		**structures** **wear-out elements**	**car bodies** **automobile tires**

Figure 1.3 — Options for changing the failure rate parameter for wear and tear failure types

4. *Design failure mode and effects analysis (DFMEA):* proactive tool to detect and correct design flaws by analysing possible failure modes, effects and mechanisms, followed by a recommendation for corrective actions. This analysis is a cross-functional approach to the reliability of the designed object at this stage of the project. *FMEA* was one of the first systematic methods of systems failure analysis. This method was developed by reliability analysts in the late 1940s to identify problems in military systems prematurely.
5. *Fault tree analysis (FTA):* logic diagram showing the relationship between a potential failure in a system and its causes. The causes at the lowest level are called basic (root) events and can be component failures, environmental conditions, human errors and normal events,

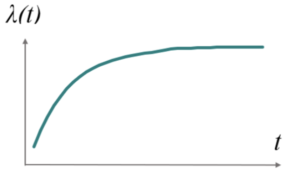
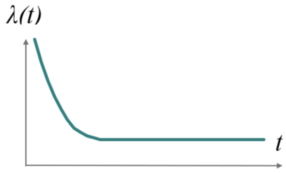
Failure behavior		General characteristics	Typical examples
D		complex machines with high-stress trials after start of operation	high pressure relief valves
E		well designed complex machines	gyro compass pressure centrifugal pump
F		electronic components complex components after corrective maintenance	computer “mother boards” programmable controls

Figure 1.4 — Options for changing the failure rate parameter for random failure types

that is, events that are expected to occur during the lifetime of the system. *FTA* was introduced in 1962 at Bell Telephone Laboratories in connection with a safety assessment of the Minuteman intercontinental ballistic missile launch control system. *FTA* is one of the most frequently used methods in risk and reliability research. As quantitative inputs for this analysis, probabilistic characteristics for facility elements (failure and recovery rate parameters) and events that are part of the hierarchical tree structure are used. The final quantitative assessment of product reliability depends on these characteristics. The methods and approaches presented in Chapter 3 allow to adequately determine these characteristics on the basis of data from already operating facilities.

6. *Life cycle costs (LCC):* calculation of the costs that are expected over the life cycle of an asset. The term refers to all costs associated with the development, acquisition and ownership of an asset. A very strong emphasis in the customer’s analysis of these calculations is placed on

the costs associated with the reliability of the product: planned and unplanned costs. Scheduled costs are related to the maintenance of the asset (inspection, diagnostics, preventive maintenance, lubrication). Optimisation of costs for this item is presented in Chapter 5. Unplanned costs arise from failures in operation: they are associated with both the direct replacement of the defective element, and with the consequences of failure (fines, reduction in the potential profit from the operation of products). The algorithm for reducing this type of cost is made up of the models presented in Chapters 2, 3, 4, 5. First, the indicators per node are correctly set in the design calculation based on the statistical criterion (Chapter 3). Then, based on the predicted time series values for each node, a quantitative estimate of failure-free performance is generated (Chapter 4). Based on these values and the optimum preventive maintenance time in Chapter 5, the replacement periods of the nodes are determined using dynamic programming apparatus.

7. *Accelerated reliability and/or durability testing:* in today's competitive environment, the time allocated to each of the project phases is sought to be reduced in a reasonable manner. Testing is one of these processes. As products become more reliable as technology advances, degradation failure research is more difficult to carry out because a large number of hours are required to simulate the ageing stage. Ignoring testing, although it will reduce costs during the verification phase of the project, may cause much higher costs in the subsequent use of the facility [41].
8. *Review of the design by reliability characteristics.* There should be several checkpoints in the reliability assurance program at which the design is reviewed for a number of characteristics. The performance of these revisions makes it possible to assess whether, and if so why, the reliability performance objectives are being met. If necessary, the

cross-functional analysis team should develop measures to improve reliability.

The next step is to check that the product being developed meets the target reliability requirements. This step decides how the product will be manufactured. Let's look at the main processes involved in this step.

1. *Compliance test.* The objective of this stage is to test the product in a minimum amount of time and with a small sample size for compliance with the specified (standardised) values. For this purpose, special programs and methods are developed for various types of tests, calculating the sample size, the supplier's and the consumer's risks. Statistical experiment planning [42], [43] are used to conduct tests competently.
2. *Analytical reliability verification.* It can be too expensive and time-consuming to carry out expensive tests in several phases of a project. In such cases, if adequate mathematical models are available that reflect changes in the product during its life cycle, verification can be performed virtually at this stage. These approaches include: finite element analysis, computer simulation (creating a digital twin) and numerical calculations.
3. *Process failure mode and effects analysis (PFMEA).* Previously there was a failure mode and effects analysis of potential design failures, but now the same procedure is performed for the manufacturing process. This work is implemented to ensure the quality of safe assembly operations, minimising the risk posed by the component manufacturing process and product assembly [44].

1. *Process control plans and charts.* In industrial statistics, there is often an emphasis on identifying process variability. This is because manufacturing variability increases the risk of «infant mortality»: a period of high failure rates early in the life of an object. To minimise the cost of early failures due to variability in manufacturing, process

control plans and maps are developed for critical operations. Also called Statistical Process Control (SPC), this trend was started by Walter Schuchart, who in 1924 dealt with the variability of lamp performance for the AT&T corporation [45].

2. *Detection of latent defects.* Many technical objects contain internal defects which can be identified by special methods. If such samples are sent to the customer, they will fail at an early stage. In order to immediately identify such items, manufacturers use 5 types of screening, and the interpretation of the results is also done using industrial statistics.
3. *Acceptance inspection.* This stage decides the rejection/acceptance of a batch of products based on various measurements. If batches that do not meet the specified specifications are not rejected, defective products will cause customer dissatisfaction and adversely affect future performance.

The next step starts the direct operation of the technical system. This step is usually accompanied by a large information load: fast tracking of failures during the running-in period, handling of customer complaints, verification of compliance with reliability requirements for purchased components. However, this phase is very important for using primary data about the behavior of the technical object for future reliability management activities. Let's take a look at the main components of this phase.

1. *Reliability assessment during initial field usage.* The information to be processed for the calculations must be as detailed as possible: the type of failure, the external conditions, the load level, the exact start time of the failure, and the repair time must be known. This is further complicated by the fact that the maintenance personnel are only getting acquainted with the new object, and the information is often distorted by the large number of intermediate links before it

is received by the reliability specialist. This problem has been partly addressed in [8], [6].

2. *Analysis of warranty costs.* In order to calculate product life cycle costs, it is necessary to understand how the actual warranty costs correspond to those predicted in the LCC design phase. In addition, a rapid and high-quality generation of corrective actions will help to optimise this type of cost. Correct processing of failure statistics as discussed in Chapter 3, constructing an accurate forecast of failure rates for future periods of use as proposed in Chapter 4, and accounting for human error in repairs using the algorithm in Chapter 5 can minimise warranty costs at this stage of the project.
3. *Reasoning of reliability calculations and list of corrective measures for the customer.* All activities in the controlled operation phase are coordinated between several companies. In the rail component industry, the manufacturer, the customer and the operating organisation are often the main actors in the dialogue. The models, methods and approaches used must be adequately justified so that all parties can agree on various technical solutions.

1.3 Calculation of Failure-Free Parameter of Recoverable Systems on Markov Models

For a long time in the history of industrial development, the various components of technical products were manufactured almost individually – at production sites on a smaller scale than the current ones. The increase in mass production and the build-up of industrial capacity has led to the need to standardise components, which has only relatively recently led to the ability to reliably estimate component failure rates due to the accumulation of data on the repeatability of failures of each component type. This process is accompanied by an ongoing exchange of information between the manufacturer and the customer, leading to the development of new methodologies for the evaluation and verification of reliability indicators.

In recent years, there has been a development of interaction between suppliers and customers of rolling stock components. This makes it necessary to consider reliability analysis from both the technical system designer and the operating organisation. As a consequence, the task of calculating a failure rate for a set of doors has arisen, although it has always been a per-unit calculation.

Chapter 2 looks at a number of structural reliability diagrams that reflect the way the doors function as part of a wagon. Structural reliability analysis methods have long been used for complex technical systems. This method helps to show the logic of interconnection of component parts and product functioning and to modify the system: to complicate or simplify it in order to eliminate the effect of excessive redundancy. However, this type of calculation usually assumes that the failure mechanisms of the components in question operate independently of each other – the failure of one element does not affect the functioning time of another. Such an assumption cannot be applied to the organisation of an electronic component structure because, for example, a control unit failure could overload the gearmotors. As a consequence of this fact, a shift from structural diagrams to Markov analysis was made in order to calculate the reliability of rolling stock components. Structural studies based on graph theory are discussed in [39]. A detailed application of Markov analysis for various reliability purposes is given in [46]. The results of this approach are described in Chapter 2, published in [5].

1.4 Analytical Criteria for Comparing Reliability Characteristics of Two Samples

In reliability theory and industrial statistics, the application of statistical hypothesis testing criteria plays a major role. This part of the study applies to all technical system development blocks from scheme 1.1. This is due to a number of advantages:

1. **System reliability assessment.** Statistical criteria allow to define many characteristics necessary for reliability calculation: groups of peer units, laws of distribution of operating time before failure, operational characteristics, repair and maintenance conditions, intensity of object use and many other factors, required to provide the necessary level of technical system reliability.
2. **Decision-making.** Often at different stages of the product life cycle it is necessary to find the optimum maintenance, repair or upgrade strategy. This requires a comprehensive approach to analysis, including a good comparison of different statistical data based on statistical criteria. This is due to the fact that reliability is an interdisciplinary field, and an expert opinion on an issue may not be sufficient for making a decision.
3. **Quality improvement in production.** Statistical criteria for hypothesis testing can identify bottlenecks at various stages of the production process, reduce rejection rates and identify key factors affecting the quality of manufactured products.
4. **Economic effect.** Hypothesis testing helps to identify suitable component suppliers, determine appropriate product destinations, and rule out adverse factors in the operation of technical systems.

Because of these factors, Chapter 3 analyses the recommended statistical criteria in reliability theory [15], [22], [28], [37], [47], [48], and a wider range of criteria [49], [50], [51] are considered. These hypothesis testing methods belong to several types of classes and can be applied for different purposes. We present a new type of nonparametric criterion, not previously prevalent in the reliability analysis of technical systems of the category «location-scale» – a simultaneous test for equality of central tendency and variance parameters. It has been tested on variable reliability problems and its results have shown the priority of the criterion by several characteristics.

1.5 Predicting Reliability Based on Time Series and Survivability

The most common approach in reliability theory is to refine probabilistic analysis methods by applying theoretical distribution laws. After identifying the form of the distribution law, several numerical characteristics of the corresponding theoretical distribution of the reliability function are calculated. An important advantage of this approach is the simplicity of calculations and not very difficult assessment of the reliability of the obtained results. On the basis of these methods of statistical evaluation of reliability properties one can find out whether or not the chosen theoretical distribution law of mean time between failures contradicts the empirical data. Most often the set of statistical data on failures of components and assemblies of access systems in train cars is relatively insignificant. Often, even well-studied components of rolling stock can behave differently from the expected theoretical distribution due to a number of factors: new operating conditions, installation errors, incorrect compliance with operating instructions, etc. This imposes restrictions on the use of theoretical laws of distribution of operating time between failures when prescribing measures for repair and improvement of reliability of elements and systems as a whole. In other words, asymptotically during the life cycle of an object we can approximate its reliability by some theoretical distribution law, but in the short term, when some factors affect it during periods of short duration (months and quarters), their applicability for prediction is very limited.

The application of theoretical distribution laws is of great importance in reliability theory. The exponential distribution law is particularly common. The use of this approach greatly simplifies statistical processing of both bench and in-service test results. However, an important limitation is its suitability only for aging-free systems in the period of «normal operation», when random failures prevail (Fig. 1.5). At the same time, some authors [52] say that this type of failure distribution is not typical for software reliability, due to the fact that they are usually caused by design flaws. Accordingly, the period of «normal operation» in this case cannot be approximated in the same way as in technical systems, and the exponential distribution law for this type of problems has a number of limitations.

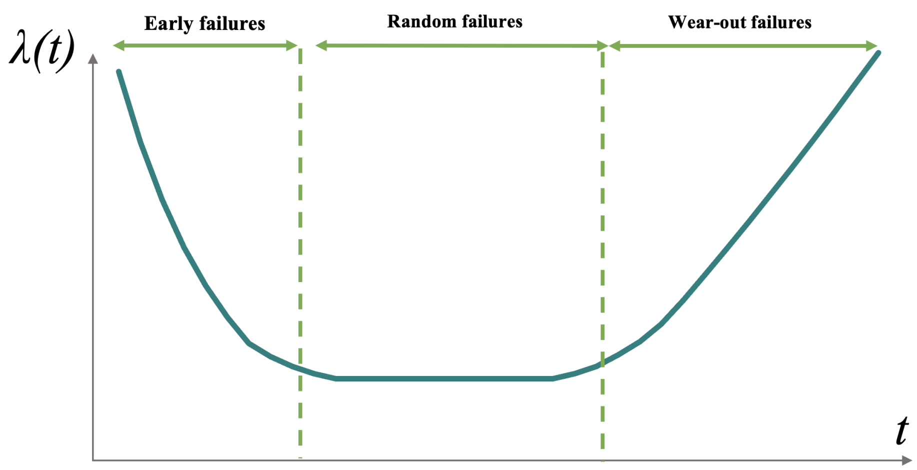


Figure 1.5 — Options for changing the failure rate parameter for wear and tear failure types

The exponential distribution is not suitable for describing the runtime of products in the presence of in-service failures or in the event of a period of wear and tear [29]. In addition, with the changing component composition of a complex technical system, the general patterns of behaviour of the whole product during even normal operation change, so the application of the exponential distribution is often limited even in a given part of the lifecycle of many products [16]. Moreover, there are studies that suggest that the widespread use of this type of distribution law is a major cause of inadequate solutions to reliability problems. This is particularly true for predicting the durability of technical objects [23]. In particular, in [13] an example is given that works that are devoted to testing hypotheses about the type of distribution law deserve special attention, because very often errors occur due to the assumption that an exponential law is applied for the object in question, when in fact the Weibull distribution law takes place.

There are a number of computer programs for reliability and security analysis, both domestic and foreign. One of the most common software products of this type is RAM Commander. In spite of an extensive set of tools for reliability research (reliability block diagrams, database of reference rates of failures, graphs of states and transitions, trees of failures and events), the given software complex excludes possibilities of short-term forecasting of reliability indicators taking into account various influencing factors (season, change of maintenance periodicity, changes of fleet, etc.). In this case short-term forecast of reliability indicators is a necessary auxiliary tool for planning costs of corrective measures, determining production load, optimal allocation of resources to components of one system, which behaviour in time significantly differs. This problem is particularly acute at a time of rapidly changing conditions of industrial sector development. At the moment the issues of technical condition prediction and reliability are still little used in applied engineering practice. Although a large number of theoretical models are available, few of them are actually applied in practice.

To solve the arising problem of short-term forecasting of failure rates for automatic door components in Chapter 4, an analysis was carried out to select a model for predicting the parameter of the failure rate of a given component of a Moscow commuter train. The Holt – Winters method was selected as the forecasting method after a number of validations of other models (Brown, Holt) [53], [54].

The application of the time-of-life analysis apparatus is described in great detail in [55] and [56]. This type of study belongs to one of the newer areas of statistical analysis and is gaining in popularity, expanding the fields of application. Examples of implementations can be found on its basis such as predicting the probability of customer departure, estimating the rate of patient mortality, identifying factors affecting uptime at a certain stage of the life cycle, and much more. This is because Survival Analysis is a statistical technique that is used to analyse the time to a particular event, such as the failure of a technical system. This method is widely used in reliability theory to determine the reliability of a system and to estimate the probability of its failure in the future.

The basic idea of survivability theory is that a system can "survive"or fail at any point in time, and the time to failure can be different for each system. The purpose of time-to-failure analysis is to determine the reliability characteristics of a system, such as mean life, mean time between failures, etc.

Survivability theory is well applicable in the field of reliability analysis because it takes into account the time to failure and not just the fact of failure, which allows us to estimate the probability of future system failure [40]. In addition, survivability theory allows us to account for data censoring, which occurs when the time to failure is unknown for a number of objects in a sample.

Thus, survival theory is a powerful tool for analysing the reliability of technical systems and estimating the probability of their future failure.

In Chapter 4, elements of survival theory are applied to optimise time series parameters. As a result, a model for short-term prediction of the failure rate parameter based on the Holt – Winters method is built, whose adaptation parameters are derived from Kaplan – Meier estimates.

As a result, it is possible to determine whether an entire technical system (wagon doors) will meet the reliability requirements of an enlarged system (wagon/unit) on the basis of the predicted unit values, using the method for determining the normalised reliability figures proposed in Chapter 2.

1.6 Optimisation of System Maintenance with Consideration to Human Factor

Because technical systems are built, assembled, installed and subsequently operated by humans, it is impossible to exclude the human influence on reliability. Human error can affect the reliability and safety of a technical system in different ways. Some of them will manifest themselves quickly, some of them will appear en masse after some time of operation of the object, and some of them will appear with a constant frequency. Besides, the consequences of various types of errors are very different in different industries. For example, an incorrect tightening torque on a bolted connection can cause misalignment and noise in one instance and the removal of a vehicle from service in another.

This area is a separate block of reliability analysis – Human Reliability Analysis (HRA). It was developed around the middle of the 20th century because of the development of high-risk systems (aviation, rocket science, military industry).

Work [7] analyses the reliability links with other structural units, as each category of allied professionals contributes to the reliability of the future system during the product design and maintenance phase.

Scientists in fields such as biometrics, psychology, medicine and sociology are mainly involved in research on the «human-machine» relationship. Together they can be used to create models for evaluation, management and optimisation of reliability. For example, [27] gives examples of models, which could be used to describe the performance and reliability share of human activities. The author mentions that this component varies according to a number of factors: from physical and psychological components to the level of adaptability to the work performed.

A detailed explanation of HRA in the field of transport systems is given in [33]. The author describes the basic tools of reliability analysis, which is based on the «human-machine» relationship in the railway and shipbuilding industry. In addition, the monograph describes a wide range of proactive measures to reduce human errors in these industries.

People also play a key role in nuclear power generation, and their impact on reliability has become an important subject for study because human error can lead to disasters such as those at Three Mile Island (1979) and Chernobyl (1986). In addition, studies by Licensee Events Reports (LERs) and the Nuclear Regulatory Commission (NRC) have shown that about 65% of nuclear system failures are directly or indirectly due to human error. For this reason, a major contribution to the development of HRA is made by this industry [32].

The challenge of balancing system reliability against the cost of maintaining it is one of the most popular challenges in today's industry. A deliberately high level of reliability can result in enormous and unjustifiable maintenance costs. Conversely, a low level of reliability may not incur expensive maintenance costs, but the consequences of such a failure can involve both human life and health hazards as well as huge economic costs. In high-risk industries, the optimum level of cost is necessarily also considered with safety in mind. In [57], for example, the author not only talks about the importance of the human factor in aircraft fail-safety, but also suggests that a high level of safety must be factored into the costing of aeronautical engineering projects. After all, safety depends to a large extent on how humans interact with the technical system.

Thus, the task of developing a model for finding specific maintenance costs that takes into account both the characteristics of the technical system itself and the influence of the human factor on it arises. Optimisation problems for determining the maintenance interval have been formulated [16]. The development of this model is presented in a more recent study [58]. The model is also partly given in [52] for evaluating control system maintenance problems.

Based on the analysis of existing models for finding the optimal period of preventive measures and the analysis of the reliability domain due to human factor, in Chapter 5, a new model is proposed. It is based on the construction of a specific cost function based on conditional probabilities considering human-caused failure (based on the Weibull distribution function model [33]) and technical-caused failure in the inter-preventive period (based on the exponential distribution [37]).

The second objective in this part of the study is to find the optimal replacement time for a system component based on the accumulation of the method already proposed in Chapter 5 with the models and algorithms developed in Chapters 3, 4:

a Determination of missing node data for the analysis (estimated operating profile, recovery time, list of peer nodes, key differences in operation of similar systems, etc.) based on the statistical criterion from Chapter 3.

b Predicted failure-free value for the near future based on the model proposed in 4.

c Calculated value of optimum preventive maintenance time using the algorithm developed in Chapter 5.

Based on these system reliability characteristics, a function is constructed to determine the optimum replacement time for the component in question. This will make it possible to determine the timescale for purchasing/producing this type of component to ensure that the necessary spare parts are available for the initial operational period in the case of new development, and to determine the life cycle cost of the product.

The results of this study are as follows:

1. An algorithm for determining the optimum preventive maintenance period for a technical system has been developed.
2. The behaviour of the resulting cost function as a function of the Weibull distribution parameters is investigated.
3. A model for calculating the optimum replacement time is proposed, taking into account the results of this study described in Chapters 3, 4.

1.7 Chapter Conclusions

This chapter analyses the sources that have been used to select and build models for reliability analysis and prediction. The topic of reliability analysis encompasses a wide range of mathematical methods used in different fields – engineering, as well as economic and social fields, and biomedical statistics. This made it possible to compare the effectiveness of the methods used, to study the literature of diametrically opposed fields, and to trace the evolution of these fields over the last half century. In addition, a general overview of the methods used in each of the chapters is provided, as well as a review of the specifics of their applications. The functional relationship of these chapters to each other is also given, taking into account the stages of development illustrated in the diagrams 1.1, 1.2.

Work [7] analyses the links of reliability with other structural units.

CHAPTER 2. Estimation of Door Carriage Set Failure Rate Parameter Based on Structure Diagram and Markov Process Analysis

2.1 Initroduction

The current challenge is to ensure a high level of rolling-stock reliability. Passenger safety is directly related to the assessment of rolling-stock component failure rates. Suppliers of components for passenger coaches are required to carry out reliability assessments over a period of time agreed with the customer (the rail coach manufacturer). Factories, in turn, report to transport companies by evaluating the reliability performance of their products, i. e. rail coaches. Hence there is a difference in approaches to the quantitative assessment:

1. Manufacturers estimate values per coach, while their suppliers calculate figures for individual units and components;
2. The methodology for assessing reliability performance for coaches differs from the one for rolling-stock components due to the different levels of operation and design. As a consequence, an exponential law will always be applicable to a coach when assessing its reliability, while, for example, for footboards, the exponential, normal, and Weibull distribution laws may be used, depending on the design.

Passenger coach manufacturers consider the reliability figures for the entire coach door set (4–6, depending on the model) according to the following rule: failure rate (or MTBF – Mean Time Between Failures) of the door is multiplied by its number in the coach, which, in terms of structural reliability, indicates an elementary sequential structure, where a door failure equates to a coach failure, which is debatable.

