Review on the Dissertation presented by Li Yin,
entitled "Solutions of Spanning Tree Games"
submitted for the Degree of Candidate of Physical and Mathematical Sciences
in Scientific Specialization 1.2.3. "Theoretical Informatics, Cybernetics"

Research relevance Li Yin's dissertation is devoted to the investigation on the dynamic stability of different cooperative solutions in dynamic cooperative games with spanning trees. Several game models of various types constructed in this dissertation allow us to study stochastic and non-stochastic, two-stage and multi-stage, as well as simple and complex network topologies. This research combines a dynamic network game model with a minimum spanning tree game model. A conducted research permits players to be engaged in a conflict-controlled process and manages to construct a cost allocation scheme, which satisfies the principle of dynamic stability. The importance of the presented research is confirmed by the relevance in economic and social life in which many dynamic networks with spanning trees exist, such as dynamic gas pipeline transmission networks or electricity transmission networks.

The theoretical significance and scientific novelty of this dissertation are determined by the author's academic achievements on the development of a dynamic game model with spanning trees and the construction of a dynamic stable cooperative cost allocation. The author succeeds in extending the game model with a spanning tree to the dynamic case in stochastic and non-stochastic settings. Also, the author considers a two-stage stochastic game with spanning forests. A related cooperative solution is considered for each constructed game model, and dynamic stability property is studied. In this dissertation, the author investigates on how cooperative solutions are constructed in games with spanning trees throughout the entire dynamic process when the network structure changes randomly and considers the methods of allocating cooperative payoff and different allocation properties. In addition, based on the dynamic logistics network problem for perishable goods, the stochastic game model containing perishable goods is developed, and related cooperative solutions and dynamic stability are examined. In the non-stochastic case, the author presents a tool for transforming the cost matrix using the Hadamard product of matrices.

The following key results of this dissertation should be singled out:

1. A two-stage spanning tree game with shock is constructed and studied. At the second stage, a particular player leaves the game with some probability depending on his position in the network built by all players at the first stage. The cooperative behavior of all players in the two-stage spanning tree game with shock is determined.

In addition, the characteristic function and the dynamic Shapley value satisfying time consistency property are constructed using the imputation distribution procedure.

2. The model with several sources in the network is considered, and a two-stage spanning forest game with shock is proposed. At the second stage, a particular player leaves the game with some probability depending on his position in the network built by all players at the first stage. The cooperative behavior of all players in the two-stage spanning forest game with shock is determined. The characteristic function is constructed, and the dynamic Shapley value satisfying time consistency property in the game is determined using the imputation distribution procedure.

3. A new α-matrix is defined for modifying the network built by the players in the one-stage spanning tree game.

4. A mult-istage spanning tree game with complete information is defined using the α-matrix. The cooperative behavior of all players in this game is determined, and the characteristic function is constructed. A new characteristic function is defined based on the characteristic function of one-stage game. The new characteristic function is adopted for constructing the core satisfying strong time consistency property.

Practical significance. The mathematical model of designing a spanning tree or spanning forest network proposed by the author and the solution of the cost allocation problem for dynamic networks can be used in practical applications for minimizing cost in economic and social-economic networks. For example, cost allocation problems commonly arise in transportation. A two-stage spanning tree game with shock and perishable goods can serve as a mathematical model for planning the transportation of perishable goods in dynamics. The mathematical models considered in this dissertation are also beneficial for analyzing the problems of building networks and allocating cost in other fields with similar features.

In supply chain management problems, the dynamic game model with perishable goods is also actively used. In a dynamic spanning tree game with perishable goods, it is necessary to consider the rates of deterioration and loss in value when distributing such goods through transport and logistics networks. According to practical experience, the greater the transportation cost the higher the transportation speed, the better the protection of perishable goods the smaller the loss in value. Besides the minimum spanning tree cost, all these losses in value are part of the total cost on transport and logistics network. Based on such a realistic game model, Section 1.3 constructs a two-stage game with perishable goods, which exists a particular player who leaves the game at the second stage with a probability that depends on the behaviors of the players at the first stage and the amount of perishable goods that all players need. Hereby Section 1.3 constructs a two-stage game with perishable goods, which exists a particular player who leaves the game in the second stage with a probability that depends on the behaviors of the players at the first stage and the total cost of perishable goods that all players need.

The **reliability** of the results of this dissertation is determined by the rigour of the mathematical proofs, the statements formulated in the work, and the approbation of the main statements of this dissertation at the international and Russian conferences.

However, there are several **remarks** on this research:

1. In Sections 1.2, 1.3, and 1.4, is it possible to consider the case of multi-stage games with spanning tree?

2. Is it possible to use other approaches to define the characteristic functions?

3. There are some typos, mistakes, or grammatical mistake. For example, Definition 1.3.1 misses a negative symbol "-".

The above comments do not detract from the overall positive impression of Li Yin's dissertation. This dissertation completes scientific research at a high level and is undoubted of theoretical and practical interest. The results, conclusions, and recommendations are new, reliable, and substantiated. The results of this dissertation are published in 6 scientific papers, five of which are published in the publications indexed in Scopus.

Li Yin's dissertation "Solutions of Spanning Tree Games" fulfills the requirements established by the Decree №11181/1 dated November 19, 2021 "On the Procedure for Awarding Academic Degrees at St. Petersburg State University", and Li Yin deserves to be granted with a Degree of Physical and Mathematical Sciences (Scientific Specialization 1.2.3 "Theoretical Informatics, Cybernetics")

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January 28, 2022