

**DEPARTMENT OF COMPUTER AND INFORMATION
SCIENCE**

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REPORT

By a Member of the Dissertation Committee for the Dissertation

**by Ovanes Petrosian “Dynamic and Continuous Information Updating in Models of
Conflict Control”**

submitted in satisfaction of the requirements

**for the degree of Doctor of Sciences in Physics and Mathematics
(specialization 2.3.1. System analysis, information control and processing)**

The presented thesis is devoted to the approach of dynamic and continuous updating for conflict multi-agent control models. Overall, the work contains an Introduction and five chapters. There various types of information updating about motion equations and agents payoff functions are studied. Moreover, the new types of the game theoretical optimality principles especially developed for the information updating approach are proposed, such as the Nash equilibrium, cooperative strategies and related cooperative solutions. The convergency issue for the Nash equilibrium, cooperative strategies related controls and corresponding trajectories in the case of dynamic updating and continuous updating is investigated. In order to define the controls and related trajectories, modified optimality conditions for controls with continuous updating are derived in the form of Hamilton-Jacobi-Bellman equations, The Pontryagin’s Maximum Principle and special conditions for the Linear Quadratic case models.

The scientific novelty of the thesis is primary due to the presence of results related to the construction of new type of controls or strategies with dynamic and continuous updating, construction of the related optimality conditions in the form of the Hamilton-Jacobi-Bellman equations, Pontryagin’s Maximum Principle and the linear quadratic optimality conditions. Other important theoretical results are the proof of convergence theorems for the controls or strategies and corresponding

trajectories for the autonomous and non-autonomous linear quadratic models with continuous updating, as well as the study of new cooperative solutions and characteristic functions with continuous updating and the proof of the corresponding theorems.

The theoretical significance of the thesis is in developing a new approach with dynamic and continuous information updating, construction of the corresponding basic theory of information updating, namely the description of controls or strategies, related strategies and trajectories construction approach and analysis of the resulting controls and trajectories.

The practical significance of the thesis is determined by the construction of the inverse problem of optimal control based on the approach with continuous information updating, and automatic identification of the value of the information horizon or window.

The validity and reliability of the scientific results presented in the thesis is supported by the rigorous mathematical proofs of the formulated statements, approbation of the described results at numerous international conferences, publications in peer-reviewed Russian and international publications. The author published 38 scientific papers; 28 scientific papers are devoted to the thesis research topic, of which 22 scientific papers were published in venues indexed in the international databases Scopus or WoS.

Chapter 1 deals with a new cooperative solution IDP-core that is applied for multi-agent control models with dynamic and continuous updating. The proposed solution is defined using the list of the newly defined axioms of time-consistency and IDP non-dominance. It is important to notice that the time-consistency property and the notion of the IDP already existed before, but the author used these properties as the axioms to define the cooperative solution for the first time. In addition, it is important to notice that up to my knowledge the constructed this way cooperative solution for dynamic and differential game models IDP-core is the only existing cooperative solution for differential games defined using the set of axioms. The explicit form of IDP-core cooperative solution is derived based on the state axioms, it is defined using the set of linear constraints for the imputation distribution procedures. Another

practical result is related to the construction of the non-emptiness properties of the IDP-core based on the linear programming technique.

Chapters 2 and 3 present the approach of dynamic updating applied for the discrete and differential multi-agent control models or discrete and differential games. Presented results are devoted to the general class of cooperative and non-cooperative discrete and differential game models with dynamic updating and special classes of models with dynamic updating represented by the stochastic forecast about the unknown dynamics and random horizon approach for informational window of the agents. Additionally for the differential model a detailed study of the near-real oil market models with dynamic updating are considered in detail. The attempt was performed to use the historical dataset to reconstruct the game model and do the conclusion about the future possible actions of the agents on the oil market. In chapters 2 and 3 the concept of equilibrium and cooperative controls with dynamic updating is introduced and the algorithm for constructing the corresponding equilibrium and cooperative trajectory with updating is given. The concept of a cooperative solution with dynamic updating and two approaches for constructing it are based on the imputation distribution procedures, also a special type of characteristic function for the dynamic updating case is introduced. The strong Δ -time consistency for an arbitrary cooperative solution is proved and the correspondence between the cooperative solutions defined in the interval of information horizon and with dynamic updating is studied.

Chapter 4 is devoted to the continuous updating approach applied to the continuous time differential multi-agent models of conflict control. The chapter begins with a description for the assumptions of continuous updating: at each current time instant the agents or players have or use the information about the motion equations and payoff functions only on the interval with the length equal to the value of the informational horizon; current time instant t continuously changing as the model time evolves. A detailed description of non-cooperative and cooperative models with continuous updating first proposed by the author is provided. The concept and definitions of cooperative and equilibrium controls for the continuous updating case are proposed based on the assumptions defined above. An important theoretical result are the related optimality conditions for the controls with continuous updating represented in the form of the Hamilton-Jacobi-Bellman equations, Pontryagin's

Maximum Principle and a special linear-quadratic case conditions. For the linear-quadratic model autonomous and non-autonomous cases are considered and for a special logarithmic resource extraction game model, a uniform convergence of the controls with dynamic updating and controls with continuous updating as the updating interval tends to zero is proved. Related results regarding the pointwise convergence for trajectories with dynamic and continuous updates are provided as well. Considering the cooperative setting of continuous updating multi-agent control models a strong time-consistency property of an arbitrary cooperative solution is proved. Additionally to a large number of model examples, a detailed description of the resource extraction model with continuous and dynamic updating is provided.

Chapter 5 presents the inverse optimal control problem with continuous updating. The considered problem itself is in some sense an application of the continuous updating approach to the inverse optimal control theory. It is important to notice that the formulation of the inverse optimal control problem with continuous updating itself is new, including the approach to find the optimal control. Another important fact is that the approach with continuous updating finds its application in the inverse optimal control problems where the interaction between a person or an agent and various technical systems such as a car takes place. The reason is that a person or an agent usually considers a truncated informational window to perform the action. Additionally in this chapter an applied linear mathematical model of a driving car is considered. It is supposed that the driver has a quadratic payoff function that defines his priorities for driving the car depending on the controls and state of the car. The meaning of the inverse optimal control problem solution is in the analysis of driver type based on observations of the actions used or the identification of the quadratic payoff function of the driver.

While there is little to criticize in this thesis, I mention that it might be also important in future to extend the practical approbation of the proposed continuous updating approach for the inverse optimal control or identification of the driver in the real simulation. Table 5.1 provides the car description, but not a description of the driver itself; however, the conclusion of the section 5.5 refers to the decision maker (the driver). The part of the driver parameters and the car parameters are loosely connected, the driver may fall asleep or be drunk, for example. Maybe in future it would be possible to modify the continuous updating technique to study or take into

account such things.

In conclusion, the Thesis “Dynamic and Continuous Information Updating in Models of Conflict Control” by Ovanes Petrosian satisfies the requirements established by Order № 11181/1 at 19 of November 2021, “On the Procedure for Awarding Degrees at Saint Petersburg State University”, and exceeds the high standards for Doctor Theses. Ovanes Petrosian fully deserves to be awarded the degree of Doctor of Sciences in Physics and Mathematics (specialization 2.3.1. System analysis, information control and processing). Clause 11 of the aforementioned Order was not broken by the author of the Thesis.

Sincerely,



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