A structural approach has been considered to correctly determine the functioning of the doors within a coach, taking into account the hierarchy of connections. The Reliability Block Diagram (RBD) is a graphical representation of the functional state of a system. The RBD shows the logical relationship of the functioning components necessary for the successful operation of the system [59]. However, RBD based modelling methods are designed for non-recoverable systems where the order of failure occurrence is irrelevant. For systems where the order of failure occurrence must be taken into account and for recoverable systems, other methods such as Markov analysis are more suitable. In the research process, the structural approach was used to represent the overall system functioning, with the help of which a transition to the state transition diagram for the Markov analysis [59] was carried out, taking into account the recoverability of the system.

2.2 Problem Formulation

The aim of the work is to calculate the failure rate of a coach with a 6-door set based on the analysis of possible reliability structural diagrams, followed by transition to state transition diagrams. In general, at the initial stage, in a step-by-step manner it is presented as follows:

1. On the basis of the data on the algorithm of the doors functioning in the car set, determine the key criteria affecting the failure-free operation of the car set of doors.
2. Develop several options for structural reliability schemes.
3. Based on an interdisciplinary analysis, select the scheme that most adequately reflects the actual conditions of functional inclusion in the wagon and select the types of redundancy.

4. Select the method of calculation to move from the reliability index values for the elements of the scheme to the index for the whole system.

2.3 Reliability Calculation Based on the Logical and Probabilistic Approach

The exterior doors are designed for the equipment of all types of passenger coaches with a structural speed of up to 200 km/h (Fig. 2.1). The doors provide a comfortable and safe environment:

– end doors – transition between coaches, side doors – entrance to the coach and exit from the coach to the outside;
– no exposure to sudden changes in pressure and temperature;
– preventing dust and moisture from entering the coach;
– ensuring that the coach is soundproofed and thermally insulated in all operating modes of the train.

Figure 2.2 shows the layout of the different types of doors in relation to the maximum possible coach door set.

The structural scheme of rail coach door set reliability can be considered in a variety of ways. The simplest representation is a non-redundant system, where the failure of any element leads to the failure of the whole system. Then the probability of failure-free operation is calculated according to the known formula:

$$P_{c1}(t) = \prod_{i=1}^{N} P_i(t),$$

where P_i – failure-free probability of the i-th element,

N – number of elements in the system.

A second possible representation of the structural diagram of the reliability of a coach door set is a redundant system. Here we will consider two variants:

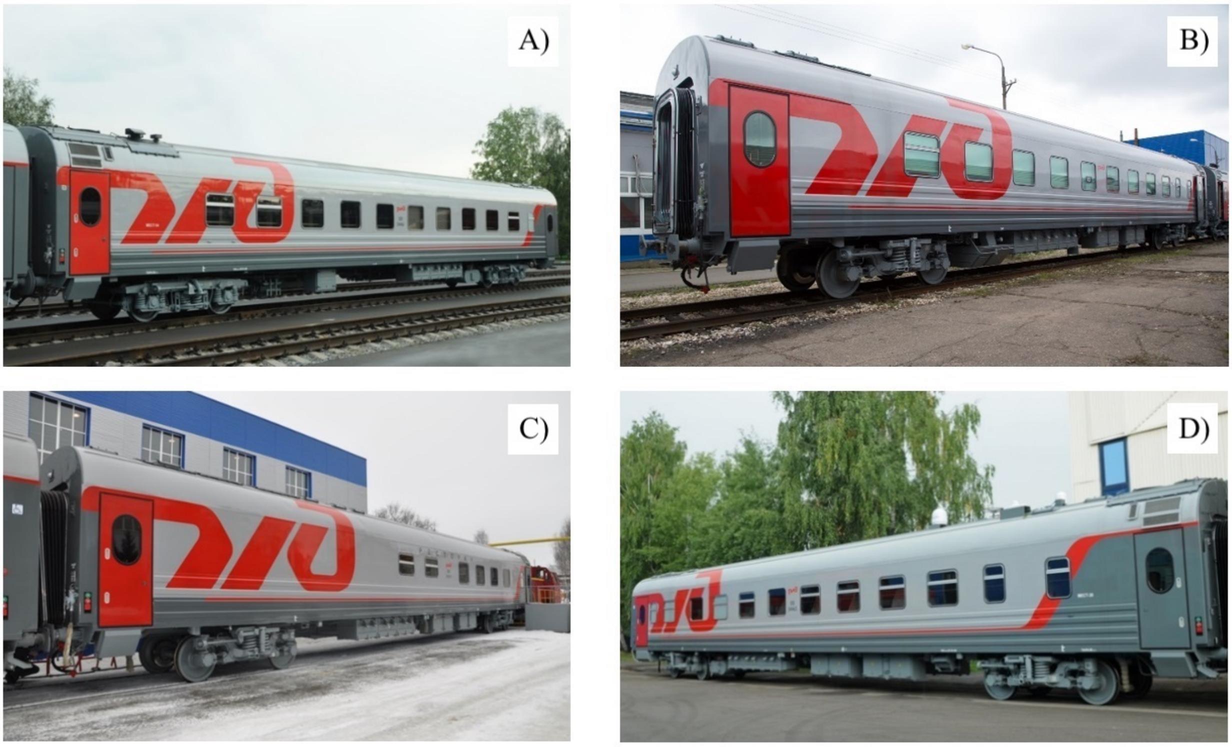


Figure 2.1 — Models of passenger coaches considered in relation to structural reliability schemes:
A) model 61-4447 (non-compartment); B) model 61-4462 (compartment);
C) model 61-4460 (dining-car); D) model 61-4445 (staff compartment) [60]

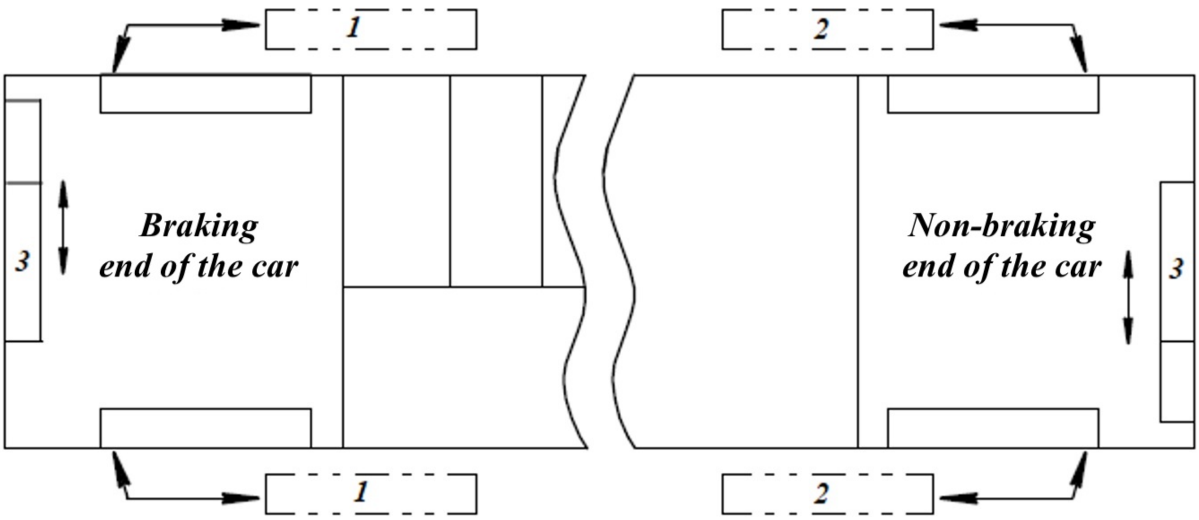


Figure 2.2 — Passenger coach door arrangement scheme: 1 – electromechanically operated side single wing door; 2 – manually operated side single wing door; 3 – electromechanically operated end single wing door

– reserving the manual door with a second manual door (Fig. 2.4a);
– reserving side automatic doors with manual ones (Fig. 2.4b).

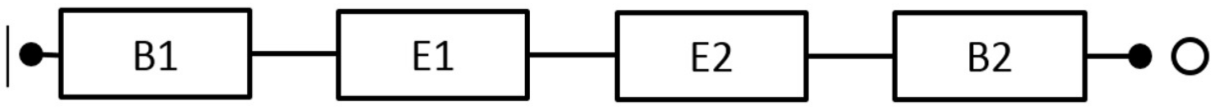


Figure 2.3 — Linear block diagram of coach door set reliability for a passenger coach, where Ei is an electromechanically operated door and Bi is an end door

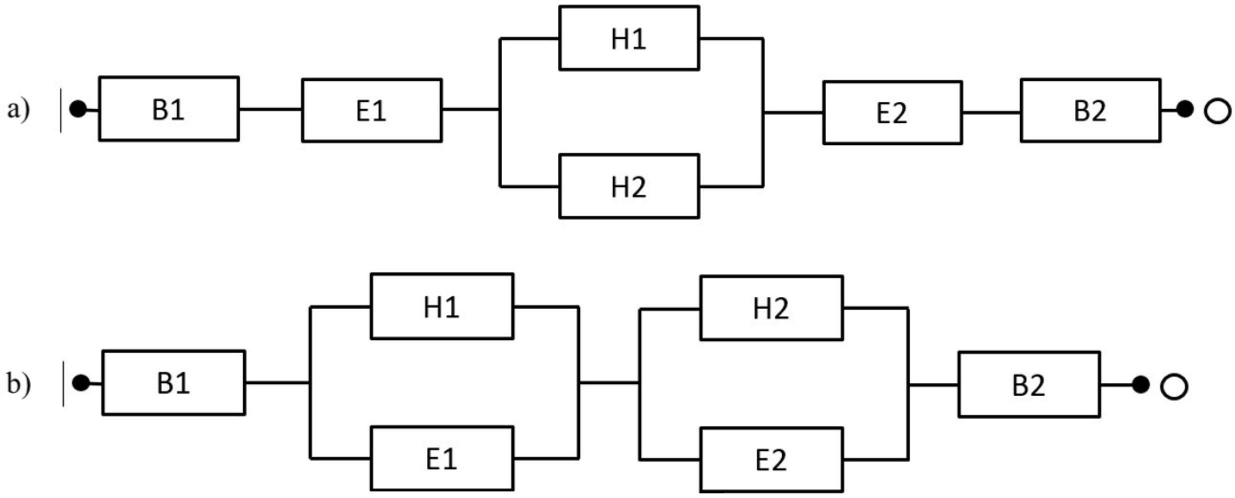


Figure 2.4 — Redundant block diagrams of coach door set reliability for a passenger coach, where Ei is an electromechanically operated side door; Bi is an end door, Hi is a manually operated door

This results in two variants of a mixed system with element-by-element redundancy of individual units.

The probability of failure-free operation with a redundant manually operated door (Fig. 2.4a) is calculated:

$$P_{c2}(t) = P_{B1}(t)P_{E1}(t)P_{B2}(t)P_{E2}(t)(1 - P_{H1}(t))(1 - P_{H2}(t)).$$

The probability of failure-free operation with electromechanically operated side doors redundant with manually operated doors (Fig. 2.4b) is calculated:

$$P_{c3}(t) = P_{B1}(t)P_{B2}(t)(1 - P_{E1}(t))(1 - P_{H1}(t))(1 - P_{E2}(t))(1 - P_{H2}(t)).$$

Another possible approach to structural evaluation is to consider a coach set as an «m of n» structure. A system of this type can be regarded as a variant of a system with parallel connection of elements, the failure of which will occur if less than m elements out of n elements ($m < n$), connected in parallel, are operational.

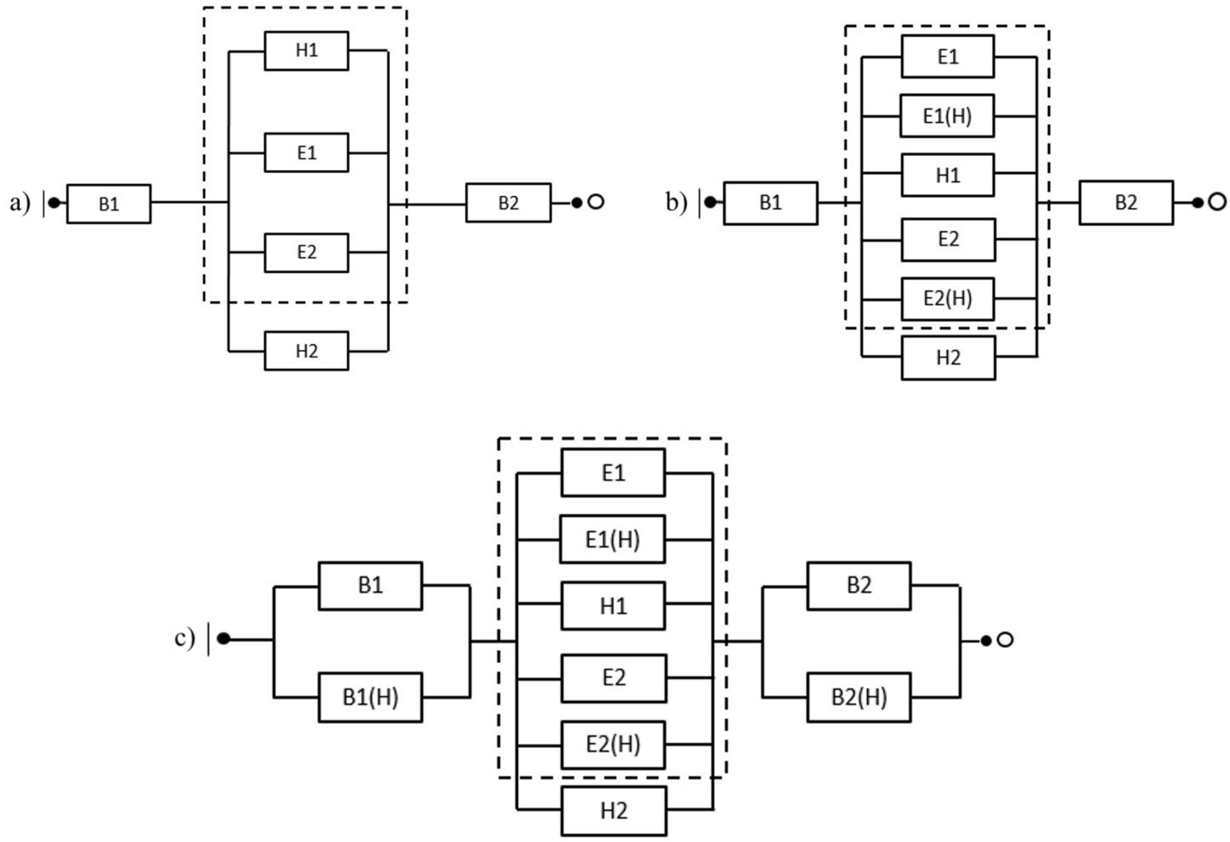


Figure 2.5 — Block diagrams «m of n» to assess reliability of passenger coach door set, where Ei – electromechanically operated door, $Ei(H)$ – electromechanically operated door in manual mode, Bi – end door, $Bi(H)$ – end door with electromechanical drive in manual mode, Hi – door with manual drive

Here are three options to consider:

– system «3 of 4», which is considered functional when doors $H1$, $E1$ and $E2$ are working (Fig. 2.5a);

– system «5 of 6», which takes into account the possible operation of electromechanically operated doors in manual mode $Ei(H)$ (Fig. 2.5b);

– system «5 of 6», which takes into account the possible operation of electromechanically operated side and end doors in manual mode $Ei(H)$ and $Bi(H)$ (Fig. 2.5c).

The two structures presented are mixed: sequentially connected or element-by-element reserved end doors are added to the «m of n» structure.

Then, assuming all the elements that make up the «m of n» structure are equally reliable, the probability of failure-free operation of the structure, shown in Figure 2.5a, will be equal to:

$$P_{c4}(t) = P_{B1}(t)\,P_{B2}(t)\left(4P_m(t)^3 - 3P_m(t)^4\right),$$

where $P_m(t)$ is an element of the «m of n» structure.

The probability of failure-free operation of the structure shown in Figure 2.5b, under the same conditions, will be equal to:

$$P_{c5}(t) = P_{B1}(t)\,P_{B2}(t)\left(6P_m(t)^5 - 5P_m(t)^6\right).$$

Finally, the probability of failure-free operation of the structure shown in Figure 2.5c, where both manual and automatic operations of the end door are considered, will be equal to:

$$P_{c6}(t) = (1 - P_{B1}(t))\,(1 - P_{B1(H)}(t))\,(1 - P_{B2}(t))\times$$
$$\times\,(1 - P_{B2(H)}(t))\left(6P_m(t)^5 - 5P_m(t)^6\right).$$

Let us estimate the failure rate for the structures shown in Figures 2.4b, 2.5a–c. The MTBF of the system can be represented as:

$$T = \int\limits_0^\infty P(t)\,dt = \int\limits_0^\infty \left(1 - \left(1 - e^{-\lambda t}\right)^n\right)\,dt.$$

Let's perform a variable substitution:

$$1 - e^{-\lambda t} = x \Longrightarrow t = \frac{1}{\lambda}\ln\frac{1}{1-x} \Longrightarrow dt = \frac{1}{\lambda(1-x)}\,dx.$$

Then:

$$T = \frac{1}{\lambda}\int\limits_0^\infty \frac{1-x^n}{1-x}\,dx = \frac{1}{\lambda}\int\limits_0^\infty \frac{-(x-1)\left(x^{n-1} + x^{x-2} + \ldots + x + 1\right)}{1-x}\,dx =$$
$$= \frac{1}{\lambda}\left(\frac{x^n}{n}\bigg|_0^1 + \frac{x^{n-1}}{n-1}\bigg|_0^1 + \ldots + \frac{x^2}{2}\bigg|_0^1 + x|_0^1\right) = \frac{1}{\lambda}\left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}\right).$$

Thus, for the «m of n» structure we get:

$$T = \frac{1}{\lambda}\left(\frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2} + \ldots + \frac{1}{m}\right). \tag{2.1}$$

By substituting numerical values into (2.1), Table 2.1 gives a range of values for the failure rates depending on which structural reliability scheme can be adopted for examination. The group of experts, dealing with this type of doors at different stages of the life cycle, has adopted the «3 of 4» scheme (Fig. 2.5a) for calculation.

Due to the fact that the logic-probabilistic approach, although quite easy to calculate, does not take into account the door recovery process, the version of estimating the failure rate using Markov processes was considered.

2.4 Reliability Calculation Based on Markov Random Processes

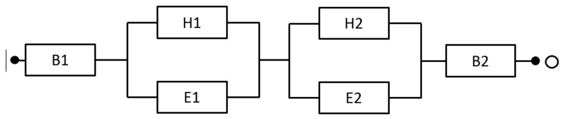
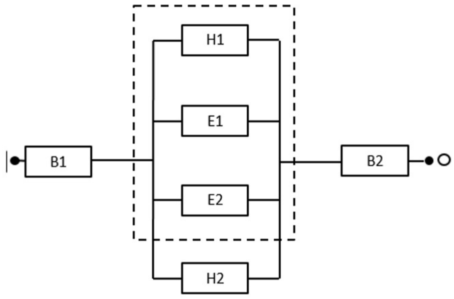
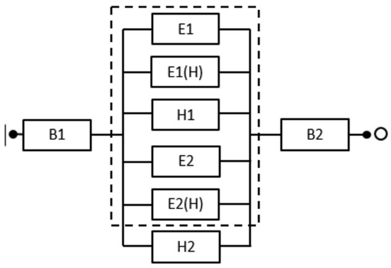
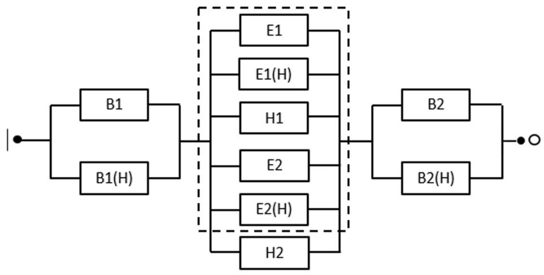
We evaluate the reliability parameters of the «3 of 4» structure with end doors connected sequentially using Markov analysis. After defining the state of the system, by means of transition probability densities λ and μ known for the doors, we construct the state transition diagram for the given structure (Fig. 2.6). The state labels here reflect the following information:

– identification «1» denotes an operable state and «0» denotes a failed state;
– the six values per row are the identification of the failure/operational status of each of the six doors in the graph states (the first 4 values relate to the side doors of the car and the last 2 to the end doors).

The first state «111111»: all six doors are in working order at the initial moment. The «011111» state indicates that the system has entered the phase where the first side door has failed and the others (3 side doors and 2 end doors) are functioning normally. The other states are interpreted similarly.

Next, we compose a system of Kolmogorov differential equations (2.2), where the number of equations in the system will be equal to the number of

Table 2.1 — Results of calculation of the failure rate for different rail coach door set layouts

Scheme type	**Diagram representation**	**Failure rate λ, $1/km$**
Reserving side automatic doors with manual ones		$\lambda_c = 5{,}5566{\cdot}10^{-6}$
System «3 of 4» with end doors connected sequentially		$\lambda_c = 5{,}5566{\cdot}10^{-6}$
System «5 of 6» which takes into account the possible operation of electro-mechanically operated doors in manual mode		$\lambda_c = 7{,}88 \cdot 10^{-6}$
System «5 of 6» which takes into account the possible operation of electro-mechanically operated side and end doors in manual mode		$\lambda_c = 6{,}7694{\cdot}10^{-6}$

states. The initial data are single door failure rate $\lambda = 1{,}667 \cdot 10^{-6}\,1/km$ and recovery rate $\mu = 0{,}041\,1/km$.

