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REPORT ON

SOLUTIONS OF COOPERATIVE STOCHASTIC GAMES WITH TRANSFERABLE PAYOFFS

Presented by Elena Mikhailovna Parilina

Thesis submitted in conformity with the requirements for the degree of
Doctor of physico-mathematical Sciences

Discrete mathematics and mathematical cybernetics

University of Saint Petersburg

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This report first provides an overview of the class of problems studied in the thesis presented by Dr. Parilina. It then proceeds with an evaluation of each of the four main chapters of thesis. Finally, the report concludes with a global assessment of the work.

Overview of the thesis

When players involved in a strategic situation wish to cooperate in a static (one-shot) game, they must first coordinate their strategies to achieve the best feasible collective outcome and, second, decide how to share this outcome. Typically, this is done by adopting one of the many available solutions concepts of cooperative games, e.g., the Shapley value or the core. When the game is dynamic, then the process does not end there and a question of formidable difficulty arises, namely, how to ensure that the players abide by the agreement signed at the initial date until its maturity.

The literature has adopted essentially two approaches to deal with the sustainability of a dynamic cooperative agreement. A first approach consists of providing the cooperative solution with an equilibrium property. In such cases, the sustainability problem is empty, since no player can improve her payoff by unilaterally deviating from cooperation, that is, the agreement is self-supported. A second approach seeks to render the initially signed agreement *time consistent*, so that at any intermediate instant of time, the cooperative payoff-to-go dominates its noncooperative counterpart. To implement a time-consistent solution, one needs to introduce some so-called *imputation distribution procedure* (IDP). The thesis presented by Dr. Parilina is concerned with the design of time-consistent cooperative-equilibrium solutions for stochastic games, which belong, by any measure, to one of the most mathematically challenging class of games.

In Chapter 1, the focus is on stochastic games of fixed duration, while in Chapter 2 the games are of infinite duration. Chapter 3 deals with the class of dynamic games played over event trees, where the transitions are not affected by players' actions but are the work of nature. Chapter 4 presents a series of applications of stochastic games. In addition to these four main chapters, the thesis proposes an introduction where the literature in the area is surveyed and the content and contribution of the thesis are highlighted. It also includes a conclusion wrapping up the main results and methods.

Chapter 1: Cooperative Stochastic Games with Finite Duration

In this chapter, the candidate first introduces the main ingredients of a stochastic game of finite duration. She then presents the different alternative definitions of a characteristic function, which is the main tool in cooperative game to assess the strategic strength (or value) of all possible coalitions. Three cooperative solutions are retained in this chapter, namely, the Shapley value and the nucleolus (which are values) and the core (which is a set of imputations if not empty).

The main results are related to:

- The design of IDPs that are subgame consistent, with illustrations in the case of the Shapley value and when a specific imputation is selected from the core.
- The regularization of IDPs to ensure positivity of its components. Regularization procedures are of crucial importance when, for some reason, the players insist in getting positive payoffs in each node or time period. They can be considered a refinement of an IDP.
- The design of strongly subgame consistent solutions, which are needed when the adopted cooperative solution is not a value, but a set.

The chapter contains some examples to illustrate the concepts and their implementation.

Chapter 1 contains a series of nice results that have been published by the candidate in very good scientific journals, which attest of their quality. Interestingly, all the obtained results are valid for any choice of characteristic function. The examples provided are nicely drafted and help the reader in seeing the implementation of the different steps needed to obtain the desired outcomes. As a final (very minor) remark, I find that an opening paragraph to set the stage (like the one in Chapter 2) would be a nice addition.

Chapter 2: Cooperative Stochastic Games with Infinite Duration

As the title suggests, this chapter extends the framework introduced in Chapter 1 to games of infinite duration. Moreover, the stochastic game considered in Chapter 2 involves a set of states that can be realized at any stage to be finite and that does not change over time. In the first sections, the author introduces some main definitions and results for this class of game. The author then shows that there exists a subgame-perfect Nash equilibrium that implements a cooperative solution for a suitably defined regularized game. These results establish a nice link between cooperative and noncooperative games in the framework of stochastic games à la Shapley. A condition is provided to ensure an irrational-behavior-proof property in stochastic games with infinite duration à la Yeung. The author also deals with the existence of stable cooperative solutions in stochastic games with infinite duration. An example of competition between two asymmetric firms is studied, where each of the two firms can choose to either collude or compete. Dr. Parilina shows how the cooperative solution (e.g., the Shapley value) can be strategically supported, that is, be part of a subgame Nash equilibrium. The irrational-behavior-proof condition is also illustrated. The chapter includes a fully worked-out example of a stochastic game with one absorbing state. Finally, strong subgame consistency of the core is analyzed.

