REPORT

By a Member of the Dissertation Committee on the dissertation "Dynamic network games" presented by Artem Aleksandrovich Sedakov to obtain the degree of Doctor of Physico-Mathematical Sciences, Specialization 01.01.09 "Discrete Mathematics and Mathematical Cybernetics"

Artem Aleksandrovich Sedakov presented the thesis on the dynamic network games and applications to economic management and social networks. The thesis provided some insightful and thoughtful game models and approaches based on the features of the corresponding problems. Drawing on a notably wide range of the relevant academic literature, the thesis offers original models, methods, existence conditions, and results/conclusions for dynamic network game problem investigation.

The thesis began by surveying the important development of dynamic network games and pointed out the research limitations. Thereby the author believes that it is important to develop methods for studying equilibrium behavior in noncooperative dynamic network games and mechanisms implementing cooperative solutions and supporting cooperation in cooperative dynamic network games. Therefore, the research of the thesis is of importance and use in theory and practice. Particularly, the thesis studied the time consistency and strong time consistency of cooperative dynamic network games, which are of extra significance. The thesis has 611 pages in total, which are presented in full in two languages, Russian and English. The thesis consists of an introduction, five chapters, conclusions, and a bibliography. The five chapters include the main results and original works of the thesis, which are simply reviewed as follows.

The first chapter presented definitions of dynamic network games and cooperative dynamic games. Based on the constructed characteristic functions of cooperative dynamic games, the author proposed cooperative game solutions and imputation distribution procedures, studied the time consistency and strong time consistency of the cooperative

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game solutions, and given sufficient conditions for the strong time consistency of the core and an imputation distribution procedure for a core imputation as well as discussed the irrational-behavior-proof and cooperative regulatory conditions. Particularly, the author studied the strong time consistent imputation distribution procedures, iterative transformation rule, and limiting properties under a linear transformation of the characteristic function of the cooperative dynamic game. Moreover, the author studied cooperative dynamic games with discounting and given sufficient conditions for the strong time consistency of the imputation distribution procedure for the core imputation and the Shapley value.

In Chapter 2, the author deeply discussed two-stage network games for undirected networks through considering three types of players' behavior (Nash equilibrium, cooperation at both stages, and its mixture) and extended them to repeated network games and multistage network games. In particular, the author proposed the theory of multistage network games with shock. Further, the author proposed and proven the sufficient conditions for the time consistency and strong time consistency of cooperative solutions and strategic support of cooperation and discussed sufficient conditions for the existence of the subgame consistency of cooperative solutions and imputation distribution procedures for the core, τ-value, and Shapley value.

The third chapter proposed the mathematical model of the reachability of a consensus in the network with two principals through considering their two different influence modes. In addition, the author discussed the consensus reachability of the social network with heterogeneous agents as well as some limiting properties of the beliefs of network members. Further, the author constructed a mathematical model of coordinated influence in a social network and developed methods for finding cooperative strategy profiles for both open-loop and feedback classes of strategies and cost allocation schemes based on the constructed characteristic function of the cooperative dynamic game.

In Chapter 4, the author constructed the mathematical model of sequential formation of multistage network games with perfect information and studied the subgame perfect

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equilibrium and indifferent equilibrium with players' payoffs represented by the Shapley value. Moreover, the author proposed several solutions of dynamic cooperative network games with restricted cooperation through extending the Myerson value, AT-solution, and centrality rewarding Shapley and Myerson values. Specifically, the author studied a special class of multistage network games with a major player under a communication graph. When players' payoffs cannot be exactly given by real numbers, the author studied multistage network games with interval payoffs using the Moore' interval operations and proposed the sufficient conditions for the time consistency of the interval Shapley-like value and imputation distribution procedures for the median element of the interval Shapley-like value.

The fifth chapter is devoted to cooperation and equilibrium in dynamic games played over event trees. The author described the basic setting of linear-state dynamic games played over event trees for multidimensional states and control variables and proposed explicit expressions of equilibrium strategies and payoffs as well as cooperative controls and payoffs for players. In addition, the author discussed the S-adapted equilibrium using Pontryagin maximum principle. In particular, the author given the explicit value of the price of anarchy and conducted a sensitivity analysis for a pollution control dynamic game model over a binary event tree with homogeneous players.

The scientific novelty of the thesis is primarily due to the simplification of the characteristic function in cooperative dynamic network games by reducing minimax problems to maximization problems for both directed and undirected networks and sufficient conditions for the time consistency and the strong time consistency of cooperative solutions, which also imply the fulfillment of the irrational-behavior proof and cooperative regulatory conditions.

