Dr. K. Avrachenkov,
Inria Sophia Antipolis,
2004, Route des Lucioles,
06902, France,
K.Avrachenkov@inria.fr

REPORT

By a member of the dissertation committee of the dissertation of KOROLEV Alexei Vasilevich on the theme: "Mathematical Models of Control for Economic Systems with a Network Structure", submitted in conformity with requirements for the degree of Doctor of Physico-Mathematical Sciences in Speciality 01.01.09 — Discrete Mathematics and Mathematical Cybernetics.

The thesis of Alexei Korolev is dedicated to game theoretic and economic models on networks of agents. This is a very contemporary topic as many modern technological, economical and sociological systems (e.g., online social networks or inter-connected economic systems) can be adequately described by network models.

The thesis of Alexei Korolev consists of 5 technical chapters as well as of Introduction, Conclusions and Bibliography. In the Introduction, the candidate carefully described the motivation of the work, the relation of the work with the state of the art, the main contributions of the thesis and the main methodologies.

Chapter 1 sets the stage for the dissertation development and introduces several important concepts. In particular, a novel concept of typology is introduced, which together with the type adjacency matrix, appears to be a very important tool in the analysis of network centralities and network games. Specifically, Theorem 1.4.4. provides a way to calculate network centralities from the type adjacency matrix. This provides a clear computational gain, since often the type adjacency matrix is much smaller than the original adjacency matrix. It is a bit pity that the proofs of Theorem 1.4.4. and related results are too brief (something like "this follows from the definitions"). More formal proofs would be welcome. On the other hand, the author carefully discusses the conditions of existence of various path-based centrality measures. In Chapter 1 the author provides an extensive discussion of the economic and marketing implications of his and related works. Since my research background is not in economics, I cannot comment on the

RESEARCH CENTRE
SOPHIA ANTIPOLIS - MÉDITERRANÉE
2004 route des Lucioles - BP 93
06902 Sophia Antipolis Cedex France
Phone: +33 (0)4 92 38 77 77
Fax: +33 (0)4 92 38 77 65

www.inria.fr

importance of this part of Chapter 1. In Section 1.7.1.2 the author discusses the Lotka–Volterra system of differential equations with delays but then, unless I missed something, this model is not used in the thesis.

In Chapter 2 the author introduces a two-stage game-theoretic model on networks with production and externalities. The agent in a network node has to decide who much he or she consumes and how much he or she invests into knowledge in the first stage. The utility depends not only on the efforts of a player but also on the knowledge environment created by the player's neighbors. Basically, the cost function is quadratic. In this game the network structure influences significantly the type of equilibrium. I like very much the fact that in the analysis of this game, the expressions for network centralities come up naturally. Also, as in the first chapter the concept of node typology helps a lot in the calculation of various equilibria. It could be of interest to the author that a random walk network centrality with non-homogeneous restarts have been studied in Avrachenkov, K., van der Hofstad, R., & Sokol, M. "Personalized PageRank with node-dependent restart", Springer LNCS v.8882, 2014.

Chapter 3 analyses the stability of equilibria of the network game models introduced in the previous chapter. Specifically, the convergence conditions of "better response" dynamics are investigated if some agents slightly deviate from the equilibria strategy or network structure is modified. I find that the question of stability of a game-theoretic model under structural changes such as the changes in network topology is very original. The author has considered both discrete-time and continuous-time settings. In addition, the author has investigated the case when the production parameters are stochastic processes. This makes the models to be quite realistic.

Chapter 4 is dedicated to the control and competition of opinions in social networks. There are several influencing agents, who can act either cooperatively or competitively. Firstly, the case of a long-run opinion formation is considered and the problem is formulated as a static game. Then, the author considers the transient behavior and dynamics in both discrete and continuous times. Then, the presence of a coordinated principal is considered and the equilibrium is analyzed in the sense of Stackelberg. Furthermore, the author draws implications of his models on marketing strategies.

In Chapter 5 the author extends the models of Chapter 4 by addition of state-space constraints. These additional constraints are motivated by sustainable management of active economical systems and the technical extensions of the developed methods come in natural generalizations. The author deals with both cases of right-hand stochastic and left-hand stochastic influence matrices. I like the clever variable transformation which allowed to transfer the results from one case to the other. Again, the author motivates well his models from economic and marketing points of view.

This thesis of Alexei Korolev contains really impressive amount of material and significantly advances the topic of networking games. The results also provide very interesting interpretations in economics and marketing. The results of the thesis have been presented in important conferences on games and optimization (such as *GTM*, *WCGO*, *EURO*) and in high-level journals (such as Automation and Remote Control, Mathematical Game Theory and its

Applications, Journal of Dynamics and Games, Mathematics). Many publications are indexed by Scopus and Web of Science.

For all the above reasons, I conclude that the dissertation of KOROLEV Alexei Vasilevich on the theme "Mathematical Models of Control for Economic Systems with a Network Structure" meets the requirements established by Order No. 11181/1 of 19 November 2021, "On the procedure for awarding academic degrees at Saint Petersburg State University", and the candidate KOROLEV Alexei Vasilevich deserves the award of the degree of Doctor of Physico-Mathematical Sciences in the Speciality 01.01.09 — Discrete Mathematics and Mathematical Cybernetics. Clauses 9 and 11 of the aforementioned Order by the author of the thesis is not broken.

Member of the Dissertation Council,
Konstantin Avrachenkov,
Doctor of Philosophy (PhD),
Habilitation à Diriger des Recherches (HDR),
Director of Research at Inria,
30 December 2021.