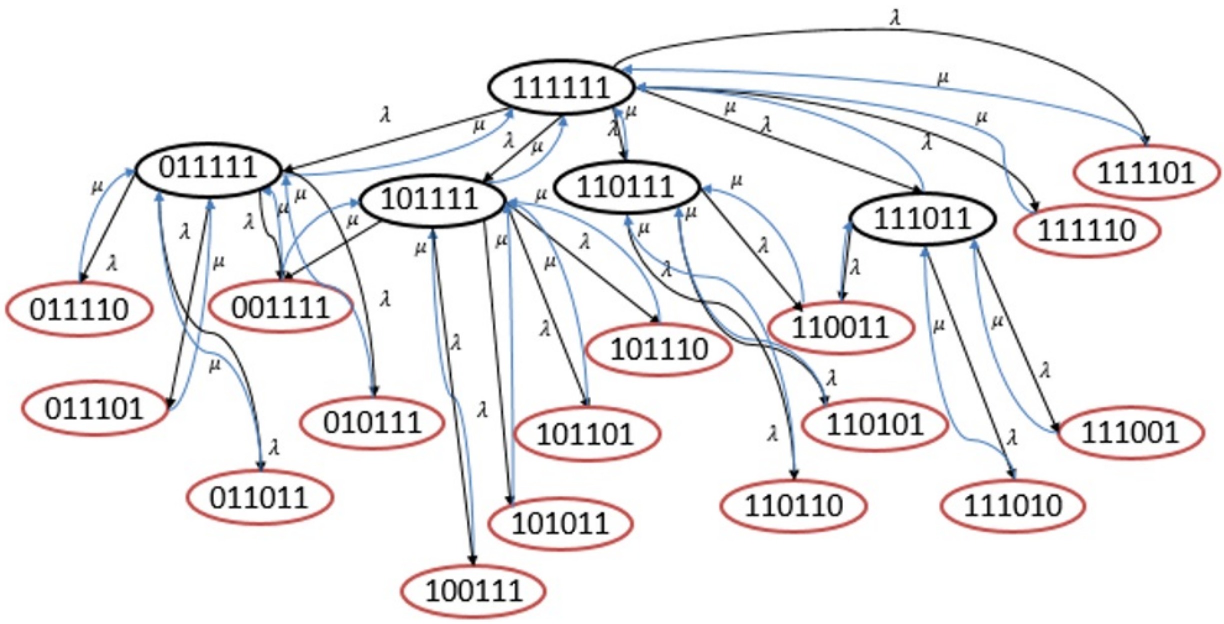


Figure 2.6 — State transition diagram for structure «3 of 4» with end doors connected sequentially (λ – failure rate, μ – repair rate)

Using MATLAB, we estimate the probability of failure-free operation of the system. We obtain $P = 0.9917$. This probability value is calculated for a period equal to the service life of the doors in question.

$$
\begin{cases}
\frac{dP_{111111}}{dt} = -6\lambda P_{111111}(t) + \mu \left[P_{011111}(t) + P_{101111}(t) + \right. \\
\qquad \left. + P_{110111}(t) + P_{111011}(t) + P_{111101}(t) + P_{111110}(t)\right] \\
\frac{dP_{011111}}{dt} = -\left(5\lambda + \mu\right) P_{011111}(t) + \mu \left[P_{011110}(t) + P_{011101}(t) + \right. \\
\qquad \left. + P_{001111}(t) + P_{010111}(t) + P_{011011}(t)\right] + \lambda P_{111111}(t) \\
\frac{dP_{101111}}{dt} = -\left(5\lambda + \mu\right) P_{101111}(t) + \mu \left[P_{001111}(t) + P_{100111}(t) + \right. \\
\qquad \left. + P_{101011}(t) + P_{101101}(t) + P_{101110}(t)\right] + \lambda P_{111111}(t) \\
\frac{dP_{110111}}{dt} = -\left(5\lambda + \mu\right) P_{110111}(t) + \mu \left[P_{010111}(t) + P_{100111}(t) + \right. \\
\qquad \left. + P_{110110}(t) + P_{110101}(t) + P_{110011}(t)\right] + \lambda P_{111111}(t) \\
\frac{dP_{111011}}{dt} = -\left(5\lambda + \mu\right) P_{111011}(t) + \mu \left[P_{011011}(t) + P_{101011}(t) + \right. \\
\qquad \left. + P_{110011}(t) + P_{111010}(t) + P_{111001}(t)\right] + \lambda P_{111111}(t) \\
\frac{dP_{111101}}{dt} = -\mu P_{111101}(t) + \lambda P_{111111}(t) \\
\frac{dP_{111110}}{dt} = -\mu P_{111110}(t) + \lambda P_{111111}(t) \\
\frac{dP_{001111}}{dt} = -2\mu P_{001111}(t) + \lambda \left[P_{011111}(t) + P_{101111}(t)\right] \\
\frac{dP_{010111}}{dt} = -2\mu P_{010111}(t) + \lambda \left[P_{011111}(t) + P_{110111}(t)\right] \\
\frac{dP_{011011}}{dt} = -2\mu P_{011011}(t) + \lambda \left[P_{011111}(t) + P_{111011}(t)\right] \\
\frac{dP_{011101}}{dt} = -\mu P_{011101}(t) + \lambda P_{011111}(t) \\
\frac{dP_{011110}}{dt} = -\mu P_{011110}(t) + \lambda P_{011111}(t) \\
\frac{dP_{100111}}{dt} = -2\mu P_{100111}(t) + \lambda \left[P_{101111}(t) + P_{110111}(t)\right] \\
\frac{dP_{101011}}{dt} = -2\mu P_{101011}(t) + \lambda \left[P_{101111}(t) + P_{111011}(t)\right] \\
\frac{dP_{101101}}{dt} = -\mu P_{101101}(t) + \lambda P_{101111}(t) \\
\frac{dP_{101110}}{dt} = -\mu P_{101110}(t) + \lambda P_{101111}(t) \\
\frac{dP_{110011}}{dt} = -2\mu P_{110011}(t) + \lambda \left[P_{110111}(t) + P_{111011}(t)\right] \\
\frac{dP_{110101}}{dt} = -\mu P_{110101}(t) + \lambda P_{110111}(t) \\
\frac{dP_{110110}}{dt} = -\mu P_{110110}(t) + \lambda P_{110111}(t) \\
\frac{dP_{111001}}{dt} = -\mu P_{111001}(t) + \lambda P_{111011}(t) \\
\frac{dP_{111010}}{dt} = -\mu P_{111010}(t) + \lambda P_{111011}(t)
\end{cases}
\tag{2.2}
$$

Let us use the Laplace transform to calculate MTBF. To do this, we write a system of equations for all operable states of a technical object [61] (2.2).

$$\begin{cases} \dfrac{dP_{111111}}{dt} = -6\lambda P_{111111}(t) + \mu\left[P_{011111}(t) + \right. \\ \qquad\qquad \left. + P_{101111}(t) + P_{110111}(t) + P_{111011}(t)\right] \\ \dfrac{dP_{011111}}{dt} = -\left(5\lambda + \mu\right) P_{011111}(t) + \lambda P_{111111}(t) \\ \dfrac{dP_{101111}}{dt} = -\left(5\lambda + \mu\right) P_{101111}(t) + \lambda P_{111111}(t) \\ \dfrac{dP_{110111}}{dt} = -\left(5\lambda + \mu\right) P_{110111}(t) + \lambda P_{111111}(t) \\ \dfrac{dP_{111011}}{dt} = -\left(5\lambda + \mu\right) P_{111011}(t) + \lambda P_{111111}(t) \end{cases} \tag{2.3}$$

We apply Laplace transformations to obtain the numerical value of the failure rate. For $P(t)$ we have the following:

$$P(z) = \int_0^\infty P(t)\, e^{-zt}\, dt.$$

Generally speaking, the MTBF is

$$T_i = \int_0^\infty P(t)\, dt.$$

Since $T_i = P(z)$ at $z = 0$, we can transform (2.3) given that the probability of the first state at $t = 0$ is equal to one and the probabilities of other states of the system at $t = 0$ are zero (the condition that the system is in an operable state at the initial time), we obtain the following equations:

$$\begin{cases} -1 = 6\lambda T_{111111} + \mu\left[T_{011111} + T_{101111} + T_{110111} + T_{111011}\right] \\ 0 = -(5\lambda + \mu)T_{011111} + T_{111111} \\ 0 = -(5\lambda + \mu)T_{101111} + T_{111111} \\ 0 = -(5\lambda + \mu)T_{110111} + T_{111111} \\ 0 = -(5\lambda + \mu)T_{111011} + T_{111111} \end{cases}$$

Thus $\lambda_{syst} = 3{,}331 \cdot 10^{-6}\,\dfrac{1}{km}$ when $\lambda = 1{,}667 \cdot 10^{-6}\ {}^{1}/_{km}$ and $\mu = 0{,}041\ {}^{1}/_{km}$.

Thus, we obtained the value of the normalized uptime index $\lambda_{syst} = 3{,}331 \cdot 10^{-6}$ for the car set of doors in the railcar based on the transition from the selected structural scheme to the state transition diagram with further Kolmogorov system solution and Laplace transformation. The calculated value is three times less than the originally proposed value from the assumption that the failure of any door equates to the failure of the whole car set of six doors. Had this proposal been approved as a standardised reliability indicator, the failure rate of the doors would have accounted for an unacceptable proportion of potential failures. This could have led to an erroneous customer requirement to underestimate the standardised indicator per door (instead of constructing the type of functional interaction, changing the characteristics of the indicators of the components that make up the system exclusively).

As a result of constructing an adequate structural diagram and calculation on the basis of Markov analysis, it was possible to determine a correct value of a standardised reliability index, which adequately reflects the failure-free rate of the door set as a part of the wagon. The developed approach with the corresponding calculated values was used for the reconciliation of reliability indicators according to operational data between the customer and the manufacturer. The methodology can also be adapted to the structural diagrams and functioning algorithms for other modifications of products as part of complex systems.

2.5 Chapter Conclusions

The chapter elaborates various structural schemes for calculating the failure-free performance parameters of a passenger train door car set. In contrast to the logic-probabilistic approach in determining the failure rate, the method using Markov processes is a priority, as it takes into account the repair factor.

Thus, the last value of the system failure rate is applicable for the considered structural scheme of reliability «3 of 4» for further monitoring of the car door set during the operational tests.

The developed method is an effective tool for the important product planning phase (block №1 in figure 1.1). The definition of normative indicators and the harmonisation of values has a significant impact on project reliability for several reasons:

1. Since the exact operating and functioning conditions are not yet known at the planning stage, a correct structural diagram with a further calculation will allow the most detailed model version of the system links to be shown.
2. In the case of a rough calculation of the normative values, an incorrect reliance on this value will occur later on in the operation of the facility: the actual values from the door results will be compared with the normative value, which is knowingly over/underestimated as a result of the simplified approach.
3. An inadequate value of the standardised safety factor can lead to disagreements with the customer, resulting in extended project completion times and possible erroneous changes to the safety values of a system unit (door) in the technical documentation due to a lack of correct representation of the structural links and functional algorithms.

CHAPTER 3. Analytical Criteria for Comparing the Reliability Characteristics of Two Samples

3.1 Introduction

There are a large number of application problems where empirical information of various kinds, which is of a random nature, needs to be analysed. In the field of reliability research a large variation of these problems can be found. Here are examples of some of them according to the criterion of reliability performance evaluation:

1. Is the newly delivered unit the same as one that has already been tested in operation?
2. Has there been a change in the reliability of the control units since the software upgrade?
3. Can a standardised reliability rating for an improved product be set at the same value as the predecessor version?

Thus, one of the most important and common assumptions a researcher must test is whether or not two samples belong to the same population. The problem is usually solved by means of statistical criteria chosen by the expert according to a number of determinants, depending on the type of problem posed.

In the case where there is an understanding of the functional structure based on available empirical data or expert judgement, the task is reduced to testing the hypothesis of equality of distribution parameters. Empirical data are obtained from the actual operation of the facilities or from test results. The first option is a more reliable option for analysis, however, when developing completely new technical systems, the second method is necessary for the purpose of preliminary assessment of reliability. The fact that there is less confidence in conclusions based on test data often leads to a more careful choice of method for statistical processing of the available information. For example, if researchers know that the law of distribution of operating time before failure of a technical product is normal, they often use Student's t-criterion in case the condition of equality of dispersions is observed. However, this criterion is not quite correct when the number of outliers is large. If the variances are unknown, it is possible to use the modified Student's test, Welch's test, Cochran – Cox test, etc.

When these kinds of questions arise, the reliability analyst needs to make a decision about the present state of the observed process, but there is no predetermined right and only one solution. In any statistical study, several types of errors are considered:

1. **Type I error** is an error that results in rejecting a true hypothesis H_0, provided it is in fact true («false alarm»). In reliability – the probability of rejecting a workable batch.
2. **Type II error** is an error that results in H_0 not being rejected even though it is not actually true («missed detection»). The probability of missing an unreliable batch of technical devices.

3.2 Application of Non-parametric Methods in Reliability Analysis

One of the most popular trends in statistical hypothesis estimation is the development and investigation of non-parametric methods. This is due to the fact that, unlike parametric methods, they are more loyal in terms of the necessary assumptions about the data, as well as due to a simpler application algorithm. These factors explain their popularity in application areas: biometrics, medical statistics, marketing, sociology, psychology, reliability, industrial analytics. In addition, the use of various statistical software tools is increasing, which also facilitates the testing of various statistical hypotheses.

For a detailed review of the different statistical hypotheses on bias and scale, see *Practical Engineering, Process, and Reliability Statistics* [28].

Most methods for calculating reliability indicators are based on approaches where the distribution law takes on a particular form. Most often the exponential distribution law is used and less often normal, lognormal, Weibull, etc. In this approach, parametric methods of hypothesis testing are used, since it is assumed that the failure data are characterised by one or more unknown population parameters (mean, median, standard deviation, etc.) Accordingly, hypotheses are formulated as equations/inequalities with respect to the unknown parameters related to the problem. The applicability of non-parametric criteria depending on the type of analysis is presented in Table 3.1.

For example, the hypothesis of position may be formulated as equality of means, and the hypothesis of scale – equality of variance. Thus, parametric methods are based on a number of assumptions that sometimes unrealistically characterise empirical data. This fact entails the following problem: critical values for statistics are calculated according to an assumed distribution, and when it is theoretically identified incorrectly (in terms of reflecting a real process), this also causes inaccuracies in the interpretation of statistical test results. In such cases, non-parametric methods that do not take into account the true distribution of the data are used. Their use is appropriate when it is not possible to observe the assumptions characteristic of parametric tests. For these reasons, the use of non-parametric criteria is now widely used [62], [49].

When reviewing the reliability literature, the most common hypotheses that can be observed are about the distribution law of the uptime. The most common criteria are Kolmogorov – Smirnov, Cramer – von Mises. Information on their applicability can be found in the basic fundamental reliability theory literature of the 20th century [63], [15]. In addition, the same criteria are used in

Table 3.1 — Applicability of non-parametric and parametric criteria depending on the type of analysis

Analysis Type	Non-parametric Criterion	Parametric Criterion
Comparison of two dependent samples	Wilcoxon signed-rank test	t-test for dependent samples
Comparison of two independent samples	Mann – Whitney U-test and two-sample Kolmogorov – Smirnov test, Cramer – von Mises test	t-test for independent samples
Comparison of three or more related samples	Friedman test	Repeated measures, analysis of variance (ANOVA)
Comparison of three or more unrelated samples	Kruskal – Wallis H test	One-factor ANOVA analysis of variance
Comparison of categorical data	Chi-square test and Fisher's exact test	–
Comparison of two ranked variables	Spearman correlation coefficient	Pearson correlation coefficient
Comparison of two variables, one of which is of binary type (discrete case)	Point-biserial correlation coefficient	Pearson correlation coefficient
Comparison of two variables, one of which is of binary type (continuous case)	Biserial correlation coefficient	Pearson correlation coefficient
Checking the randomness of the sample	Runs test	–

more recent editions [48], [64], [47], [65], [66] and government standards [67]. The importance of research into the development of statistical criteria for hypothesis testing is also noted in [13]. The authors give the example that errors are very common due to the assumption that an exponential law is used for the object in question, when in fact the Weibull law of distribution is in place.

In reliability theory it is quite common to encounter data where it is difficult to make assumptions about the distribution law due to insufficient samples. This happens both because of the short period of observation of the test objects, and because of unreliable and incomplete data on equipment failures. The most common non-parametric test is the Wilcoxon test (Mann - Whitney). The same criterion, judging by a number of facts, is described in GOST R MEK 60605-6-2007 «Criteria for checking constancy of failure intensity and failure rate parameter» [68] as applied to large samples. However, the Wilcoxon – Mann – Whitney criterion has a number of drawbacks, which are extensively discussed in the statistical literature [69], [70].

The problem of applicability of the Wilcoxon – Mann – Whitney test is found in many fields. In particular, in the field of mathematical psychology [71] there have been studies which formulate a number of limitations for Student's and Mann – Whitney tests. In particular [48] only the Wilcoxon test for comparing two samples is given.

In addition to comparing central trends, there is the task of comparing variability. In particular, a strong emphasis on minimising and controlling variance is realised in the area of industrial quality control.

However, there are also criteria for testing location and scale changes together. Most often in the literature they are so called – «location-scale tests». These methods are relevant because in many technical situations (as in biomedical research), measures and modifications of designs, changes in the maintenance and repair process, can lead to an impact on both the location parameter and the scale parameter. Such verification methods exist, for example, [72]. However, in terms of transparency and speed of implementation of the calculation, this procedure is very difficult for practitioners to implement.

In the statistics literature, non-parametric tests for joint determination of differences in location and scale are based on a combination of two tests: one for the shift, and one for variability. Most often the combination is realised by sum of squares of standardised test statistics, and this is the case with the Lepage algorithm [73]. This rank test is the best known for testing the «location-scale» hypothesis. It consists of a combination of Wilcoxon and

Ansari – Bradley statistics. However, a number of recent studies in the last decade have investigated the Cucconi test [69]. The criterion is rather obscure, but attracts interest because, unlike other «location-scale» tests, it is not a combined quadratic form combining a location-based criterion and a scale-based criterion. While Lepage's «location-scale» criterion was well known, Cucconi's criterion was published a few years earlier in an Italian economic publication. Paper [50] provides for the first time exact critical values for this criterion, as well as a detailed study for simulated samples from different families of distributions. The solution presented by Cucconi differs in that, instead of the quadratic form combining the shift and scale criteria, it implements the problem using rank squares and counter-ranks. An important advantage is the fact that to implement the Cucconi test, one has to compute the ranks of the observations in the pooled sample, whereas in the Lepage algorithm, one has to compute the Ansari – Bradley estimates.

One of the biggest problems in reliability assessment is insufficient data. Often there are tasks to estimate the provisional reliability of a new product when observations are available for a very short period of time. Objectives may be formulated in different ways. Here are a few of them:

1. Can supplier A's sensors be classified at the same level of reliability as supplier B's?
2. Are there significant differences in repair times at the repair plant №1 with those at the repair plant №2?
3. Can we say that after three months of using the new type of software, there has been a change in the uptime of the underground doors?
4. Is there any difference in the daily mileage of locomotive traction cars for March 2023 as compared to March 2022?

A number of these questions in the field of engineering analytics present several difficulties: small samples for analysis and computational intensiveness.

In [50], an example of the application of several criteria to estimate the difference in lung capacity of two small groups of subjects is provided.

In contrast to the Cucconi and Lepage tests, the Kolmogorov – Smirnov and Cramer – von Mises criteria do not allow the null hypothesis of no difference in mean expiratory volume rate between subjects to be disproved at the significance level of 0,05. This comparison is made because the latter two criteria are widely used for the common task of the two samples. In addition, as mentioned above, the latter two are the most popular in the field of reliability theory.

Goodness of Fit Test

The Kolmogorov – Smirnov and Cramer – von Mises tests allow us to determine whether there are any differences between the underlying population distributions of the samples in question. These tests are sensitive to all possible types of differences between the two distribution functions, including shape differences (kurtosis and skewness).

Kolmogorov – Smirnov Test

The most common situation considered is when the estimated distribution functions are continuous. The criterion itself is based on the differences between the empirical distribution functions $EDF_s(x)$ of two samples. Let $F_1(x)$, $F_2(x)$ be the distributions corresponding to samples 1 and 2, respectively. Denote by $EDF_1(x)$ and $EDF_2(x)$ the empirical distribution functions of random variables X_1 and X_2 underlying the samples:

$$EDF_j(x) = \frac{1}{n_j} \sum_{i=1}^{n_j} \mathbb{I} \left(X_{1i} \leqslant x \right), \quad j = 1{,}2.$$

The type of criterion statistics depends on whether the hypothesis is one-sided or two-sided. For the two-sided version, this is

$$H_0 : \{F_1(x) = F_2(x) \ \forall x \in \mathcal{R}\}$$

and its alternative is

$$H_1 : \{F_1(x) \neq F_2(x) \text{ for at least one } x \in \mathcal{R}\} .$$

The test statistics is calculated as follows:

$$KS = \sup_{x \in \mathcal{R}} \left| EDF_1(x) - EDF_2(x) \right|.$$

Large statistics values indicate a deviation from the main hypothesis towards the alternative hypothesis.

Although the Kolmogorov – Smirnov criterion is very common, it has a number of drawbacks. In particular, the author in [22] says that when using this criterion to approximate an empirical function to a theoretical one, overestimation of the significance level may occur. This means that there is a risk of accepting as plausible the hypothesis that is poorly consistent with the real data.

Cramer – von Mises Test

Another common criterion for the general two-sample problem is the Cramer – von Mises test. Some researchers prefer it to the Kolmogorov – Smirnov test, but it is inferior to the latter in terms of computational simplicity. The Cramer – von Mises criterion is used to test the hypothesis

$$H_0 : \{F_1(x) = F_2(x) \ \forall x \in \mathcal{R}\}$$

and its alternative

$$H_1 : \{F_1(x) \neq F_2(x) \text{ for at least one } x \in \mathcal{R}\}.$$

Consider a version of the criterion based on the following statistics

$$CVM = \frac{(n_1 n_2)^{1/2}}{n^{3/2}} \left(\sum_{i=1}^{n_1} \left| EDF_1(X_{1i}) - EDF_2(X_{1i}) \right| + \right.$$
$$\left. + \sum_{i=1}^{n_2} \left| EDF_1(X_{2i}) - EDF_2(X_{2i}) \right| \right).$$

The essence of the Cramer-von Mises criterion is similar to the Kolmogorov-Smirnov test: $EDF_1(x)$ and $EDF_2(x)$ are estimates of $F_1(x)$

and $F_2(x)$, while statistics CVM is a function of distances between $EDF_s(x)$. According to the alternative hypothesis, the more different $F_1(x)$ and $F_2(x)$ are, the higher the value of statistics CVM.

Location-Scale Test

Typically, non-parametric criteria for joint determination of differences in location (central tendency) and scale (variability) for two samples are based on a combination of two tests: for location shift and for scale. Typically, this combination is achieved using the sum of the squares of the standardised statistics. Consider just such a case.

Lepage Test

In general, the hypothesis of non-parametric criteria for joint detection of change in location and scale is formulated as follows:

$$H_0 : \{\mu_1 = \mu_2 \cap \sigma_1 = \sigma_2\} \text{ and its alternative } H_1 : \{\mu_1 \neq \mu_2 \cup \sigma_1 \neq \sigma_2\}.$$

This criterion is based on a combination of two tests. This is achieved on the basis of the sum of two standardised test statistics squared:

$$LEP = \frac{(W - \mathbb{E}_0(W))^2}{\mathbb{V}_0(W)} + \frac{(AB - \mathbb{E}_0(AB))^2}{\mathbb{V}_0(AB)}.$$

$$\mathbb{E}_0(W) = \frac{n_1(n+1)}{2}, \quad \mathbb{V}_0(W) = \frac{n_1 n_2(n+1)}{12},$$
$$\mathbb{E}_0(AB) = \frac{n_1(n+2)}{4}, \quad \mathbb{V}_0(AB) = \frac{n_1 n_2(n+2)(n-2)}{48/(n-1)}$$

when n is an even number,

$$\mathbb{E}_0(AB) = \frac{n_1(n+1)^2}{4/n}, \quad \mathbb{V}_0(AB) = \frac{n_1 n_2(n+1)\left(n^2+3\right)}{48/n^2}$$

when n is an odd number.

