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**Report**

of a member of the dissertation council on the PhD thesis presented by Mikhail VOLOSHIN

*Analysis of the asymptotic behavior of solutions and the synthesis of stabilizing controls for nonlinear nonstationary difference systems*

submitted for the Candidate of Physical and Mathematical Sciences degree with specialization 05.13.01 – System analysis, information control and processing (physical and mathematical sciences)

Investigation of stability conditions for dynamical systems is one of the core problems of the theory of control and the system analysis. The most complicated this problem becomes in the cases when the model is nonlinear and time-varying, having commutations, as it is studied in the thesis. The difficulty comes from the absence of constructive applied approaches for stability examination, then the focus can be shifted to particular classes of systems, where an advancement can be achieved by utilizing their features, as it is done by the author by considering the homogeneous and the Persidskii models. Since for implementation or simulation, a control or estimation algorithm has to be realized in digital devices (computers, or controllers), the problem of preservation by a discretized system the stability properties of the original dynamics should be additionally explored, which is the main topic considered in the thesis of M. Voloshin. The investigated in the thesis problems are definitely challenging, and the obtained solutions are original and widely examined during the last decades. Therefore, the thesis is devoted to an actual problem of the theory of control, whose solution is demanded in many applications.

The manuscript is clearly written, and well organized. The author clearly explains the complexity of the studied stability and stabilizations problems, demonstrating a good understanding of the field and existing methods and tools.

The manuscript contains five chapters. In Chapter 1 the preliminary definitions and used results from the literature are summarized. In Chapter 2, first, the continuity properties of sequences of homogeneous functions are investigated; second, the problem of analysis of uniform stability for an explicit Euler discretization of a nonlinear time-varying homogeneous system is formulated, under the assumption that its averaged time-invariant version is asymptotically stable; third, a solution to this problem is given, which is based on the Lyapunov function method. In Chapter 3 these results are extended to control design for homogeneous time-varying discrete-time systems, whose averaged dynamics can be rewritten as the chain of integrators. Two special classes of systems are additionally considered: with the models described by the Rayleigh's equation, and with the models in the Persidskii form with odd-power nonlinearities higher than 1 and a linear part. In Chapter 4 the time-varying Persidskii systems are analyzed with odd-power nonlinearities (higher and smaller than 1), but without linear part. Chapter 5 is devoted to analysis of stability in switched systems, where each subsystem is presented by a discretized version of time-invariant Persidskii model with globally Lipschitz nonlinearities. Two cases are considered, with exponential and rational convergence of Lyapunov functions for individual subsystems.

The presented results are given on a high scientific level, and the numerous publications clearly indicate the good mathematical background of M. Voloshin.

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Nevertheless, I have the following comments:

1. Discretization of a homogeneous continuous-time dynamics by the Euler method preserves partially the stability properties of the original system (convergence to a set containing the origin for a negative degree, and local asymptotic stability for a positive one). In case studied in the thesis the time-varying dynamics can be presented as a sum of the averaged part (homogeneous continuous-time system) and the respective deviations, which are integrally bounded (or something like this). Could you comment, please, on such an interpretation of the considered problem?

2. Analysis of stability for homogeneous systems in continuous or in discrete time (time-invariant and time-varying cases) is a rather popular topic of research nowadays. There are different results on discretization of this class of systems, and unfortunately, all these results are not presented in the state of the art part of the thesis. Such a discussion would better highlight the actuality of problems and solutions presented in the thesis.

3. Before Theorem 5.1 it is shown that each subsystem is exponentially stable (it has an exponentially converging Lyapunov function), then a usual condition of convergence for the switched dynamics is formulated through (averaged) dwell-time constraint on the times between commutation. How the condition \chi(m,p)\to\infty for p\to\infty, which is imposed in Theorem 5.1, is related with the dwell time?

4. It seems that (8) is ISS with respect to error of realization of the virtual control, the same for the dynamics of y, then the result follows from the conventional ISS arguments under small gain constraints, and it was probably obtained before [64]?

5. In theorem 1.1 and 1.2 there is a typo: “x+”.

6. Lemma 2.1 has been proven previously, for example, in Polyakov, 2020.

7. In the formulation of theorem 3.1 and 3.2, \tilde{F}(x) has to include u(x)?

Despite the above mentioned remarks and questions, the thesis clearly deserves to be positively evaluated.

The thesis of M. Voloshin “Analysis of the asymptotic behavior of solutions and the synthesis of stabilizing controls for nonlinear nonstationary difference systems” fulfills the requirements established in the decree 01.09.2016 № 6821/1 on “Order of Granting Degrees in St. Petersburg State University”. Mikhail Voloshin deserves to be granted with the degree of Candidate of physical and mathematical sciences (Specialization 05.13.01 – System analysis, information control and processing). Clause 11 of the said decree has not been violated by the candidate for the degree.

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