The theoretical significance of the thesis is to developing the noncooperative dynamic network game theory and constructing equilibria for these games as well as developing the cooperative dynamic network game theory and designing mechanisms, which support cooperation over a planning horizon in these classes of games using the time consistency, strong time consistency, and subgame consistency of the redistribution of players' payoffs.

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The practical significance of the thesis is determined by the applicability of the considered classes of dynamic network games in various areas such as environmental management and social networks.

The thesis contains several original and key results as follows:

(1) Formalizing dynamic network games.

(2) Constructing superadditive characteristic functions of two-stage network games for both undirected and directed networks and proposing sufficient conditions for the time consistency and strong time consistency of cooperative solutions and their adaptation for two-stage network games with pairwise interactions.

(3) Constructing superadditive characteristic functions and subgame consistency imputation distribution procedures of the Shapley values for a class of dynamic network games with shock.

(4) Proposing sufficient conditions for the strong time consistency of the core and the imputation distribution procedure of a core element for cooperative dynamic games. Proving the convergence of an iterative process of the transformation mechanism of the characteristic functions in cooperative dynamic games, which imply the strong time consistency of both the imputation distribution procedure of a core element and the imputation distribution procedure for a core element constructed by the limiting characteristic function.

(5) Finding Nash equilibriums, characteristic functions and the Shapley value for the discrete-time opinion dynamics network games and proposing sufficient conditions for reaching a consensus in opinion dynamics models in networks with two centers of influence.

(6) Constructing subgame perfect Nash equilibriums in dynamic network games with perfect information and players' payoffs represented by the Shapley value and finding cooperative solutions for dynamic games with a major player.

(7) Constructing the interval Shapley-like value and proving sufficient conditions for the time consistency of its elements in cooperative dynamic games with interval payoffs and perfect information played over tree graphs.

(8) Proposing explicit expressions for the price of anarchy.

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The accuracy of the obtained results or conclusions is confirmed by strict mathematical proofs. The applicability of the developed models, methods, and algorithms is confirmed by concrete applications and numerical examples. The author published more than 40 related research papers.

Some remarks on the thesis are given as follows:

(1) The normalization and usages should be enhanced and properly corrected. For example, Line 3 below the Proof of Theorem 1.12 in Page 367, the eigenvectors of the matrix should be written in the vector format rather than the set format.

(2) There are some typos, mistakes, or errors which should be carefully checked and corrected. For example, the left hand of Eq. (1.66) in Page 372, Lines 4 and 5 in Page 422, the last line 3 from the bottom of Page 425.

(3) There are some repeated contents. For example, the concept of an imputation is repeated several times, e.g., Page 345 and Page 413.

(4) The thesis used the Moore's interval operations to study the cooperation in multistage games with interval payoffs and the time consistency of the interval Shapley-like value. The interval Shapley-like value is involved in the Moore's interval subtraction, which is irreversible. Such a kind of interval operations may lead to unreasonable results. The author is suggested to refer to the following monograph and articles to improve the research work of the thesis.

[1] Deng-Feng Li, Yin-Fang Ye, Wei Fei. Extension of generalized solidarity values to interval–valued cooperative games. Journal of Industrial and Management Optimization, 2018, doi:10.3934/jimo.2018185.

[2] Wei Fei, Deng-Feng Li, Yin-Fang Ye. An approach to computing interval-valued discounted Shapley values for a class of cooperative games under interval data. International Journal of General Systems, 2018, 47(8): 794-808.

[3] Deng-Feng Li, Yin-Fang Ye. An approach to computing interval-valued egalitarian Shapley values of interval-valued cooperative games with coalition monotonicity-like. Economic Computation and Economic Cybernetics Studies and Research, 2018, 52(3): 73-84.

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[4] Deng-Feng Li. Models and Methods of Interval-Valued Cooperative Games in Economic Management. Springer, Cham, Switzerland, 2016.

Despite the above indicated remarks, which actually do not reduce the overall excellent impression on the thesis's study, the dissertation deserves a positive assessment. Undoubtedly, the thesis is an accomplished scientific research with new fundamental results. The obtained results are of significant theoretical and applied importance, which allows us to qualify them as a scientific achievement. The topic of the thesis is new and attractive and absolutely corresponds to the mentioned specialty.

The dissertation of Artem Aleksandrovich Sedakov entitled "Dynamic network games" fulfills the requirements established by Decree № 6821/1 of 01.09.2016, "On the procedure for awarding academic degrees at Saint Petersburg State University", while Artem Aleksandrovich Sedakov deserves to be awarded his Doctor of Physico-Mathematical Sciences, Specialization 01.01.09 "Discrete Mathematics and Mathematical Cybernetics". Clause 11 of the aforementioned Decree was not broken by the author of the thesis.

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