Cucconi Test

A well-known criterion for the task of determining change in central tendency and variability is the Lepage test discussed earlier. Many tests have been proposed from this area, almost all of which are Lepage-type tests: a combination of a location test and a variation test. Marozzi [50], [51] investigated and compared a number of non-parametric criteria between themselves and with the Cucconi test [69]. This criterion is not as well known, but is noteworthy for several reasons:

1. Historically, it appeared earlier than the Lepage test.
2. It does not combine the two tests: location and variation, as other common criteria in this family do.
3. A distinctive positive feature is the power and probability of type I error.

The hypothesis for the Cucconi and Lepage criteria is often formulated as in the previous paragraph [49], and also as follows in [50]. Let X_1 and X_2 be continuous random variables belonging to two general populations, and let F_1 and F_2 be their distribution functions. The general procedure for comparing two samples is as follows:

$$H_0 : X_1 \overset{d}{=} X_2 \text{ and its alternative } H_1 : X_1 \overset{d}{\neq} X_2,$$

where $X_1 \overset{d}{=} X_2$ means that $F_1(t) = F_2(t)\ \forall t \in \Re$, and $X_1 \overset{d}{\neq} X_2$ means that $\exists A \subset \Re : F_1(t) \neq F_2(t),\ t \in A$ with $\Pr(A) > 0$.

$$F_1(t) = G\left(\frac{t-\mu_1}{\sigma_1}\right) \text{ and } F_2(t) = G\left(\frac{t-\mu_2}{\sigma_2}\right),$$

where $G(\cdot)$ is the distribution function of a continuous variable with expectation 0 and standard deviation 1, μ_1 and μ_2 (σ_1 and σ_2) location (scale) parameters of populations 1 and 2, respectively. Let observations $X_{11}, \ldots, X_{1n_1}$ and $X_{21}, \ldots, X_{2n_2}$ be random samples from the general populations 1 and 2, respectively. For the task of comparing central tendency and variance together, Cucconi [69] proposed a rank test based on the statistics

$$C = \frac{U^2 + V^2 - 2\rho UV}{2\left(1-\rho^2\right)},$$

where

$$U = \frac{6\sum\limits_{i=1}^{n_i} W_{1i}^2 - n_1(n+1)(2n+1)}{\sqrt{\dfrac{n_1 n_2 (n+1)(2n+1)(8n+11)}{5}}},$$

$$V = \frac{6\sum\limits_{i=1}^{n_i} (n+1-W_{1i})^2 - n_1(n+1)(2n+1)}{\sqrt{\dfrac{n_1 n_2 (n+1)(2n+1)(8n+11)}{5}}},$$

$n = n_1 + n_2$, W_{ji} denotes rank of X_{ji} of pooled sample $\underline{X} = (X_{11}, \ldots, X_{1n_1}, X_{21}, \ldots, X_{2n_2}) = (X_1, \ldots, X_{n_1}, X_{n_1+1}, \ldots, X_n)$ and

$$\rho = \frac{2\left(n^2-4\right)}{((2n+1)(8n+11))} - 1,$$

$$\lim_{n\to\infty} \rho = \lim_{n\to\infty} \frac{2n^2-8}{16n^2+30n+11} - 1 = -\frac{7}{8} = \rho_0.$$

In his study, Cucconi observed that for samples not differing greatly in size, with more than 6 items, the convergence to the norm was very good. In addition to being superior to the Lepage test in terms of power, probability of type I error, and ease of implementation, the Cucconi test is also suitable for the case of related samples. The unbiasedness and robustness of this test is shown in [69], critical values are given in [50].

3.3 Application of Non-parametric Criteria to Solving Reliability Problems

It is often a challenge to compare reliability performance before and after retrofitting. A similar task arises when evaluating changes in performance in different versions of the same design. In this example, the basic criteria previously discussed have been calculated: Kolmogorov – Smirnov, Cramer – Mises, Cucconi.

3.3.1 Conversion of Standardised Reliability Index for Newly Developed Product from Cycles to Kilometres

The Moscow Central Diameters (MCDs) operate several types of electric trains. At the same time, a modified version of the electric train is launched on each of the diameters when it is opened. As mentioned earlier, reliability indicators for rolling stock components are specified in several units of measurement. The most common options are kilometres travelled, cycles (opening/closing of a functional element) and hours.

For a new model of rolling stock components, it is possible to determine the failure-free index in cycles on the basis of bench test data. However, in order to determine an equivalent value in kilometres travelled, data from in-service testing is often needed, which is not yet available at the time of approval of the design brief due to the fact that the diameter has just opened and the trains have not been operated on it to any significant extent.

At the same time, data on the reliability and performance of the rolling stock components for MCD-1 and MCD-2 are available. In addition, stations have already been marked on the map, the distances between which can be determined.

Final description of the task: it is required to determine the value of standardised mean time between failures for a new product (to be operated at MCD-3). The normalised value of this indicator in cycles is known, the conversion factor from cycles to kilometres needs to be reasonably determined using data from products already in service, but on other paths (MCD-1 and MCD-2) (Figure 3.1).

Result: based on the collection and comparison of station distance data at MCD-1, MCD-2, MCD-3 using Cucconi statistical criterion, it was shown that there are no significant differences between the operational profiles of the considered facilities. As a consequence, based on the distribution characteristics

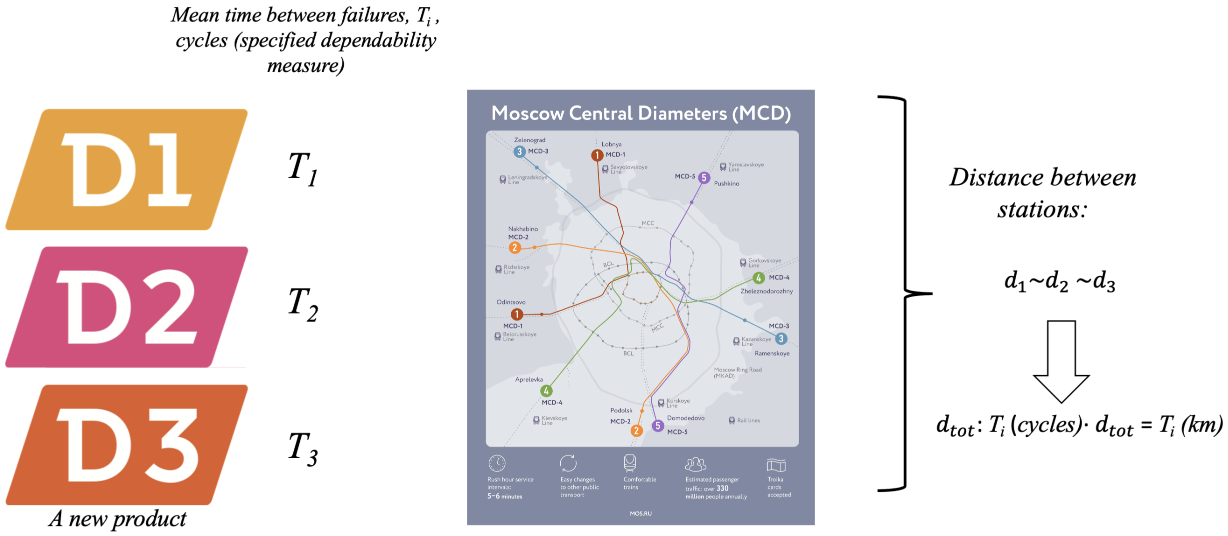


Figure 3.1 — Conversion of a standardised reliability index for a newly developed product from cycles to kilometres

of the total population, a conversion from cycles to kilometres was made for the standardised reliability indicators for the newly developed product (see Table 3.2).

Table 3.2 — Results of the p-value for the task of converting a standardised reliability index for a newly developed product from cycles to kilometres

	p-value (Cucconi)
MCD1/MCD2	0,81
MCD1/MCD3	0,72
MCD2/MCD3	0,45

3.3.2 Setting Recovery Time When Designing

Often, reliability indicators are a tool for detecting differences in the operating and maintenance conditions of technical facilities. The identification of these differences is necessary to identify the facilities that are the closest equivalents. The criteria for analogues can be different: in terms of the nature of maintenance, key design features, operating procedures, service conditions, etc.

Here we consider the task of identifying the closest analogue to a lifting device for the disabled in terms of recovery time. However, the closest analogue exists (in figure «Device 1») for several factors, but the sample size for the recovery time of this object is small to determine the central value. However, there are two other analogues of the lifting device (in Diagram 3.2 «Device 2» and «Device 3»), they are not as close to the object of interest as the first, but for them there is more statistical information on recovery times.

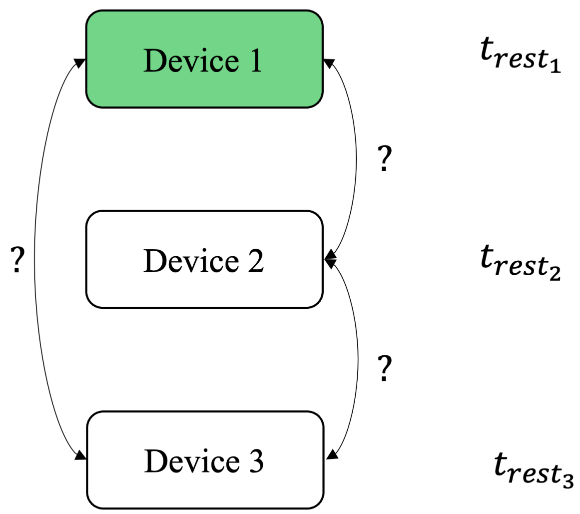


Figure 3.2 — Setting the recovery time in the design

Let us formulate the following problem: determine from the statistical data of the three devices the closest analogues in terms of the recovery time factor for calculating the standardised value of the average recovery time for technical conditions.

The following p-values were obtained from a number of criteria (see Table 3.3).

At a significance level of 0,05, the following conclusions can be drawn:

1. No significant differences in recovery time were found between Devices 1 and 3 by either criterion;
2. Between Devices 1 and 2 significant differences are revealed only by the Cucconi and Cramer – von Mises tests (when strictly comparing the p-values with the significance level value);

Table 3.3 — Setting the recovery time in the design

	Test	p-value
№1/№3	Kolmogorov–Smirnov	0,60
	Cucconi	0,74
	Lepage	0,25
	Cramer–von Mises	0,58
№1/№2	Kolmogorov–Smirnov	0,15
	Cucconi	0,04
	Lepage	0,68
	Cramer–von Mises	0,05
№2/№3	Kolmogorov–Smirnov	0,01
	Cucconi	0,03
	Lepage	0,00
	Cramer–von Mises	0,00

3. Significant differences were found between Devices 2 and 3 in the results of all four criteria considered.

Let's turn to the comparison and identify how, on a subject level, recovery time values of lifting devices can be so different. In expert analysis, it was found that Device 2 differs from 1 and 3 in terms of maintenance. The fact is that it has the peculiarity of having several sensors as part of its design. In case of repair or maintenance the time to get access to them increases. Devices 1 and 3 do not have this feature.

As a result of this analysis, the decision was made to use Device 1 and Device 3 as the basis for determining the standardised average recovery time for the new hoist design.

3.3.3 Justification for Choosing MTBF Value for Import Substitution

The process of replacing technical device suppliers is quite complex and time-consuming in several respects. The main ones are:

1. Changes in delivery times, hence the timing of the finished product;
2. Changes in the price of the final product;
3. Modification of the technical documentation;
4. Recalculation of reliability indicators;
5. Coordination of all process changes with the customer.

In terms of reliability analysis in the import substitution process, it is often a question of justifying the choice of a new element in a renewed product composition in terms of reliability characteristics.

This section deals with the same new product as in 3.3.2 – a lifting device for the disabled. In reviewing the composition of the product, the design office suggested replacing one type of sensor with another. When a design element is replaced, the overall reliability value of the finished product changes. In order to recalculate the total reliability characteristic for the whole technical object, it is necessary to know what value of mean time between failures (MTBF) the newly introduced sensor has.

The sensors proposed in the new design have not previously been used on these units. However, they have been installed on a similar product, but the sample of operational data for this product is small. Yet there are operational statistics for this type of switch in the side door design of long haul trains. The question arises, is it possible to translate the failure-free value of this type of door-mounted sensors for an elevating device? These technical objects in a general sense have different operating conditions, design characteristics, but the same operating profiles.

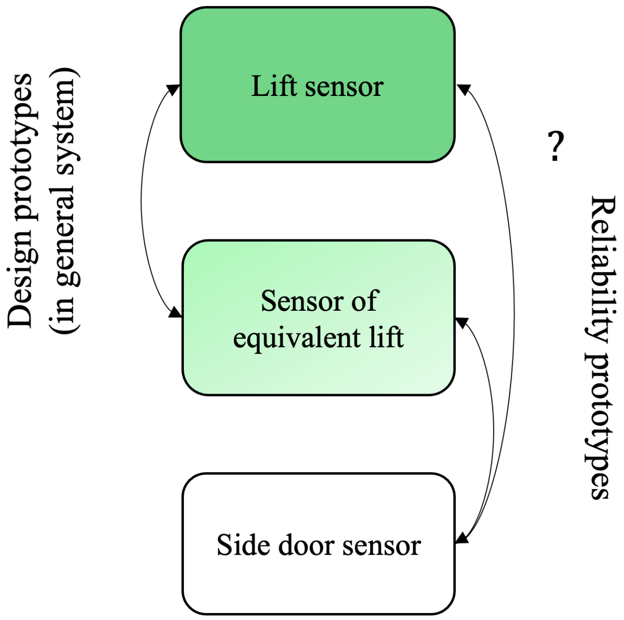


Figure 3.3 — Justification for the choice of mean time between failures for import substitution

Thus, one could formulate the task as follows: on the basis of the data on the operating time between failures (in days) of the sensors included in the lift unit analogue and in the car side door, to determine whether there are significant differences in these samples. The conclusions obtained are needed in order to use a known value of the reliability index of a given element in the door for the design value of the reliability index in the lifting device.

The result of applying the Cucconi criterion was p-value = 0,59, which is greater than 0,05, meaning that there is no significant difference in the MTBF of the sensor fitted to the side door and the sensor fitted to the analogue lift unit. Thus, by constructive analogy, we can extend this value from the lift counterpart to the new design under development.

Conclusions on the results of projects 3.3.1 – 3.3.3

Three applied problems, which arise in reliability analysis and for which the use of statistical criteria for hypothesis testing is necessary, are solved as a result of this research. The results of using Cucconi's criterion and a number of criteria frequently used in reliability analysis are presented. The goals of these problems can be varied: search and verification of peer components, determination of equivalent operating conditions, identification of significant differences in repair times of technical systems. According to the results of testing several types of statistical criteria, the Cucconi test reveals differences even in small samples, and is also easy to implement from the point of view of the calculation procedure.

3.4 Chapter Conclusions

Statistical criteria play an important role in the field of reliability analysis. By applying them correctly, it is possible both to determine system parameters unknown at design time and to check already in operation production facilities for similarity in terms of both design characteristics and operating profile. Of particular interest in this chapter are the statistical tests for the joint determination of location and scale by Lepage and Cucconi. These tests allow the estimation of the central tendency and the scale of the sample simultaneously using ranking statistics. These criteria have a number of advantages:

1. **Distributional independence:** Both criteria do not require the determination of the type of distribution of the sample in question, which makes them applicable to different types of data in the field of reliability analysis. This aspect is particularly important in the development of new technical systems, as in the early stages of data collection it is often not possible to determine the distribution law.
2. **Resistance to outliers:** «location-scale tests» are based on rank statistics, which makes them robust to anomalous values in samples. Given that errors in diagnostics due to both hardware and human error interfere with the accumulation of reliability information, outliers occur in almost every sample. For this reason, robustness to outliers is

another important advantage in the application of this type of criterion in the field of reliability performance.

We will highlight the additional benefits of applying the Cucconi criterion:

- **Convenience of calculation:** the algorithm for implementing the test is quite simple. This factor allows it to be used without much difficulty in most statistical packages, which is important for engineering services, as the variability of software tools of this type used in industrial enterprises is rather small.
- **Universality of applicability:** can be applied to both small and large sample sizes and is suitable for both independent and dependent samples.
- **Power:** the Cucconi test has good power for detecting differences in both central tendency and variability, especially when sample sizes are medium to large.

The main advantage of the Cucconi criterion is its great practical relevance. The three production problems in 3.3 have shown in which data sections and for which range of purposes this approach can be applied.

A test study was also carried out on data from the operation of two different types of doors: one class is more obsolete and the other modernised. As a result of 741 pairwise comparisons, 21 cases were recorded in which only the Cucconi test revealed differences (when they are objectively present: the samples in these cases are from different classes), while other tests (Welch, Student, Wilcoxon – Mann – Whitney, Lepage) did not detect them.

This section of the study refers to varied tasks at different stages of the product life cycle (see Figure 1.1): hypothesis testing on production data is needed both in the first three stages during product development (processing of chronological data, interpretation of test results, selection of components) and in the last three stages (defect identification in batches, operational data processing from operation and determination of effectiveness of reliability improvement measures).

In general, the Cucconi test is an excellent tool for comparing the central tendency and variability of two samples, especially when the data do not follow a normal or other distribution or when outliers are observed.

These advantages make this statistical criterion applicable to the specific nature of the data used in reliability analysis.

4.1 Introduction

The prediction of technical condition and reliability is an urgent task, as the timely detection of faults has become one of the key objectives of any industrial sector. To solve practical reliability problems, theoretical distribution laws are most often used, the arsenal of which is not very large [23]. The most common is the exponential distribution. A frequent cause of inadequate solutions to reliability problems is its use in cases where it is not actually applicable.

On the one hand, such popularity of this method of reliability prediction can be explained by the fact that this approach simplifies solving many reliability estimation tasks, but on the other hand, it imposes a number of limitations on the model and makes the calculation roughly approximate. For example, reliability forecasting by means of exponential distribution does not take into consideration ageing and wear-out processes. The most convenient characteristic of the reliability is the parameter of failure rate, because it allows easy enough to calculate the quantitative characteristics of a complex system and clearly reflects the object's behaviour in time. The approach based on the exponential distribution of operating time is characterised by an erroneous assumption about constancy of the parameter of failure rate with the fact that the real value of this parameter changes several tens of times during the considered operating time intervals. This is due to several factors:

1. In general, the life cycle of a complex system can be visualised in the form of a bathtub reliability curve (see Fig. 1.5). The assumption of an exponential law will be appropriate when evaluating on the sections of the segment characterising the period of «random» failures.

However, it will not adequately describe the change of reliability during degradation failures as well, as it does not take into account the ageing and wear and tear processes.

2. If one divides all technical objects into several classes, it is possible to imagine variations in the failure rate parameter as in Figures 4.1, 4.2.

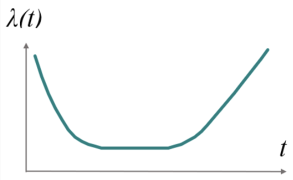
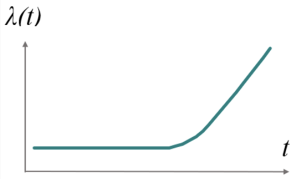
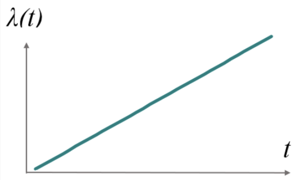
Failure behavior		General characteristics	Typical examples
A		**"bathtub curve"**	**old steam engine (late 18th to early 19th century) Various modern complex systems**
B		**simple devices complex machines with bad design (one single dominating type of failure)**	**car water pump shoelace, cords Chevrolet Vega engine (1974)**
C		**structures wear-out elements**	**car bodies automobile tires**

Figure 4.1 — Options for changing the failure rate parameter for wear and tear failure types

However, in times of rapid development of innovative technologies, it is often difficult to determine whether a product belongs to one group or the other, and the selection of a theoretical model of reliability from a number of distribution laws is therefore difficult due to the small amount of data on the functioning of the elements in the sample.

3. Any stage of the life cycle is accompanied by the influence of many random factors: external conditions, seasons, upgrades, imported components, adaptability of personnel to maintenance, features of technological equipment and many others. This significantly limits the application of theoretical distribution laws for reliability prediction for short-term periods.

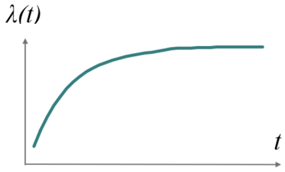
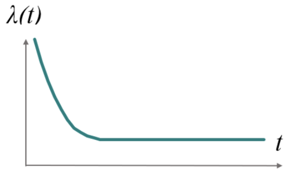
Failure behavior		General characteristics	Typical examples
D		**complex machines with high-stress trials after start of operation**	**high pressure relief valves**
E		**well designed complex machines**	**gyro compass pressure centrifugal pump**
F		**electronic components complex components after corrective maintenance**	**computer "mother boards" programmable controls**

Figure 4.2 — Options for changing the failure rate parameter for random failure types

Although the exponential law is often recommended as a priority for electronic devices and complex technical systems, sometimes its use in predicting mean time between failures leads to overestimation of individual elements/units with small numbers of components and underestimation of predicted reliability of large systems (over 10^5 elements) [23].

In the process of technical systems exploitation, there is a necessity to forecast the values of quantitative reliability parameters in order to plan the costs for maintenance and repair, to revise the range and volume of maintenance works in the operational documentation. Also, the nature of changes in operational reliability parameters is used to design analogous units.

An overview of prediction methods in the field of reliability analysis is given in [74].

When using the traditional mathematical apparatus based on the exponential distribution, it is not always possible to make accurate short-term forecasts. More credible estimates of reliability can be made by using adaptive forecasting tools. Adaptive methods can be used for forecasting both macroeconomic parameters and for describing changes in technical and economic characteristics of products and variables of chemical processes, studying the behavior of the equipment failure rate depending on its service life.

In this paper reliability calculation and statistical analysis methods are applied to model failure rate (FR) of locomotive traction car door unit and to predict the value of this reliability parameter for several periods. The relevance of the research clearly stems from the necessity of determining the FR value for further use in developing analogous products, predicting reliability of rolling stock component units and subsequently possible optimization of maintenance and repair schedules [53].

A common reason for inadequate solutions to reliability problems is the use of a one-parameter exponential distribution. On the one hand, this approach simplifies the solution of many reliability estimation problems, but on the other hand, it imposes a number of constraints on the model and makes the calculation roughly approximate. For example, reliability prediction using exponential distribution does not take into account the processes related to aging and wear and tear. It is also worth mentioning that this approach to calculating reliability parameters is characterized by the assumption of constancy of the failure rate under the fact that the real value of this parameter changes by several tens of times during the considered operating time intervals.

Due to these peculiarities, it was decided to try to carry out prediction of the failure rate with the help of adaptive methods, namely on the basis of the Holt – Winters model with the use of the survival theory apparatus.

4.2 Problem Formulation

One of the promising directions in the development of short-term forecasting is related to adaptive methods. This direction makes it possible to build self-correcting models that can promptly respond to changing conditions. Adaptive methods take into account different value of series levels: novelty and obsolescence of information - which is an important feature for forecasting reliability parameters. In addition, during the year, the character of failure rate changes from season to season, as well as depends on the intensity of rolling stock operation and the implemented corrective measures aimed at improving the reliability of equipment. Such processes can be difficult to describe and predict, and adaptive models give the best result for processes with a changing trend.

The purpose of this study is to describe, model and predict the process of change in the door control unit failure rate from January 2019 to February 2022 and predict 3 steps ahead.

4.3 Implementation of Prediction Algorithm Based on Holt – Winters Model and Lifetime Theory

Using experimental data, the failure rate is usually determined by the formula [63], [24], [75]::

$$\lambda(t) = \frac{\Delta n(t)}{N(t)\Delta t},$$

where $\Delta n(t)$ is the number of failures in operating time interval Δt, $N(t)$ is the number of objects (studied parameters) operable at time Δt.

The Holt – Winters method is a modification of exponential smoothing for series with seasonality. It results in equations with three smoothing constants:

1. Smoothed series:

$$L_t = k\frac{Y_t}{S_{t-s}} + (1-k)(L_{t-1} + T_{t-1});$$

2. Trend component:

$$T_t = b(L_t - L_{t-1}) + (1-b)T_{t-1};$$

3. Seasonal component:

$$S_t = q\frac{Y_t}{L_t} + (1-q)S_{t-s}.$$

The forecast for p steps forward is as follows:

$$\widetilde{Y}_{t+p} = (L_t + pT_t)S_{t+p-s},$$

where s is the number of phases in a complete seasonal cycle, k, b, q are adaptation parameters, k, b, $q \in [0, 1]$.

Assessing the reliability of a technical system from operational data can be represented as a survival analysis problem. Survival data analysis methods have continued to evolve vigorously in recent decades [56]. Applications of these methods are scaling from their use in cancer and reliability research to business, criminology, epidemiology, and the social and behavioral sciences. It is the application of the survival function in biomedical tasks that is well suited to the analysis of survival data because it directly describes the survival experience of the cohort under study. In the case of this approach in the field of reliability analysis, the cohort is a sample of running times before/between a failure/failures.

Generally, survival analysis is a set of statistical procedures for analysing data in which the outcome variable of interest is the time before an event occurs. The event usually refers to death, illness, recurrence – for direct survival analysis in the field of medical statistics. In reliability analysis, events are failures or certain types of failures. In reliability analysis, lifetimes are runtimes. Thus, from the time series of operating time to failure, one can construct Kaplan – Mayer estimates for further prediction.

To optimize the adaptation parameters, we constructed an estimate of the survival function for the actual series data:

$$\widehat{S}(t) = \prod_{t_i \leqslant t}\left(1 - \frac{d_i}{R_i}\right),$$

where d_i is the sum of the «dropped out» objects for runtime t_i, R_i is the sum of «survivors» up to the moment t_i, excluding «dropouts».

Similarly, survival function $\widetilde{\widehat{S}}(t)$ was constructed for the simulated series of failure rate.

The optimization parameters are set to minimize the mean squared error (MSE) for the survival function:

$$MSE = \frac{1}{T} \sum_{t=1}^{T} \left(\widehat{S}(t) - \widetilde{\widehat{S}}(t) \right)^2,$$

where T is the total number of running times.

As a result, we get the plots shown in Figures 4.3 and 4.4.

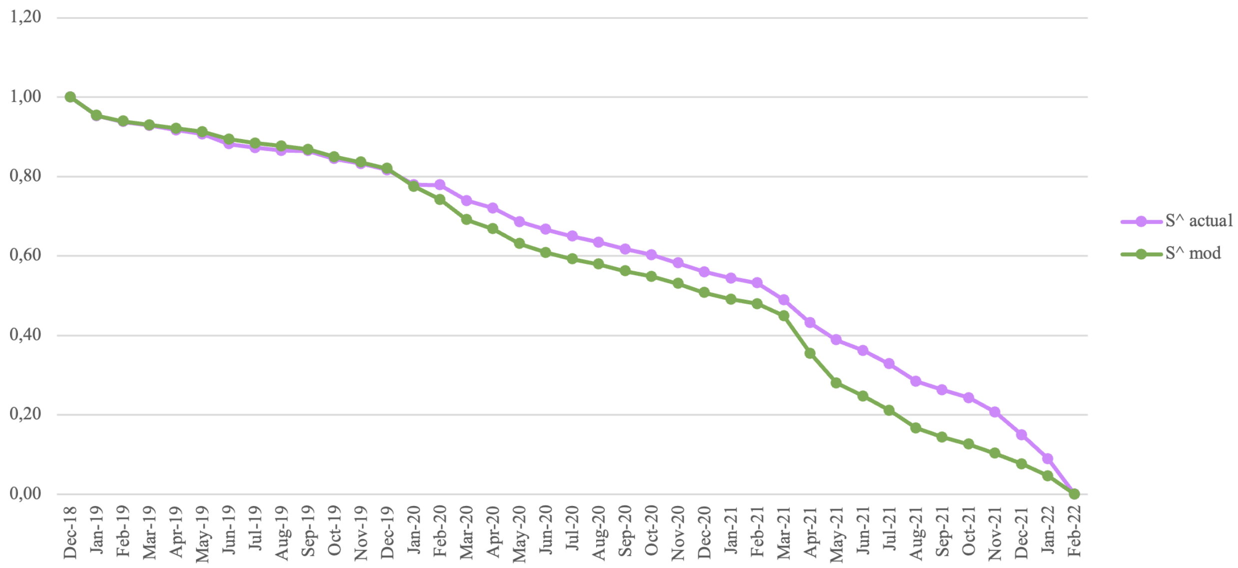


Figure 4.3 — Survival function plot (control unit)

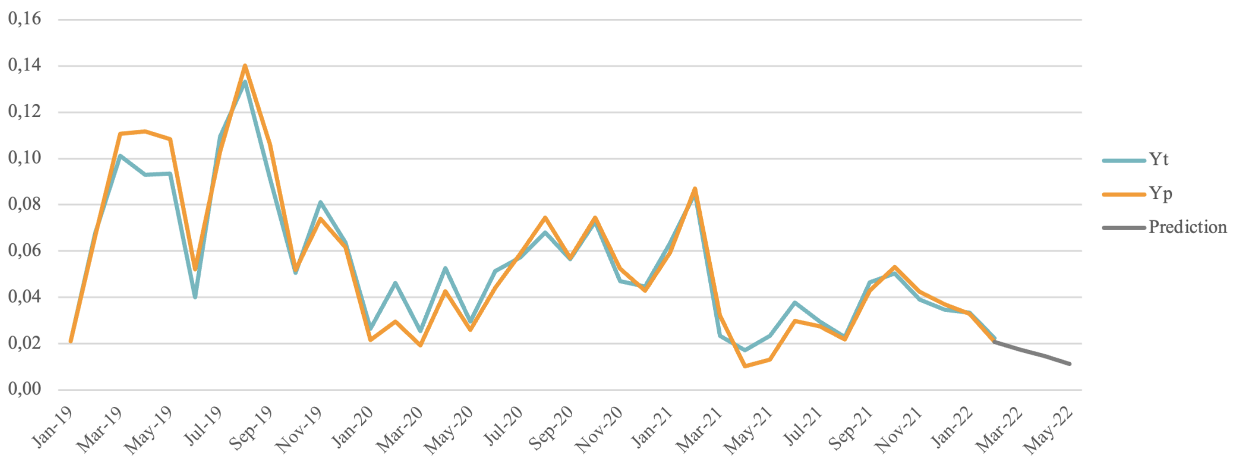


Figure 4.4 — Failure intensity λ plot of the control unit

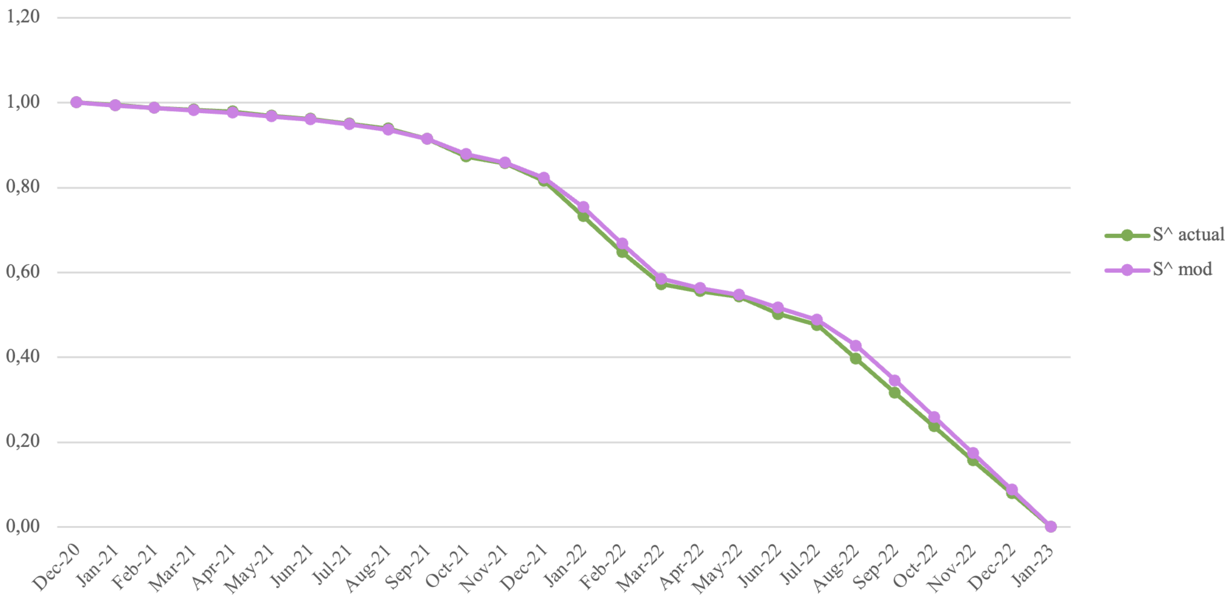


Figure 4.5 — Survival function plot (inductive sensor)

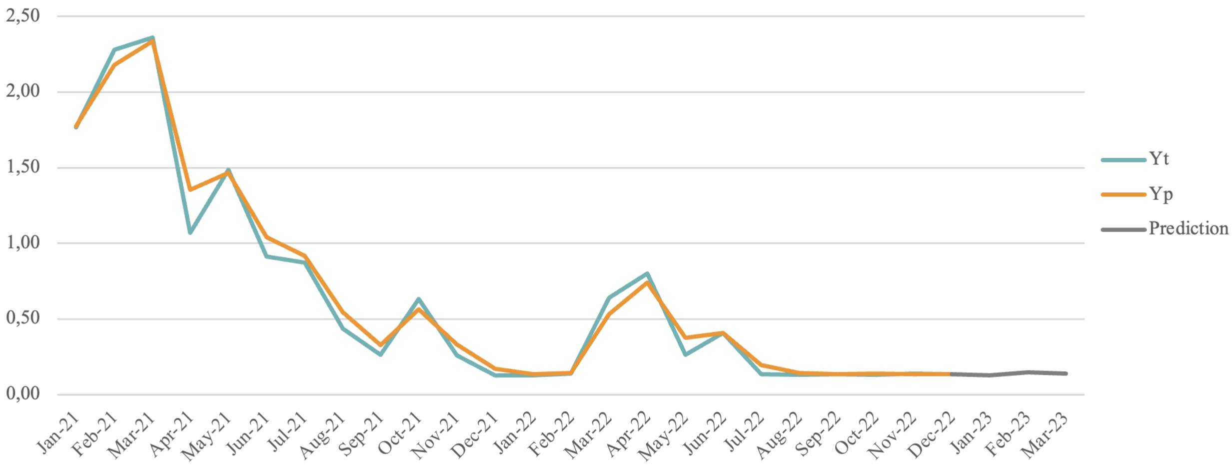


Figure 4.6 — Failure intensity λ plot of the inductive sensor

The MSE of the prediction for the time series of the failure rate parameters was $4.2 \cdot 10^{-5}$ for the door control unit failure rate prediction example. The same method was used to construct a prediction for another element, the inductive sensor from 3.3.3, for the case of a longer observation period. The results are shown in Figures 4.5, 4.6, and the predicted values will be used for the peer node whose selection was justified in 3.3.3.

4.4 Chapter Conclusions

This prediction method involves a combination of several approaches:

– use of survivability analysis, which correctly takes into account the intricacies of the technical systems' functioning;

– application of the Holt – Winters model, which allows making accurate short-term forecasts.

Reliability forecasting for the short term occupies an important place in many management and reliability processes at different stages of project development. This is driven by a number of operational needs:

1. During the operation of a technical facility, it is necessary to monitor the conformity of actual reliability values with those specified in the technical documentation for the product. Forecasting for nodes for several time periods in advance allows to determine whether the future values of reliability assessment will exceed the control values and to take corrective measures in time, as well as to plan the work of the necessary staff for this task, depending on the type of failures that cause non-compliance with the standardized indicators (Block №5 in the scheme in Fig. 1.1).
2. Forecasted values of component reliability allow to determine optimal composition of spare parts both for already operated objects, and for analogous products under development (Block №2 in the scheme in Fig. 1.1). Production of components just for the demand from operation will allow to use production capacities in an optimal way without shortage of elements when used by the customer and without overproduction at industrial sites (Block №4 in the scheme in Fig. 1.1).
3. Determining failure flow values several months in advance allows the maintenance and repair procedure on a product to be adjusted, taking into account changing circumstances and seasonal factors. Prompt

changes in activities will ensure a reduction in the Life Cycle Cost (LCC) (Block №2 in the diagram in Fig. 1.1).

The developed methodology for the analysis and prediction of node reliability data is applicable to the analysis of other rolling stock component nodes with possible correction of time series parameters (seasonal components, adaptation coefficients, etc.). The functional relationship of this part of the study with other chapters is shown in Fig. 1.2 and is an important component for the algorithms to determine the optimal timing of component replacements developed in Chapter 5. The results obtained were presented at conference

CHAPTER 5. Optimisation of System Maintenance with Consideration of Human Factor

5.1 Introduction

The influence of the human factor can be highlighted at every stage of the technical system life cycle. In recent years there has been a particular growth in the popularity of Human Reliability Analysis (HRA) and there are national standards for this area of reliability [76].

The HRA trend has developed rapidly in the last two decades [77], however, mentions of it can be found even in 20th century literature. Thus, in [31], the author focuses on describing the factors that make up the interaction between the operator and the complex technical system. There is also a listing of methods by which it was possible to quantify the impact of humans on the operation of industrial facilities. In the late 1950s, Williams pointed out [78] that reliability due to the human component had to be included in predicting the overall reliability of systems. Otherwise, the calculated total value would not reflect the real picture. Around the same time, Shapero and a group of [79] researchers drew attention to the fact that 20 to 50% of equipment failures were due to human error. Here are some historical facts which relate directly or indirectly to human errors in maintenance, repair or operation [77], [33], [32]:

– A study of 126 significant human error events in 1990 in nuclear power generation found that 42% of the problems were related to maintenance and modifications;
– Maintenance and inspection were found to be factors in approximately 12% of major aircraft accidents;
– According to studies by the US Nuclear Regulatory Commission (NRC), 65% of nuclear system failures were due to human error;

- A study of more than 4,400 maintenance history records covering the period from 1992 to 1994 concerning a Boiling Water Reactor (BWR) nuclear power plant showed that about 7.5% of all failure records could be classified as human errors related to maintenance activities;
- More than 80% of maritime accidents are due to human and organisational factors;
- A study showed that more than 20% of all system failures in fossil fuel-fired power plants are due to human error, and maintenance errors account for about 60% of the annual loss of electricity due to human error.

People play a key role in the design, manufacture, operation and maintenance of equipment. The extent of their role can vary according to a number of factors, both biometric (vision, hearing) and psychological (stress levels, uncertainty of action have a significant impact on the likelihood of human error [80]). Some mistakes have minor consequences, while others can cause great economic losses or destruction and environmental pollution.

The period between the First and Second World Wars saw the significant development of disciplines such as industrial engineering and industrial psychology. By 1945, human factors engineering was recognised as a specialist discipline. In the 1950s and 1960s, military and space programmes further increased the importance of human factors in systems design.

In addition to the already explored variations in the causes of human fail [81], new ones are emerging about which there is not much data yet. With the proliferation of controls, there are new factors affecting the performance of human operators, e. g. alert fatigue.

The nuclear industry currently plays an important role in electricity generation worldwide, accounting for around 16% of the world's total. At the same time, this industry is dangerous on a planetary scale [82]. Humans are a major factor in nuclear power generation and their impact on safety has become an important subject of study since human error can lead to disasters such as Three Mile Island (1979), Kyshtym (1957) and Chernobyl (1986). In addition, studies by Licensee Events Reports (LERs) and the Nuclear Regulatory Commission (NRC) have shown that about 65% of nuclear system failures are directly or indirectly due to human error [32].

Thus, human-dependent reliability assessment will have an active development in conjunction with the improvement of production and information systems. The last decade has seen a particular focus on the field of Reliability Centred Maintenance (RCM) in large-scale manufacturing plants, which includes maintenance methodologies to balance the cost of over-maintenance and the cost of the consequences of inadequate maintenance [83].

There are many types of human error, and most commonly they can be divided into several main groups:

1. **Design errors:** The most common cause is a design flaw in the construction of the model of the future product. In the field of rolling stock components, examples include incorrect strength calculations, failure to consider aspects of installation, the level of temperature conditions, vibrations, etc.
2. **Maintenance and repair errors:** They occur already under operating conditions of the technical system due to incorrect actions of the maintenance personnel. This category includes the following types of actions: wrong frequency of a particular type of actions; tampering with the mechanism design and incorrect return to operating condition; violation of correct adjustment of components during intervention; use of consumables not provided for in the operating regulations, etc. Moreover, depending on the type of life cycle, the probability of occurrence of these errors may increase due to increasing frequency of maintenance (for example, during the ageing of equipment).
3. **Assembly errors:** This type of error occurs during the production process, the quality of which is affected by external physical factors

(lighting, temperature, vibration, noise), as well as by poorly designed personnel procedures and poor quality drawings.

4. **Installation errors:** These cases are caused by incorrect installation instructions, incorrect reading of drawings, missing the correct markings on parts. For example, when installing doors on train carriages, it often happens that parts for two symmetrical wings get mixed up with each other, which results in the door not functioning correctly afterwards.
5. **Operating errors:** During the use of the facility, errors often occur due to the operator's fault or due to shortcomings in the manuals and instructions describing the process. For example, a failure in the door control unit due to improper actions by the operator at the control panel.

The specifics of the human factor in the repair and maintenance of machines and mechanisms are also described in [84]. The author attaches great importance to the adaptability of machines to carry out preventive and maintenance work, especially the human function to control and maintain technical systems.

Maintenance occupies an important place in the reliability of production systems. Depending on the type of system and the criticality of its failures, different schemes are chosen to organise its maintenance and repair activities. In this chapter, a model is constructed that takes into account both the human-driven reliability and the fault tolerance of the technical system.

A schematic representation of the maintenance options for technical items is shown in Figure 5.1.

In this chapter, great emphasis is placed on maintenance processes for the following reasons:

a) in-service maintenance and repair costs account for more than 50% of the lifecycle cost of a technical system [41];

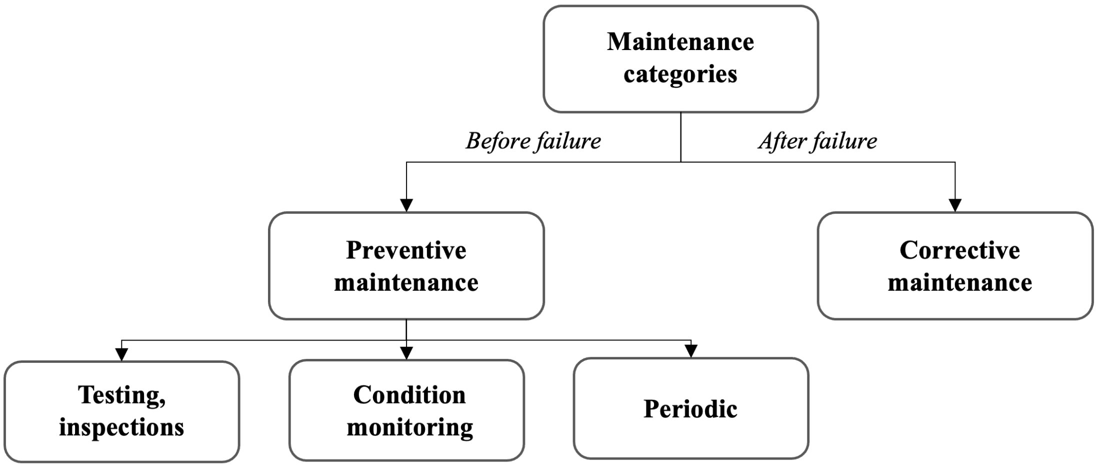


Figure 5.1 — Types of maintenance and repairs

b) more than 50% of all equipment fails prematurely after maintenance activities [77];
c) a study of electronic equipment showed that around 30% of failures were the result of operating and maintenance errors [77].

5.2 Weibull Distribution Law for Describing Reliability Caused by Human Factors

Authors of fundamental works on reliability theory often refer to the advantages of the Weibull distribution law. This applies both to describing the behaviour of technical systems in the general case, and to highlighting the influence of the human factor, i.e. in the HRA domain. In [85], an analysis of system behaviour depending on the shape parameter in the law describing it is given. If the shape and scale parameters are chosen, a better fit of the model to the empirical data can be obtained as compared to the exponential law. The characterisation of human-operator errors in the field of functional reliability of information systems is detailed in source [86].

Famous work [13] gives an example of the fact that studies that focus on testing hypotheses about the type of distribution law deserve special attention, because it is very common to see errors due to the assumption that an exponential law applies to the object in question, when in fact a Weibull distribution law is in effect. Also in work [33] this type of distribution is justified by a US Air Force study. An experiment was conducted in which human errors of several types were recorded. The results of the experiment were aggregated in a large-scale database, which led to the conclusion that the Weibull distribution is one of the most common and correct ones for estimating reliability with regard to human factors.

The field of human reliability science combines research in several areas: mathematical statistics, industrial statistics [87], engineering psychology [81], mathematical psychology [88], biometrics [89], psychology [90], risk theory [91], sociology [92].

Waloddi Weibull worked in Sweden and investigated the fatigue characteristics of materials. In 1939 he proposed a mathematical expression that can represent a large range of failure characteristics through changes in two parameters.

The reliability theory literature also suggests the use of variants such as the exponential distribution and the Rayleigh distribution [33], [27] to describe HRA functions. These distributions also refer to a Weibull distribution with shape parameters $k = 1$ and $k = 2$ respectively.

In earlier research [9] the problem arose of finding the Weibull distribution parameters to estimate the mean time to failure.

As with the attempt to calculate the parameters of a normal distribution, the ratios from [93] recommended to obtain the Weibull distribution parameters caused difficulties for some reliability experts, so another way of identifying them was proposed.

For N objects, consider a set of MTBFs t_i, arranged in non-decreasing order:

$$t_1, \ldots, t_i, \ldots, t_n < T,$$

where T is the test period, n is the number of failures in T. Then the expression for the probability with duration constraint of T is

$$P = f(t_1) \times \ldots \times f(t_i) \times \ldots \times f(t_n)\Delta t_1 \times \ldots \times \Delta t_i \times \ldots \times \Delta t_n (1 - F(T))^{N-n},$$

which is equivalent to

$$\frac{P}{\Delta t_1 \times \ldots \times \Delta t_i \times \ldots \times \Delta t_n} = \prod_{i=1}^{n} f(t_i)(1 - F(T))^{N-n}.$$

Then the likelihood function will have the following form:

$$L = \ln \frac{P}{\Delta t_1 \times \ldots \times \Delta t_i \times \ldots \times \Delta t_n} = \sum_{i=1}^{n} \ln f(t_i) + \ln(1 - F(T))^{N-n} \Leftrightarrow$$

$$\Leftrightarrow L = \sum_{i=1}^{n} \ln f(t_i) + (N - n) \ln(1 - F(T)). \tag{5.1}$$

The likelihood function in general form is (5.1) and we rewrite it using the expressions for the Weibull distribution density [57]

$$f(t) = \frac{k}{t_0} t^{k-1} e^{\frac{-t^k}{t_0}}$$

and the distribution function

$$F(t) = 1 - e^{\frac{-t^k}{t_0}},$$

where t_0 characterises the scale of the distribution curve, and k characterises the shape of the density curve.

The parameter estimates t_0 and k will be determined from conditions

$$\frac{\partial L}{\partial t_0} = 0, \quad \frac{\partial L}{\partial k} = 0.$$

We obtain the following equations for finding the Weibull distribution parameters

$$\frac{\partial L}{\partial t_0} = \sum_{i=1}^{n} \frac{\partial}{\partial t_0} \ln f(t_i) + (N - n) \frac{\partial}{\partial t_0} \ln(1 - F(T)) = 0, \tag{5.2}$$

$$\frac{\partial L}{\partial k} = \sum_{i=1}^{n} \frac{\partial}{\partial k} \ln f(t_i) + (N-n)\frac{\partial}{\partial k} \ln(1 - F(T)) = 0. \tag{5.3}$$

Modify expression (5.2):

$$\frac{\partial L}{\partial t_0} = \sum_{i=1}^{n} \frac{\partial}{\partial t_0} \ln\left(\frac{k}{t_0} t_i^{k-1} e^{\frac{-t_i^k}{t_0}}\right) + (N-n)\frac{\partial}{\partial t_0} \ln\left(e^{\frac{-t^k}{t_0}}\right) \Leftrightarrow$$

$$\Leftrightarrow \frac{\partial L}{\partial t_0} = -\frac{1}{t_0} + \sum_{i=1}^{n} \frac{t_i^k}{t_0^2} + (N-n)\frac{T^k}{t_0^2} = 0. \tag{5.4}$$

Similarly simplify formula (5.3):

$$\frac{\partial L}{\partial k} = \sum_{i=1}^{n} \frac{\partial}{\partial k} \ln\left(\frac{k}{t_0} t_i^{k-1} e^{\frac{-t_i^k}{t_0}}\right) + (N-n)\frac{\partial}{\partial k} \ln\left(e^{\frac{-t^k}{t_0}}\right) \Leftrightarrow$$

$$\Leftrightarrow \frac{\partial L}{\partial k} = -\frac{1}{k} + \sum_{i=1}^{n} \ln(t_i)\left(1 - \frac{t_i^k}{t_0}\right) + (N-n)\left(-\frac{T^k}{t_0}\ln(T)\right) = 0. \tag{5.5}$$

From equations (5.4), (5.5) the required estimates of the Weibull distribution can be numerically found to calculate the MTBF. The algorithm is implemented in Appendix B.

Knowing the parameters of the Weibull distribution allows us not only to approximate the empirical distribution function, but also to determine whether there is an optimal replacement period for a node.

5.3 Constructing Preventive Maintenance Cost Function with Consideration of Human Factors

To describe the probability of failure due to human factors, we will use the Weibull distribution function, the rationale for which was given in the previous section 5.2. For a similar probability, but due to a technical system, we will use an exponential distribution. There are several reasons for this:

– Because preventive inspections are most often carried out on an entire complex technical system, a complex product is the object of the inspection;
– The intervals between two regular inspections are usually short. In this case, the system is not affected by wear and tear failures at these time intervals, and its operability varies according to an exponential law [37], [94].

There are a number of models for constructing an optimal strategy for maintenance and repair of technical facilities. Some of them are developed without taking into account the mathematical apparatus of optimization theory, i. e. they include only the technical approach to defining the maintenance and repair procedure. There are also such approaches, which are based solely on mathematical models of preventive maintenance.

Of particular interest to the author of the study are models that consider the costs of preventive measures and the costs of corrective actions when a failure occurs. One such model was considered by the authors of research [16], then it has received further modification in more recent work [58]. After analysing this model, it was decided to include a probabilistic component describing the human impact on the uptime of the product. Since the technical measures for repair and maintenance of rolling stock equipment are provided by humans, ignoring this component of reliability as part of its overall assessment reduces the adequacy of the cost model.

5.3.1 Optimality Condition in General Case

Let us introduce notations and build a model of maintenance costs, taking into account several probabilistic characteristics:

A_γ – an event: failure due to human factors;

B_γ – an event: failure due to technical reasons;

C_γ – an event: no failure;

c_A – the cost of repairing the consequences of failure due to human factors;

c_B – the cost of repairing the consequence of failure due to technical reasons;

c_C – the cost of prevention;

$p_A(\gamma)$ – the probability of A until γ;

$p_B(\gamma)$ – the probability of B until γ;

$p_C(\gamma)$ – the probability of C until γ. Then

$$p_A(\gamma) + p_B(\gamma) + p_C(\gamma) = 1 \ \forall \gamma.$$

Let $C(\gamma)$ be the repair/preventive maintenance costs for the preventive maintenance period by time γ, then

$$C(\gamma) = c_A p_A(\gamma) + c_B p_B(\gamma) + c_C p_C(\gamma).$$

Denote by $M(\gamma)$ the mathematical expectation of the end of operation (due to repair/preventive maintenance). It can be represented as

$$\begin{aligned} M(\gamma) &= \int_0^\gamma t\,(\dot p_A(t) + \dot p_B(t))\,dt + \gamma p_C(\gamma) = \\ &= -\int_0^\gamma t \dot p_C(t)\,dt + \gamma p_C(\gamma) = -\int_0^\gamma t\,dp_C(t) + \gamma p_C(\gamma) = \\ &= -\gamma p_C(\gamma) + \gamma p_C(\gamma) + \int_0^\gamma p_C(t)\,dt = \int_0^\gamma p_C(t)\,dt. \end{aligned}$$

Denote by $C_u(\gamma) = \dfrac{C(\gamma)}{M(\gamma)}$ the unit cost, then

$$C_u(\gamma) = \frac{c_A p_A(\gamma) + c_B p_B(\gamma) + c_C p_C(\gamma)}{\int_0^\gamma p_C(t)\,dt}. \tag{5.6}$$

Introduce the following values:

$P_1(\gamma) = P\left(A_\gamma \mid \overline{B_\gamma}\right)$ – the probability of a failure due to human error during period $[0;\gamma]$ when there is no failure due to technical reasons; $P_2(\gamma) = P\left(B_\gamma \mid \overline{A_\gamma}\right)$ – the probability of a failure due to technical reasons during the period $[0;\gamma]$ when there is no failure due to human error. According to the conditional probability formula, we have:

$$\begin{aligned}
P\left(\overline{A_\gamma} \mid \overline{B_\gamma}\right) &= \frac{P\left(\overline{A_\gamma} \cap \overline{B_\gamma}\right)}{P\left(\overline{B_\gamma}\right)} = \frac{P\left(\overline{A_\gamma \cup B_\gamma}\right)}{P\left(\overline{B_\gamma}\right)} = \frac{P\left(C_\gamma\right)}{1 - P\left(B_\gamma\right)} = \\
&= \frac{1 - p_A(\gamma) - p_B(\gamma)}{1 - p_B(\gamma)}; \\
P\left(\overline{B_\gamma} \mid \overline{A_\gamma}\right) &= \frac{1 - p_A(\gamma) - p_B(\gamma)}{1 - p_A(\gamma)}.
\end{aligned} \tag{5.7}$$

Let it be now:

$$\begin{aligned}
&P_1(\gamma) = 1 - e^{-\frac{\gamma^k}{t_0}} = 1 - r(\gamma)(\text{corresponds to the Weibull distribution}), \\
&P_2(\gamma) = 1 - e^{-\frac{\gamma}{m}} = 1 - q(\gamma)(\text{corresponds to the exponential distribution}).
\end{aligned} \tag{5.8}$$

Denote

$$x = x(\gamma) = p_A(\gamma),$$

$$y = y(\gamma) = p_B(\gamma).$$

Let's rewrite (5.7) with (5.8) in mind:

$$\begin{cases} \dfrac{1 - x - y}{1 - y} = r, \\ \dfrac{1 - x - y}{1 - x} = q. \end{cases} \implies \begin{cases} 1 - x - y = (1 - y)r, \\ 1 - x - y = (1 - x)q. \end{cases} \tag{5.9}$$

From where

$$(1 - x)q = (1 - y)r,$$

$$1 - x = \frac{(1 - y)r}{q}, \tag{5.10}$$

$$x = 1 - \frac{(1 - y)r}{q}. \tag{5.11}$$

Substitute (5.10) into (5.9):

$$\frac{(1-y)r}{q} - y = (1-y)r,$$
$$(1-y)r(1-\frac{1}{q}) = -y,$$
$$1-\frac{1}{y} = \frac{1}{r\left(1-\frac{1}{q}\right)},$$
$$1-\frac{1}{y} = \frac{q}{r(q-1)},$$
$$\frac{1}{y} = \frac{r(q-1)-q}{r(q-1)},$$
$$y = \frac{r(q-1)}{r(q-1)-q},$$
$$y = \frac{r(1-q)}{r(1-q)+q}. \tag{5.12}$$

Now substitute (5.12) into (5.11):

$$x = 1 - \frac{r}{q}\frac{q}{r(1-q)+q} = 1 - \frac{r}{r(1-q)+q} = \frac{q(1-r)}{r+q-rq}. \tag{5.13}$$

Given (5.13) and (5.12), we get:

$$1-x-y = \frac{r+q-rq-q+rq-r+rq}{r+q-rq} = \frac{rq}{r+q-rq}.$$

Substitute x and y in (5.6):

$$C_u(\gamma) = \frac{c_A q(1-r) + c_B r(1-q) + c_C rq}{(r+q-rq)\int\limits_0^{\gamma} \frac{rq}{r+q-rq}\,dt}.$$

Here and hereafter, the point denotes the derivative over parameter γ, which is the time derivative. Find the minimum of $C_u(\gamma)$:

$$\frac{dC_u(\gamma)}{d\gamma} = \frac{c_A\left(\dot{q}(1-r) - q\dot{r}\right) + c_B\left(\dot{r}(1-q) - r\dot{q}\right) + c_C\left(\dot{r}q + r\dot{q}\right)}{\left((r+q-rq)\int\limits_0^\gamma \frac{rq}{r+q-rq}\,dt\right)^2}\times$$
$$\times\frac{(r+q-rq)\int\limits_0^\gamma \frac{rq}{r+q-rq}\,dt}{\left((r+q-rq)\int\limits_0^\gamma \frac{rq}{r+q-rq}\,dt\right)^2}-$$
$$-\frac{c_A q(1-r) + c_B r(1-q) + c_C rq}{\left((r+q-rq)\int\limits_0^\gamma \frac{rq}{r+q-rq}\,dt\right)^2}\times$$
$$\times\frac{(\dot{r}+\dot{q}-\dot{r}q-r\dot{q})\int\limits_0^\gamma \frac{rq}{r+q-rq}\,dt + rq}{\left((r+q-rq)\int\limits_0^\gamma \frac{rq}{r+q-rq}\,dt\right)^2} = 0,$$

where

$$\begin{aligned}\dot{r} &= \frac{d}{d\gamma}e^{-\frac{\gamma^k}{t_0}} = -\frac{k}{t_0}\gamma^{k-1}e^{-\frac{\gamma^k}{t_0}},\\ \dot{q} &= \frac{d}{d\gamma}e^{-\frac{\gamma}{m}} = -\frac{1}{m}e^{-\frac{\gamma}{m}}.\end{aligned} \tag{5.14}$$

Consider the conditions under which the numerator is zero:

$$\begin{aligned} &c_A\left[\left(\dot{q}(1-r) - q\dot{r}\right)(r+q-rq) - q(1-r)\left(\dot{r}+\dot{q}-\dot{r}q-r\dot{q}\right)\right]+\\ &+c_B\left[\left(\dot{r}(1-q) - r\dot{q}\right)(r+q-rq) - r(1-q)\left(\dot{r}+\dot{q}-\dot{r}q-r\dot{q}\right)\right]+\\ &+c_C\left[\left(\dot{r}q + r\dot{q}\right)(r+q-rq) - rq\left(\dot{r}+\dot{q}-\dot{r}q-r\dot{q}\right)\right]=\\ &\qquad= \frac{rq\left(c_A q(1-r) + c_B r(1-q) + c_C rq\right)}{\int\limits_0^\gamma \frac{rq}{r+q-rq}\,dt}.\end{aligned} \tag{5.15}$$

Let's redefine:

$$\begin{aligned} z_A &= \left(\dot{q}(1-r) - q\dot{r}\right)(r+q-rq) - q(1-r)\left(\dot{r}+\dot{q}-\dot{r}q-r\dot{q}\right),\\ z_B &= \left(\dot{r}(1-q) - r\dot{q}\right)(r+q-rq) - r(1-q)\left(\dot{r}+\dot{q}-\dot{r}q-r\dot{q}\right),\\ z_C &= \left(\dot{r}q + r\dot{q}\right)(r+q-rq) - rq\left(\dot{r}+\dot{q}-\dot{r}q-r\dot{q}\right).\end{aligned}$$

Simplify z_A, z_B, z_C:

$$\begin{aligned}
z_A &= (\dot{q}(1-r) - q\dot{r})\,(r+q-rq) - q(1-r)\,(\dot{r}+\dot{q}-\dot{r}q-r\dot{q}) = \\
&= (\dot{q}(1-r) - q\dot{r})\,(q(1-r)+r) - q(1-r)\,(\dot{r}(1-q)+\dot{q}(1-r)) = \\
&= q(1-r)\,(\dot{q}(1-r) - \dot{r}q - \dot{r}(1-q) - \dot{q}(1-r)) + r\,(\dot{q}(1-r) - \dot{r}q) = \\
&= -\dot{r}q(1-r) + r\dot{q}(1-r) - r\dot{r}q = \\
&= \dot{r}\,(q(r-1) - rq) + r\dot{q}(1-r) = \\
&= r\dot{q}(1-r) - \dot{r}q, \\
z_B &= (\dot{r}(1-q) - r\dot{q})\,(r+q-rq) - r(1-q)\,(\dot{r}+\dot{q}-\dot{r}q-r\dot{q}) = \\
&= (\dot{r}(1-q) - r\dot{q})\,(r(1-q)+q)) - r(1-q)\,(\dot{r}(1-q)+\dot{q}(1-r))) = \\
&= r(1-q)\,(\dot{r}(1-q) - r\dot{q} - \dot{r}(1-q) - \dot{q}(1-r)) + q\,(\dot{r}(1-q) - r\dot{q}) = \\
&= -r\dot{q}(1-q) + \dot{r}q(1-q) - rq\dot{q} = \\
&= \dot{r}q(1-q) - r\dot{q}, \\
z_C &= (\dot{r}q + r\dot{q})\,(r+q-rq) - rq\,(\dot{r}+\dot{q}-\dot{r}q-r\dot{q}) = \\
&= rq\,(-\dot{r}q - r\dot{q} - \dot{r} - \dot{q} + \dot{r}q + r\dot{q}) + (r+q)\,(\dot{r}q + r\dot{q}) = \\
&= -rq\,(\dot{r}+\dot{q}) + (r+q)\,(\dot{r}q + r\dot{q}) = \\
&= \dot{r}\,(-rq + q(r+q)) + \dot{q}\,(-rq + r(r+q)) = \\
&= \dot{r}q^2 + \dot{q}r^2.
\end{aligned}$$

In fact, it has been proven

Lemma 5.1 (Optimality Condition in General Case)**.** *The optimality condition for period* $[0,\gamma]$ *is*

$$\begin{aligned}
c_A\,(r\dot{q}(1-r) - \dot{r}q) + c_B\,(\dot{r}q(1-q) - r\dot{q}) + c_C\left(\dot{r}q^2 + \dot{q}r^2\right) &= \\
= \frac{rq\,(c_A q(1-r) + c_B r(1-q) + c_C rq)}{\int\limits_0^\gamma \dfrac{rq}{r+q-rq}\,dt}&.
\end{aligned}$$

The statement of Lemma 5.1 follows from (5.15).

5.3.2 Optimality Condition for Equal Failure Costs

Lemma 5.2 (Optimality Condition for Equal Failure Costs). *In the case of $c_A = c_B$, the optimality condition is*

$$\frac{c_A}{c_A - c_C} = \frac{rq}{r+q-rq} - \frac{\dot{r}q^2 + \dot{q}r^2}{rq(r+q-rq)} \int_0^\gamma \frac{rq}{r+q-rq}\,dt.$$

Proof. Consider the case where $c_A = c_B$, then

$$(c_C - c_A)\left(\dot{r}q^2 + \dot{q}r^2\right) = \frac{rq\left(c_A(r+q-2rq) + c_C rq\right)}{\int_0^\gamma \frac{rq}{r+q-rq}\,dt},$$

$$(c_C - c_A)\left(\dot{r}q^2 + \dot{q}r^2\right) = \frac{rq\left(c_A(r+q-rq) + (c_C - c_A)\,rq\right)}{\int_0^\gamma \frac{rq}{r+q-rq}\,dt},$$

$$(c_C - c_A)\left(\dot{r}q^2 + \dot{q}r^2 - \frac{r^2q^2}{\int_0^\gamma \frac{rq}{r+q-rq}\,dt}\right) = c_A \frac{rq(r+q-rq)}{\int_0^\gamma \frac{rq}{r+q-rq}\,dt},$$

$$\frac{c_A}{c_C - c_A} = \frac{\dot{r}q^2 + \dot{q}r^2}{rq(r+q-rq)} \int_0^\gamma \frac{rq}{r+q-rq}\,dt - \frac{rq}{r+q-rq},$$

$$\frac{c_A}{c_A - c_C} = \frac{rq}{r+q-rq} - \frac{\dot{r}q^2 + \dot{q}r^2}{rq(r+q-rq)} \int_0^\gamma \frac{rq}{r+q-rq}\,dt, \tag{5.16}$$

Lemma 5.2 is proved.

Corollary. *Consider*

$$\begin{aligned}\frac{d}{dt}\left(\frac{rq}{r+q-rq}\right) &= \frac{(\dot{r}q+r\dot{q})\,(r+q-rq)-rq\,(\dot{r}+\dot{q}-\dot{r}q-r\dot{q})}{(r+q-rq)^2} = \\ &= \frac{rq\,(-\dot{r}q-r\dot{q}-\dot{r}-\dot{q}+\dot{r}q+r\dot{q})}{(r+q-rq)^2}+ \\ &+\frac{(r+q)\,(\dot{r}q+r\dot{q})}{(r+q-rq)^2} = \\ &= \frac{-rq\,(\dot{r}+\dot{q})+(r+q)\,(\dot{r}q+r\dot{q})}{(r+q-rq)^2} = \\ &= \frac{\dot{r}\,(q(r+q)-rq)+\dot{q}\,(r(r+q)-rq)}{(r+q-rq)^2} = \\ &= \frac{\dot{r}q^2+\dot{q}r^2}{(r+q-rq)^2}.\end{aligned} \tag{5.17}$$

Then:

$$\frac{\dot{r}q^2+\dot{q}r^2}{rq(r+q-rq)} = \frac{\dfrac{d}{dt}\left(\dfrac{rq}{r+q-rq}\right)}{\dfrac{rq}{r+q-rq}}.$$

Consequently, (5.16) will take the form

$$\frac{c_A}{c_A-c_C} = p_C(\gamma) - \frac{\frac{dp_C(\gamma)}{d\gamma}}{p_C(\gamma)}\int\limits_0^{\gamma} p_C(t)\,dt = F(\gamma). \tag{5.18}$$

For the convenience of further transformations, let us introduce *following notations:*

$$r = e^{-\frac{\gamma^k}{t_0}}, \quad \dot{r} = -\frac{k}{t_0}\gamma^{k-1}e^{-\frac{\gamma^k}{t_0}}, \quad q = e^{-\frac{\gamma}{m}}, \quad \dot{q} = -\frac{1}{m}e^{-\frac{\gamma}{m}}.$$

Consider the limit value of γ:

$$p_C(0) = \frac{r(0)q(0)}{r(0)+q(0)-r(0)q(0)} = \frac{1\cdot 1}{1+1-1\cdot 1} = 1.$$

Then in (5.18) $F(0) = 1$.

$$\begin{aligned}\lim_{\gamma\to\infty} p_C(\gamma) &= \lim_{\gamma\to\infty}\frac{e^{-\frac{\gamma^k}{t_0}}\cdot e^{-\frac{\gamma}{m}}}{e^{-\frac{\gamma^k}{t_0}}+e^{-\frac{\gamma}{m}}-e^{-\frac{\gamma^k}{t_0}}\cdot e^{-\frac{\gamma}{m}}} = \\ &= \lim_{\gamma\to\infty}\frac{1}{e^{\frac{\gamma}{m}}+e^{\frac{\gamma^k}{t_0}}-1} = 0.\end{aligned}$$

Consider the value

$$
\begin{aligned}
\frac{\frac{dp_C(\gamma)}{d\gamma}}{p_C(\gamma)} &= \frac{\dot{r}q^2+\dot{q}r^2}{rq(r+q-rq)} = \\
&= \frac{-\frac{k}{t_0}\gamma^{k-1}e^{-\frac{\gamma^k}{t_0}}\cdot e^{-2\frac{\gamma}{m}} - \frac{1}{m}e^{-\frac{\gamma}{m}}\cdot e^{-2\frac{\gamma^k}{t_0}}}{e^{-\frac{\gamma^k}{t_0}}\cdot e^{-\frac{\gamma}{m}}\left(e^{-\frac{\gamma^k}{t_0}}+e^{-\frac{\gamma}{m}}-e^{-\frac{\gamma^k}{t_0}}\cdot e^{-\frac{\gamma}{m}}\right)} = \\
&= \frac{-\frac{k}{t_0}\gamma^{k-1}e^{\frac{\gamma^k}{t_0}} - \frac{1}{m}e^{\frac{\gamma}{m}}}{e^{\frac{\gamma}{m}}+e^{\frac{\gamma^k}{t_0}}-1} = \\
&= -\frac{k}{t_0}\gamma^{k-1}\frac{e^{\frac{\gamma^k}{t_0}}}{e^{\frac{\gamma}{m}}+e^{\frac{\gamma^k}{t_0}}-1} - \frac{1}{m}\frac{e^{\frac{\gamma}{m}}}{e^{\frac{\gamma}{m}}+e^{\frac{\gamma^k}{t_0}}-1} = \\
&= -\frac{k}{t_0}\gamma^{k-1}\frac{1}{e^{\frac{\gamma}{m}}\cdot e^{-\frac{\gamma^k}{t_0}}+1-e^{-\frac{\gamma^k}{t_0}}} - \frac{1}{m}\frac{1}{1+e^{\frac{\gamma^k}{t_0}}\cdot e^{-\frac{\gamma}{m}}-e^{-\frac{\gamma}{m}}}
\end{aligned}
\tag{5.19}
$$

$$
\frac{\gamma^k}{t_0}-\frac{\gamma}{m} = \frac{m\gamma^k - t_0\gamma}{t_0 m} = \frac{\gamma}{t_0 m}\left(m\gamma^{k-1}-t_0\right).
$$

Next, consider the limit

$$
\begin{aligned}
\lim_{\gamma\to\infty}\int_0^{\gamma} p_C(t)\,dt &= \int_0^{\infty} p_C(t)\,dt = \int_0^{\infty}\frac{rq}{r+q-rq}\,dt = \\
&= \int_0^{\infty}\frac{e^{-\frac{t^k}{t_0}}\cdot e^{-\frac{t}{m}}}{e^{-\frac{t^k}{t_0}}+e^{-\frac{t}{m}}-e^{-\frac{t^k}{t_0}}\cdot e^{-\frac{t}{m}}}\,dt = \\
&= \int_0^{\infty}\frac{1}{e^{\frac{t}{m}}+e^{\frac{t^k}{t_0}}-1}\,dt.
\end{aligned}
$$

Let's find a majorant: $\dfrac{1}{e^{\frac{t}{m}}+e^{\frac{t^k}{t_0}}-1} \leqslant \dfrac{1}{e^{\frac{t}{m}}} = e^{-\frac{t}{m}} \quad \forall t \geqslant 0,\ k \geqslant 0$. Then:

$$
\int_0^{\infty} e^{-\frac{t}{m}}\,dt = -m\int_0^{\infty} e^{-\frac{t}{m}}\,d\left(-\frac{t}{m}\right) = -m\cdot e^{-\frac{t}{m}}\Big|_0^{\infty} = -m\cdot(0-1) = m.
$$

Therefore, $\lim\limits_{\gamma\to\infty}\int\limits_0^{\gamma} p_C(t)\,dt \leqslant m$. Moreover, $\lim\limits_{\gamma\to\infty}\int\limits_0^{\gamma} p_C(t)\,dt > 0$ since $p_C(t) \geqslant 0\ \forall t \geqslant 0$ and there exists t^* such that $p_C(t^*) > 0$.

Thus, the limit of the integral is finite positive value m^*, $0 < m^* \leqslant m$.

Return to $\dfrac{\frac{dp_C(\gamma)}{d\gamma}}{p_C(\gamma)}$ in (5.19) and denote

$$f(\gamma) = -\frac{k}{t_0}\gamma^{k-1}\frac{1}{e^{\frac{\gamma}{m}} \cdot e^{-\frac{\gamma^k}{t_0}} + 1 - e^{-\frac{\gamma^k}{t_0}}} - \frac{1}{m}\frac{1}{1 + e^{\frac{\gamma^k}{t_0}} \cdot e^{-\frac{\gamma}{m}} - e^{-\frac{\gamma}{m}}}.$$

Consider several cases, depending on the range of parameter k.

Case I: $k > 1$

1. Consider $\lim\limits_{\gamma\to+0} f(\gamma)$:

$$\lim_{\gamma\to+0}\left(-\frac{k}{t_0}\gamma^{k-1}\right) = 0,$$

$$\left.\begin{aligned} &\lim_{\gamma\to+0} e^{\frac{\gamma}{m}-\frac{\gamma^k}{t_0}} = 1 \\ &\lim_{\gamma\to+0}\left(-e^{-\frac{\gamma^k}{t_0}}\right) = -1 \end{aligned}\right\} \implies \lim_{\gamma\to+0}\frac{1}{1 + e^{\frac{\gamma}{m}-\frac{\gamma^k}{t_0}} - e^{-\frac{\gamma^k}{t_0}}} = 1.$$

$$\left.\begin{aligned} &\lim_{\gamma\to+0} e^{\frac{\gamma^k}{t_0}-\frac{\gamma}{m}} = 1 \\ &\lim_{\gamma\to+0}\left(-e^{-\frac{\gamma}{m}}\right) = -1 \end{aligned}\right\} \implies \lim_{\gamma\to+0}\frac{1}{1 + e^{\frac{\gamma^k}{t_0}-\frac{\gamma}{m}} - e^{-\frac{\gamma}{m}}} = 1.$$

As a result, $\lim\limits_{\gamma\to+0} f(\gamma) = -\dfrac{1}{m}$.

2. Consider $\lim\limits_{\gamma\to+\infty} f(\gamma)$:

$$\lim_{\gamma\to+\infty}\left(-\frac{k}{t_0}\gamma^{k-1}\right) = -\infty,$$

$$\left.\begin{aligned} &\lim_{\gamma\to+\infty} e^{\frac{\gamma}{m}-\frac{\gamma^k}{t_0}} = 0 \\ &\lim_{\gamma\to+\infty}\left(-e^{-\frac{\gamma^k}{t_0}}\right) = 0 \end{aligned}\right\} \implies \lim_{\gamma\to+\infty}\frac{1}{1 + e^{\frac{\gamma}{m}-\frac{\gamma^k}{t_0}} - e^{-\frac{\gamma^k}{t_0}}} = 1.$$

$$\left.\begin{array}{l}\lim\limits_{\gamma\to+\infty} e^{\frac{\gamma^k}{t_0}-\frac{\gamma}{m}} = +\infty \\ \lim\limits_{\gamma\to+\infty} \left(-e^{-\frac{\gamma}{m}}\right) = 0\end{array}\right\} \implies \lim_{\gamma\to+\infty} \frac{1}{1+e^{\frac{\gamma^k}{t_0}-\frac{\gamma}{m}} - e^{-\frac{\gamma}{m}}} = 0.$$

The result is $\lim\limits_{\gamma\to+\infty} f(\gamma) = -\infty$.

Conclusively:

$$\lim_{\gamma\to+0}\left[p_C(\gamma) - \frac{\frac{dp_C(\gamma)}{d\gamma}}{p_C(\gamma)}\int_0^\gamma p_C(t)\,dt\right] = 1 + \frac{1}{m}\cdot 0 = 1,$$

$$\lim_{\gamma\to+\infty}\left[p_C(\gamma) - \frac{\frac{dp_C(\gamma)}{d\gamma}}{p_C(\gamma)}\int_0^\gamma p_C(t)\,dt\right] = 0 + \infty\cdot m^* = +\infty.$$

We can formulate a lemma.

Lemma 5.3. *$\int\limits_0^\gamma p_C(t)\,dt \leqslant m(1-q)$.*

Proof. We have

$$p_C(t) = \frac{rq}{r+q-rq} = \frac{1}{\frac{1}{r}+\frac{1}{q}-1} = \frac{1}{e^{\frac{t^k}{t_0}} + e^{\frac{t}{m}} - 1}.$$

Since $e^{\frac{t^k}{t_0}} - 1 \geqslant 0$ $\forall t \geqslant 0$, then $\dfrac{1}{e^{\frac{t^k}{t_0}} + e^{\frac{t}{m}} - 1} \leqslant \dfrac{1}{e^{\frac{t}{m}}} \leqslant q$ $\forall t \geqslant 0$.

Then $\int\limits_0^\gamma p_C(t)\,dt \leqslant \int\limits_0^\gamma q\,dt$ $\forall\gamma \geqslant 0$. From here:

$$\int_0^\gamma p_C(t)\,dt \leqslant \int_0^\gamma e^{-\frac{t}{m}}\,dt = -m\,e^{-\frac{t}{m}}\Big|_0^\gamma = m\left(1-e^{-\frac{t}{m}}\right) = m(1-q).$$

Lemma 5.3 is proved.

Corollary. *Since $\lim\limits_{\gamma\to+\infty} q(\gamma) = 0$, then $\lim\limits_{\gamma\to+\infty}\int\limits_0^\gamma p_C(t)\,dt \leqslant m$.*

Case II: $k \leqslant 1$

Back to equation (5.18):

$$\frac{c_A}{c_A - c_C} = p_C(\gamma) - \frac{\frac{dp_C(\gamma)}{d\gamma}}{p_C(\gamma)} \int\limits_0^{\gamma} p_C(t)\,dt = F(\gamma).$$

Let's add new notations to the existing ones:

$$p_C(\gamma) = \frac{rq}{r+q-rq} \qquad \begin{array}{ll} r = e^{-\frac{\gamma^k}{t_0}}, & \dot{r} = -sr, \\ q = e^{-\frac{\gamma}{m}}, & \dot{q} = -\dfrac{1}{m}q, \end{array}$$

where $s = \dfrac{k}{t_0}\gamma^{k-1}$, therefore $\dot{s} = \dfrac{k(k-1)}{t_0}\gamma^{k-2} = \dfrac{k-1}{\gamma}s$.

$$F(\gamma) = \frac{rq}{r+q-rq} + \frac{sq + \frac{1}{m}r}{r+q-rq} \int\limits_0^{\gamma} \frac{rq}{r+q-rq}\,dt.$$

$k \leqslant 1$

1. $F(\gamma)$ is defined and continuous $\forall\gamma > 0$. However, the function is undefined at point $\gamma = 0$, because function $s(\gamma)$ is undefined at point 0 when $k < 1$. Let us show the existence of a finite limit at this point.

Lemma 5.4. $\lim\limits_{\gamma \to +0} F(\gamma) = 1$ *when* $k \leqslant 1$.

Proof. The first term:

$$p_C(0) = \frac{r(0) + q(0)}{r(0) + q(0) - r(0)q(0)} = 1.$$

Consider in detail the second term:

$$\frac{sq + \frac{1}{m}r}{r+q-rq} \int\limits_0^{\gamma} \frac{rq}{r+q-rq}\,dt = \left(s \cdot \frac{1}{\frac{r}{q} - r + 1} + \frac{1}{m} \cdot \frac{1}{\frac{q}{r} - q + 1} \right) \times$$

$$\times \int\limits_0^{\gamma} \frac{rq}{r+q-rq}\,dt.$$

At $\gamma = 0$ $\lim\limits_{\gamma\to+0} s(\gamma) = +\infty$ when $k < 1$ and $s(0) = \dfrac{1}{t_0}$ when $k = 1$.

$$\frac{1}{\frac{r(0)}{q(0)} - r(0) + 1} = \frac{1}{\frac{1}{1} - 1 + 1} = 1,$$
$$\frac{1}{\frac{q(0)}{r(0)} - q(0) + 1} = \frac{1}{\frac{1}{1} - 1 + 1} = 1,$$
$$\int\limits_0^0 \frac{rq}{r + q - rq}\,dt = 0.$$

Thus,

$$\lim_{\gamma\to+0} F(\gamma) = 1 + \lim_{\gamma\to+0}\left((s+1)\int\limits_0^\gamma \frac{rq}{r+q-rq}\,dt\right) =$$
$$= 1 + \lim_{\gamma\to+0}\left(s\int\limits_0^\gamma \frac{rq}{r+q-rq}\,dt\right).$$

We obtain an uncertainty of type $+\infty\cdot 0$. Replacing s with $\dfrac{1}{\frac{1}{s}}$, we move to uncertainty $\dfrac{0}{0}$, which can be solved by L'Hopital's rule:

$$\lim_{\gamma\to+0}\frac{\int\limits_0^\gamma \frac{rq}{r+q-rq}\,dt}{\frac{1}{s}} = \lim_{\gamma\to+0}\frac{\frac{d}{dt}\left(\int\limits_0^\gamma \frac{rq}{r+q-rq}\,dt\right)}{\frac{d}{d\gamma}\left(\frac{1}{s(\gamma)}\right)} =$$
$$= \lim_{\gamma\to+0}\frac{\frac{rq}{r+q-rq}}{-\frac{\dot{s}}{s^2}} =$$
$$= \lim_{\gamma\to+0}\left(\frac{rq}{r+q-rq}\cdot\left(-\frac{s^2}{\frac{k-1}{\gamma}s}\right)\right) =$$
$$= \lim_{\gamma\to+0}\left(\frac{rq}{r+q-rq}\cdot\left(-\frac{\gamma}{k-1}\cdot\frac{k}{t_0}\gamma^{k-1}\right)\right) =$$
$$= \lim_{\gamma\to+0}\left(\frac{rq}{r+q-rq}\cdot\left(-\frac{k}{(k-1)t_0}\gamma^{k}\right)\right) =$$
$$= \frac{1\cdot 1}{1+1-1}\cdot\left(-\frac{k}{(k-1)t_0}\cdot 0\right) = 0.$$

Hence, $\lim\limits_{\gamma\to+0} F(\gamma) = 1$.
Lemma 5.4 is proven.

2. The next step is to show the presence of an additional majorant for $\int\limits_0^\gamma p_C(t)\,dt$.

Lemma 5.5. *It is a fair constraint:*

$$\int\limits_0^\gamma \frac{rq}{r+q-rq}\,dt \leqslant \frac{1}{s}(1-r) \quad \forall\gamma \geqslant 0,\ k \leqslant 1.$$

Proof.

a) Estimate an upper bound of $p_C(\gamma)$:

$$\frac{rq}{r+q-rq} = \frac{1}{\frac{1}{r}+\frac{1}{q}-1} \overset{\frac{1}{q}-1\geqslant 0}{\leqslant} \frac{1}{\frac{1}{r}} = r \Rightarrow$$

$$\Rightarrow \int\limits_0^\gamma \frac{rq}{r+q-rq}\,dt \leqslant \int\limits_0^\gamma r\,dt.$$

b) Let's show that the formula is correct

$$\int\limits_0^\gamma r(t)\,dt \leqslant \frac{1}{s(\gamma)}\,(1-r(\gamma))$$

when $k < 1$. First, let's check point $\gamma = 0$: on the left-hand side

$$\int\limits_0^0 r(t)\,dt = 0.$$

The right-hand side:

$$\frac{1}{s}(1-r) = \frac{t_0}{k}\gamma^{1-k}\left(1-e^{-\frac{\gamma^k}{t_0}}\right),$$

at 0 we have $\dfrac{t_0}{k}\cdot 0\cdot(1-1) = 0$.
Now differentiate both sides:

$$\frac{d}{d\gamma}\left(\int\limits_0^\gamma r(t)\,dt\right) = r(\gamma),$$

$$\begin{aligned}\frac{d}{d\gamma}\left(\frac{1}{s(\gamma)}\,(1-r(\gamma))\right) &= \frac{d}{d\gamma}\left(\frac{1}{s(\gamma)}\right)(1-r(\gamma)) - \\ &- \frac{1}{s(\gamma)}\frac{dr(\gamma)}{d\gamma} = \\ &= -\frac{\dot{s}}{s^2}(1-r)+r = \\ &= -\frac{k-1}{\gamma}\cdot\frac{1}{s}(1-r)+r = \\ &= \frac{1-k}{\gamma}\cdot\frac{t_0}{k\gamma^{k-1}}(1-r)+r = \\ &= \frac{(1-k)t_0}{k}\cdot\frac{1}{\gamma^k}(1-r)+r.\end{aligned}$$

Let's compare for $\gamma > 0$:

$$\begin{gathered} r \leqslant \frac{(1-k)t_0}{k}\cdot\frac{1}{\gamma^k}(1-r)+r \\ \frac{(1-k)t_0}{k}\cdot\frac{1}{\gamma^k}(1-r) \geqslant 0\end{gathered}$$

– correct.

The result shows that at point 0 both parts are equal, and when γ grows, the function in the right side increases at least no slower than the function in the left side (it follows from the inequality for derivatives). Therefore the tested inequality is true.

c) Let's show fairness

$$\int\limits_0^\gamma r(t)\,dt \leqslant \frac{1}{s(\gamma)}\,(1-r(\gamma))$$

when $k = 1$. Calculate the integral:

$$\int\limits_0^\gamma e^{-\frac{t}{t_0}}\,dt = -t_0\,e^{-\frac{t}{t_0}}\Big|_0^\gamma = t_0\left(1-e^{-\frac{\gamma}{t_0}}\right).$$

And since at $k = 1$: $s(\gamma) = \dfrac{k}{t_0}\gamma^{k-1} = \dfrac{1}{t_0}$ and $\dfrac{1}{s} = t_0$, then

$$t_0\left(1-e^{\frac{\gamma}{t_0}}\right) = \frac{1}{s}(1-r).$$

Thus, for $k \leqslant 1$ it has been shown:

$$\int\limits_0^{\gamma} \frac{rq}{r+q-rq}\,dt \leqslant \int\limits_0^{\gamma} r\,dt \leqslant \frac{1}{s}(1-r).$$

Lemma 5.5 is proven.

Corollary. *Earlier it was shown that there is another majorant* $\int\limits_0^{\gamma} p_C(t)\,dt \leqslant m(1-q)$. *Therefore we can conclude that*

$$\int\limits_0^{\gamma} p_C(t)\,dt \leqslant \min\left\{m(1-q); \frac{1}{s}(1-r)\right\}.$$

3. Now show the boundedness of $F(\gamma)$.

Lemma 5.6. $F(\gamma) \leqslant 1$ *when* $k \leqslant 1$ $\forall \gamma \geqslant 0$.

Proof. For point $\gamma = 0$ the boundedness follows from Lemma 5.4, so we show for $\gamma > 0$:

$$\begin{aligned}
F(\gamma) &= \frac{rq}{r+q-rq} + \frac{sq+\frac{1}{m}r}{r+q-rq}\int\limits_0^{\gamma}\frac{rq}{r+q-rq}\,dt \leqslant \\
&\leqslant \frac{1}{\frac{1}{r}+\frac{1}{q}-1} + \frac{s\frac{1}{r}+\frac{1}{m}\frac{1}{q}}{\frac{1}{r}+\frac{1}{q}-1}\min\left\{m(1-q);\frac{1}{s}(1-r)\right\} \leqslant \\
&\leqslant \frac{1}{\frac{1}{r}+\frac{1}{q}-1} + \frac{s\frac{1}{r}\cdot\frac{1}{s}(1-r)}{\frac{1}{r}+\frac{1}{q}-1} + \frac{\frac{1}{m}\frac{1}{q}\cdot m(1-q)}{\frac{1}{r}+\frac{1}{q}-1} = \\
&= \frac{1+\left(\frac{1}{r}-1\right)+\left(\frac{1}{q}-1\right)}{\frac{1}{r}+\frac{1}{q}-1} = \\
&= \frac{\frac{1}{r}+\frac{1}{q}-1}{\frac{1}{r}+\frac{1}{q}-1} = 1.
\end{aligned}$$

Lemma 5.6 is proven.

Theorem 5.1 (Key Property of Model)**.** *Since* $\dfrac{c_A}{c_A - c_C} > 1$ *for real problems, and* $F(\gamma) \leqslant 1$, *there is no optimal value of* γ *at* $k \leqslant 1$.

The proof follows from Lemmas 5.1 – 5.6.

A graphical representation of finding the optimal revision period on real data is given in Appendix C.

5.3.3 Paragraph Conclusions

This part of the paper focuses on the construction and analysis of a model in order to find the optimum preventive maintenance period to ensure the reliability of technical systems. The topic of constructing a maintenance and repair programme that can strike a balance between the level of uptime and the cost involved in doing so is becoming increasingly relevant today. Furthermore, in recent years, a great deal of attention in Reliability-Centered Maintenance (RCM) methodology has been directed towards reducing the proportion of faults attributable to human operators. As a rule, this initiative is implemented on the basis of regulatory procedures mainly in terms of process technology. Mathematical models of reliability assessment with regard to human factors are being developed and are usually considered separately from system reliability.

This part of the study presents the construction of a model that takes into account both ideas proposed in [16], [58] and the actively developing field of HRA (Human Reliability Analysis) to determine the optimal frequency of preventive maintenance. A number of new results are obtained:

1. A theoretical unit cost model is constructed which takes into account the maintenance costs and failure consequences of two sources of failure: the technical component of the system and the influence of maintenance personnel.
2. A general condition for an optimum maintenance period is derived.

3. The proposed model investigates the existence of an optimal solution depending on the change of shape parameter k in the case of equal failure costs due to both system specifications and human influence. The results will help to build a strategy for reliability management using the Weibull distribution parameters found in Paragraph 5.2, and to determine the optimal period of preventive measures depending on the values of parameter k.

5.4 Determining Replacement Period for Technical System Components Based on Dynamic Programming Method

Solving the problem of determining the optimal preventive maintenance period reduces the operating costs, i.e. it affects the life cycle cost of the product (this section of the operating costs is part of Block №5 of Figure 1.1, relating to the operational phase of the product).

In addition to inspection costs, the cost of replacing components from the old to the new plays a major role: too frequent replacements can be expensive, while insufficient replacement can lead to a critical failure [95]. At the same time, with import substitution, the supply of new components has become even more important in the maintenance and repair of the technical system.

As shown in Figure 5.1, technical system maintenance and repair activities can be divided into two main categories: preventive and corrective. There are several options for accounting for monthly system maintenance costs. Let us consider the most popular one [96] and adapt this model to costs specific to the rolling stock production and maintenance area. Preventive maintenance costs can be described as follows:

$$PMC = \frac{OH \times HR \times CETPM}{SPMI}, \tag{5.20}$$

OH – the equipment operating hours per month;
HR – the hourly rate worked at the site;
$SPMI$ – the scheduled preventive maintenance interval (in this case technical inspections).

Corrective maintenance cost includes the costs associated with rectifying the failure and its consequences:

$$CMC = \frac{OH \times HR \times MTTR}{MTBF}, \tag{5.21}$$

$MTBF$ – the mean time between failures;
$CETPM$ – the customer engineer's scheduled time for performing preventive maintenance ;
$MTTR$ – the mean time to repair.

The scope and timing of the replacement of the various components and system elements for a product is usually fixed in the operating and maintenance documentation for the product. In most cases, the frequency of replacements is determined by expert observation by the operator and commission inspections.

Let's determine the optimum replacement times of the product components based on the dynamic programming method and reliability characteristic values derived from the methods and algorithms developed in Chapters 3, 4, 5. One of the main methods of dynamic programming is the recurrence relation method, which is based on the use of the optimality principle developed by Bellman [97]. The dynamic programming method solves problems in which the control process is broken down into steps. A continuous control process can be treated as a discrete process by conditionally breaking it down into time periods. The length of the step is determined by the requirements of the particular task. In the case of determining the replacement time of components of a technical object, we will choose a step of a month, since, firstly, in Chapter 4 the step is equal to one month, and, secondly, more often than this period, usually no scheduled replacements are performed in this area (except for cases of production defects or out-of-state situations).

In general, the objective of the equipment replacement task is to find a replacement period for old machines/lines within an industrial facility. The criterion for optimality is most often the profit from the operation of the equipment (productivity or value of output). The cost factor is usually the cost of maintaining and repairing the equipment depending on its age.

Let us upgrade this model to the task of optimal replacement in the rolling stock component industry: the productivity function will be the monthly profit per unit of equipment. The cost function will be the maintenance costs of a unit of rolling stock per month. The costs of maintenance in operation are mainly made up of the costs of preventive and corrective measures. Let us introduce into expressions (5.20) and (5.21) the variables derived from Chapters 3, 4, 5 (see Table 5.1). Let's describe each of them:

1. Replace $CETPM$ by t_{pr} in (5.20) and $MTTR$ by t_r in (5.21). The characteristics introduced instead of $CETPM$ and $MTTR$ can be identified based on statistical hypothesis testing from Chapter 3. By identifying peer components according to various criteria (operating profile, duration of inspection, duration of repair depending on product features, design variations), a number of parameters can be identified at the stage of technical object development, according to which characteristics on new products are set. The list of these components can be compiled by applying the apparatus proposed in Chapter 3 on the factors that are required for the study. Thus, we will not have as an initial value the planned time solely by engineering analysis evaluation, but a verified evaluation by a comprehensive approach: selection of analogues in terms of maintenance technology and design characteristics, followed by a verification based on statistical hypothesis testing.

2. $MTBF$ (which is usually treated as a constant) in (5.21) will be replaced by a value to be recalculated from month to month through λ_t, computed from actual data in past periods and modelled on the time series with the apparatus of survival theory applied to the predicted values proposed in Chapter 4.
3. $SPMI$ is one of the most uncertain parameters in the field of uptime, as it depends on many factors. The most important of these are the characteristics of the components in question, the human factor and the economic cost of the maintenance process. Because of the complex set of influencing characteristics, the scope of determining the optimality of this parameter was discussed in paragraph 5.3 and $SPMI$ is replaced by γ_{opt}, whose value is obtained by the algorithm in Chapter 5 based on substitution of real data.

Table 5.1 — Parameter correspondence.

PMC		CMC	
Replacement from/to		Replacement from/to	
$CETPM$	t_{pr} (Chapter 3)	$MTTR$	t_r (Chapter 3)
$SPMI$	γ_{opt} (Chapter 5)	$MTBF$	$1/\lambda$ (Chapter 4)

The classical problem of determining an equipment replacement strategy is set as follows to describe an N-stage process [98], [99]:

$$f_N(t) = \max \begin{cases} r(t) - u(t) + f_{N-1}(t+1), \\ r(0) - u(0) - P + f_{N-1}(1); \end{cases}$$

a one-stage process has the form:

$$f_1(t) = \max \begin{cases} r(t) - u(t), \\ r(0) - u(0) - P, \end{cases}$$

where $r(t)$ is the income from the operation of a unit of rolling stock equipment per month, $u(t)$ is the monthly maintenance costs, $f_N(t)$ is the maximum income from the operation of equipment of t months age for the remaining N months of the component under the condition of optimal strategy. At each stage of the N-stage process, a decision must be made whether to continue operating the component or to replace it with a new one. The chosen option should maximise the profit. The first rows of the above functional equations define the income from continuing to operate the component (conservation), while the lower rows are for the income from replacing the component with a new one (with a replenished failure-free rate). Given the suggestion of using table replacements 5.1, let's rewrite the maintenance cost function as follows:

$$u(t) = PMC + CMC = \frac{OH \times HR \times t_{pr}}{\gamma_{opt}} + \frac{OH \times HR \times t_r}{1/\lambda_t}. \quad (5.22)$$

Based on the implementation of this algorithm, the optimum update period for the control unit software was determined from the operational data. This update period was once every six months (see Appendix D).

Chapter Conclusions

This section presents an algorithm for constructing an optimal maintenance strategy based on the human factor influence on the maintenance process of engineering facilities in operation. Human Reliability Analysis (HRA) [76] has rapidly evolved in recent years due to the increasing complexity of production systems and variations in their operation. Many human activities are being replaced by robotic functions and various automated systems also for the purpose of reducing the risk due to human error. In order to assess this risk, it is necessary to use reliability and maintenance models that take into account human influence, both for assessing reliability dependent on real people and for creating high-tech systems based on artificial intelligence.

In this chapter, the Weibull distribution law is proposed as an approximation model for the reliability due to the human operator, and the reasoning for this proposal is given in paragraph 5.2. Relations for identifying the parameters of the distribution are given. Then a method for determining the optimal maintenance interval by the criterion of minimizing the cost of failure and preventive maintenance, taking into account both the product specifications and the component of system reliability due to the impact of personnel on the maintenance process (paragraph 5.3) is presented. On the basis of a number of transformations and consideration of limit relations, this mathematical model is investigated and conclusions are drawn about the existence of an optimum, depending on changes of the Weibull distribution form parameter.

The aim of the last step in the optimisation of the maintenance and repair process of a facility is to find the optimum replacement period for a component of a composite product (paragraph 5.4). This search was carried out on the basis of a complex method, the initial parameters for which are the following reliability characteristics:

1. *Preventive maintenance and repair times.* The replacement intervals are entered into the technical documentation during the product development phase, at a stage where the component data is not yet available (stage №2 in diagram 1.1). For this reason, the indicators are derived by applying the statistical criteria from Chapter 3 (Fig. 1.2) to select the characteristics from data of similar components and comparable operating conditions.
2. *Predicted failure-free values.* In Chapter 4, a time series model with selected adaptation parameters based on the apparatus of survival theory is constructed. An accurate prediction will determine the replacement time for an element already in service, as well as for a new design. In the first case, this method will determine the period when replacement is needed in the process of operation, which will allow planning the supply of spare parts, tools, accessories before the failure occurs (stage №5 in diagram 1.1). In the second case,

based on the actual and predicted failure rates of the peer units, identify the optimum replacement time to reduce the cost of the designed product by introducing replacement instructions for the future customer (step №2 in diagram 1.1).

3. *Optimum preventive maintenance period.* The frequency of inspections and maintenance takes up a large proportion of the total life-cycle costs of a product. By finding the optimum value for this indicator, taking into account the human factor and including this frequency value in the total cost function (5.22), the model will find the replacement times with the best ratio of uptime to maintainability in terms of the unit cost.

The approaches developed in this chapter will allow both building a maintenance and repair strategy for the designed products, and adjusting the operational manuals and repair documentation used for the rolling stock units in operation. The proposed methods ensure the reduction of costs at different stages of the product life cycle (Figure 1.1), which increases product competitiveness in the market and flexible process of adaptation in the rapidly changing conditions of the industrial market.

CONCLUSION

This paper presents different approaches for assessing and ensuring the reliability of transport system components. Reliability is a complex property not only on the technical side of the product, but also includes such an important property as safety for use, which must adequately correlate with the economic effect of ensuring a given level of all necessary indicators. Excessive reliability characteristics may cause negative economic effect in the framework of competition in the industrial sphere, while its insufficient level may lead to failures and accidents of different levels of criticality, including nuclear disasters and major man-caused accidents. For these reasons, a multi-stage and multi-factor approach to reliability analysis is applicable to every area of production, regardless of its focus.

The general idea of this study is to select and develop methods that are most suitable for each of the blocks of the reliability management system in an enterprise (see Fig. 1.2). This study allows building a reliability-oriented approach in production systems based on various mathematical models. To summarise the work done, let us highlight the main results:

1. A combined approach for setting standardised reliability indicators, based on structural and Markovian analysis and taking into account the requirements of the customer and the manufacturer, is proposed.
2. On the basis of an extensive analysis of common criteria for statistical hypothesis testing in reliability theory, their testing on real data and an investigation of their theoretical properties and differences, suggestions have been made for applicable statistical criteria and examples of their implementation on various reliability data are given. This section of the analysis applies to many reliability problems and can be an end in itself for a problem, or it can be a step for calculations, methods of implementation of which are presented in Chapters 4, 5.

3. A short-term prediction model for various technical systems has been constructed. The forecast has been built on real operational data using the Holt – Winters adaptive forecasting method with a selection of series coefficients based on survivability analysis. The results of the solution of the described problem are currently used to predict the reliability of analog nodes of different types of rolling stock components. The maintenance and repair procedures for different types of doors have also been reviewed, and corrective actions for a number of door units have been assessed and revised, taking into account the application of the above prediction methodology.
4. A method of determining the optimum frequency of preventive maintenance based on minimisation of the unit cost function and investigation of its behaviour depending on the distribution parameters has been developed. An important and distinctive feature of this approach is the consideration of the human factor, which always has an impact with any type of technical measures and significantly affects the reliability of a complex technical system in conjunction with its own uptime.

By using and modifying the methods and results used in this work, the lifecycle management of rolling stock components can be significantly improved, and production and operating costs can be reduced.

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APPENDIX A

Code for Pairwise Comparison of Samples Based on Statistical Hypothesis Testing

Input data: *Tests.xlsx* (Excel file whose columns further the samples to be compared).

Output data *results.xlsx* (Excel file containing a table with the results of the tests carried out).

The program is written in the Python 3.7 programming language. First, it reads the data and converts it to a usable data type for further work. Then the Lepage test is written, as its realisation is not available in any existing library. The next step is the implementation of the statistical criteria we are interested in. The last step is to convert the results and save them.

```python
import pandas as pd
import numpy as np
from scipy import stats
from nonparstat import (Cucconi,
                        PodgorGastwirth)
from scipy.stats import rankdata
from collections import namedtuple
from tqdm import tqdm
from joblib import (Parallel,
                    delayed)

sheet_name = "All"
df = pd.read_excel(open('Tests.xlsx', 'rb'),
                   sheet_name=sheet_name)

samples = {}
for i in range(len(df.columns)):  #creating dict from df
    index = df.columns.values[i][1:]  #is a string
    df.columns.values[i] = f"sample{index}"
```

```python
    samples[df.columns.values[i]]= df.iloc[:,i].values.tolist()
for key, value in samples.items(): #deleting "nan" values
    samples[key] = [x for x in value if str(x) != 'nan']


LepageResult=namedtuple('LepageResult',('statistic','pvalue'))

def _lepage_test_statistic(a, b, ties='average'):
    n1 = len(a)
    n2 = len(b)
    n = n1 + n2

    E_W = n1 * (n + 1) / 2
    V_W = n1 * n2 * (n + 1) / 12
    if n % 2 == 0:
        E_AB = n1 * (n + 2) / 4
        V_AB = n1 * n2 * (n + 2) * (n - 2) / 48 / (n - 1)
    else:
        E_AB = n1 * (n + 1)**2 / 4 / n
        V_AB = n1 * n2 * (n + 1) * (n**2 + 3) / 48 / n**2

    W = stats.mannwhitneyu(a, b)[0]
    AB = stats.ansari(a, b)[0]
    L = (W - E_W)**2 / V_W + (AB - E_AB)**2 / V_AB
    return L

def _lepage_dist_permutation(a, b, replications=1000,
                             ties='average', n_jobs=1,
                             verbose=0):
    n1 = len(a)
    h0_data = np.concatenate([a, b])

    def permuted_test(replication_index):
        permuted_data = np.random.permutation(h0_data)
        new_a = permuted_data[:n1]
        new_b = permuted_data[n1:]
        return _lepage_test_statistic(a=new_a,
```

```
                                          b=new_b,
                                          ties=ties)
    return sorted(Parallel(n_jobs=n_jobs,
                           verbose=verbose)(delayed(
                               permuted_test)(i)
                           for i in range(replications)))


def lepage_test(a, b, replications=1000,
                ties='average', n_jobs=1,
                verbose=0):
    a, b = map(np.asarray, (a, b))
    test_statistics = _lepage_test_statistic(a=a, b=b,
                                             ties=ties)
    h0_distribution = _lepage_dist_permutation(a=a, b=b,
                                               replications=
                                               replications,
                                               ties=ties,
                                               n_jobs=n_jobs,
                                               verbose=verbose)
    p_value = (len(np.array(h0_distribution)[
        h0_distribution >= test_statistics]) + 1) /
        (replications + 1)
    return LepageResult(statistic=test_statistics,
                        pvalue=p_value)


test_names = {"Student": stats.ttest_ind,
              "Mann-Whitney U": stats.mannwhitneyu,
              "Wilcoxon": stats.ranksums,
              "Kolmogorov-Smirnov": stats.ks_2samp,
              "Epps-Singleton": stats.epps_singleton_2samp,
              "Kruskal-Wallis": stats.kruskal,
              "Brunner-Munzel": stats.brunnermunzel,
              "Ansari-Bradley": stats.ansari,
              "Cucconi": Cucconi.cucconi_test,
              "Lepage": lepage_test,
              "Podgor-Gastwirth": PodgorGastwirth.
                                  podgor_gastwirth_test}
```

```
tests = {}
for key_i, value_i in tqdm(samples.items()): #each sample
    for key_j, value_j in samples.items():   #with each sample
        if key_i != key_j and (key_i + key_j)
        not in tests.keys() and (key_j + key_i)
        not in tests.keys():
            tests[key_i + key_j] = {}
            tests[key_i + key_j]["Welch"] = {}
            (statistic ,pval) = stats.ttest_ind(value_i,
                                                 value_j,
                                                 equal_var=False)
            tests[key_i + key_j]["Welch"][
                "statistic"] = statistic
            tests[key_i + key_j]["Welch"][
                "p-value"] = pval

            for test_name, function in test_names.items():
                tests[key_i + key_j][test_name] = {}
                (statistic, pval) = function(value_i, value_j)
                tests[key_i + key_j][test_name][
                    "statistic"] = statistic
                tests[key_i + key_j][test_name][
                    "p-value"] = pval

            tests[key_i + key_j]["Cramer-von Mises"] = {}
            res = stats.cramervonmises_2samp(value_i, value_j)
            tests[key_i + key_j]["Cramer-von Mises"][
                "statistic"] = res.statistic
            tests[key_i + key_j]["Cramer-von Mises"][
                "p-value"] = res.pvalue

results = pd.DataFrame.from_dict({(samples,test_name): tests[
    samples][test_name] #from nested dictionary to dataframe
                                  for samples in tests.keys()
                                  for test_name in tests[
                                      samples].keys()},
```

```
                                              orient='index')
results.to_excel("results.xlsx")
```

APPENDIX B

Weibull.m

The program finds the parameters of the Weibull distribution. The input is a vector of MTBF t and test duration T.

```
N = 610;

t = [17;19;25;41;137;137;230];

T = 365;

f = @(x)[-1/x(1)+sum((t.^x(2))/(x(1).^2))+(N-length(t))*(T.^x
    (2))/(x(1).^2);...
     1/x(2)+sum(log(t).*(1-(t.^x(2))/x(1)))+(N-length(t))*(-(T^x
    (2))/x(1)*log(T))];


x0 = [420, 2.858526];

x_solved = fsolve(f,x0);
```

APPENDIX C

Example of Determining Periodicity on Real Data

Based on the data on operating time before failures due to human errors in supervised operation, an optimum revision period of 13.13 days was determined on the basis of the identified values of the Weibull distribution parameters (Fig. C.1).

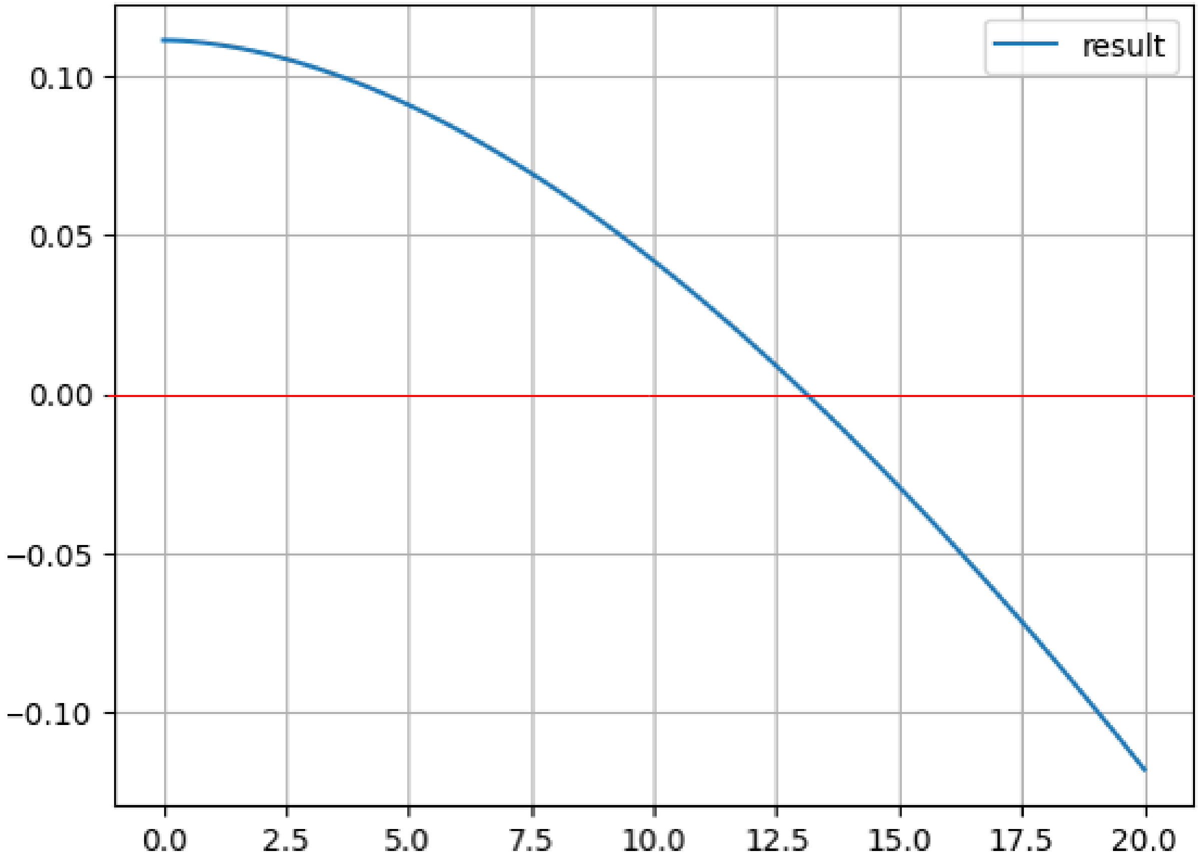

Figure C.1 — Finding the optimal revision period

APPENDIX D

Determining Optimum Time to Upgrade Door Control Unit Software

Figure D.1 shows part of the implementation of the dynamic programming algorithm from paragraph 5.4. Functions $f_N(t)$ are computed row by row and include the results of the methods from Chapters 3, 4, 5.

		Jan-21	Feb-21	Mar-21	Apr-21	May-21	Jun-21	Jul-21	Aug-21	Sep-21	Oct-21
f(t)=r(t)-u(t)		3207	2554	1933	1626	1594	1486	1484	1465	1462	1430
f1(t)	May-22	3207	2554	1933	1626	1594	1486	1484	1465	1462	1430
f2(t)	Jul-21	5761	4487	3559	3220	3080	2970	2949	2928	2893	2860
f3(t)	Jun-21	7694	6113	5153	4707	4564	4487	4487	4487	4487	4487
f4(t)	Jun-21	9320	7707	6640	6191	6113	6113	6113	6113	6113	6113
f5(t)	Jul-21	10914	9194	8123	7740	7707	7707	7707	7707	7707	7707
f6(t)	Aug-21	12401	10677	9673	9334	9301	9194	9194	9194	9194	9194
f7(t)	Jul-21	13884	12227	11267	10928	10788	10680	10677	10677	10677	10677
f8(t)	Jun-21	15433	13821	12861	12414	12274	12227	12227	12227	12227	12227
f9(t)	Jun-21	17028	15415	14347	13900	13821	13821	13821	13821	13821	13821
f10(t)	Jul-21	18622	16901	15833	15447	15415	15415	15415	15415	15415	15415
f11(t)	Jul-21	20108	18387	17380	17041	17009	16901	16901	16901	16901	16901
f12(t)	Jul-21	21594	19934	18974	18635	18495	18387	18387	18387	18387	18387
f13(t)	Jun-21	23141	21528	20568	20121	19981	19934	19934	19934	19934	19934
f14(t)	Jun-21	24735	23122	22054	21608	21528	21528	21528	21528	21528	21528
f15(t)	Jul-21	26329	24608	23540	23154	23122	23122	23122	23122	23122	23122
f16(t)	Jul-21	27815	26094	25087	24748	24716	24608	24608	24608	24608	24608
f17(t)	Jun-21	29301	27641	26681	26342	26202	26094	26094	26094	26094	26094
f18(t)	May-21	30848	29235	28275	27829	27689	27641	27641	27641	27641	27641
f19(t)	May-21	32442	30829	29762	29315	29235	29235	29235	29235	29235	29235
f20(t)	Jun-21	34036	32316	31248	30862	30829	30829	30829	30829	30829	30829
f21(t)	Jun-21	35523	33802	32794	32456	32423	32316	32316	32316	32316	32316
f22(t)	Jun-21	37009	35348	34389	34050	33910	33802	33802	33802	33802	33802
f23(t)	May-21	38555	36943	35983	35536	35396	35348	35348	35348	35348	35348
f24(t)	May-21	40150	38537	37469	37022	36943	36943	36943	36943	36943	36943
f25(t)	Jun-21	41744	40023	38955	38569	38537	38537	38537	38537	38537	38537
f26(t)	Jun-21	43230	41509	40502	40163	40131	40023	40023	40023	40023	40023
f27(t)	Jun-21	44716	43056	42096	41757	41617	41509	41509	41509	41509	41509

Figure D.1 — Implementation of the dynamic programming algorithm from Chapter 5