The main results in this chapter are extensions, to an infinite horizon context, of some of the results obtained for finite-horizon games. Note that these extensions are mathematically very challenging to obtain. What I find impressive in this chapter are the results related to the strategic support of cooperation. The regularizations introduced by the author to obtain strong subgame consistency are interesting from a conceptual as well a practical point of view. The chapter is well written, and an excellent choice of examples is provided to illustrate and explain the sophisticated mathematical results.

Chapter 3: Dynamic Games Played over Event Trees (DGPET)

The games studied in this chapter differ from the ones presented in the two previous chapters in one fundamental aspect: the transition between states does not depend on players' actions but is the work of nature, so that the stochasticity is exogenous. This chapter can be summarized by classifying its contents into three different topics, namely:

- The design of imputation distribution procedures to ensure node consistency of the cooperative solution;
- The design of Ɛ-cooperative equilibria to sustain cooperation; and
- The determination of the price of anarchy.

The first topic consists of determining IDPs in essentially three cases. In the first case, the cooperative solution retained is the Shapley value. The major contribution with respect to what has been done before in the literature is the consideration of a possible termination of the game at each node of tree, which can be caused by *force majeure*. Accounting for such a possibility complicates the model and its solving but allows for the analysis of more realistic cases. Dr. Parilina also designs IDPs for the core and analyzes the conditions under which these IDPs are strongly node-consistent. As usual, dealing with the core instead of the uniquely defined Shapley value is much more complicated.

The second topic consists of designing Ɛ-cooperative equilibria to sustain cooperation in the class of DGPET. Remarkably, the results obtained are independent of the functional forms, provided the conditions for existence of a Nash equilibrium are satisfied. A very nice illustration is provided in the framework of control pollution games, where the outcome is the determination of the amount needed to "buy" the cooperation of each player along the nodes of the event tree.

There is an abundant literature on the price of anarchy, where the focal point is to assess the collective cost when the players act non-cooperatively instead of coordinating their strategies. This vast literature has rarely dealt with dynamic games, as it typically assumes a deterministic word. For the third topic, Dr. Parilina analytically characterizes the price of anarchy for the class of linear-state DGPET and provides an illustration of the theory using an example from environmental economics.

Chapter 3 extends the theory and applications of DGPET to new territories. It contains an impressive list of new results that are mathematically challenging to obtain and are useful in many areas, such energy markets, environmental management, and renewable and non-renewable resources, to name a few.

Chapter 4: Applications of Stochastic Games

In this chapter, Dr. Parilina first discusses some models of data transmission in wireless networks. In each case, the ingredients of the game are introduced and results characterizing the existence of a Nash equilibrium are obtained. The cooperative solution is contrasted with the Nash equilibrium, using such measures as the price of anarchy and the cost of cooperation rejection, which is a less familiar concept in the literature.

The second part of the chapter introduces the concept of dynamically stable coalition structures as solutions of a stochastic game. Illustrations of two-person and three-person games are provided.

The last part of the chapter deals with a stochastic prisoners' dilemma with incomplete information about the discount factor. The mix of cooperative and noncooperative analysis is especially inspiring and so is the implementation of a learning phase. Finally, a numerical example is discussed.

This chapter differs from the others in terms of the type of problems considered. The focus of Chapter 4 is mainly on a network structure, which allows to enlarge considerably the range of applications of stochastic games, especially when one focal point is to reach a cooperative solution, or to compute the loss in efficiency when the game is played noncooperatively.

Global assessment of the thesis

This section of the report proposes a global assessment of the work submitted by the candidate along four main criteria: contribution to the advancement of knowledge, relevance, presentation, and diffusion.

Contribution to the advancement of knowledge: At the theoretical level, the thesis contains an impressive number of novel results, many of them being mathematically challenging. This work improves significantly our knowledge of stochastic games, dynamic games played over event trees and the design of mechanism to sustain cooperation over time in a large variety of situations.

Relevance: At the applied level, the thesis offers relevant and timely applications in networks, competitive markets and environmental economics. Solving environmental problems is an urgent matter for all societies and the design of agreements that can be durable over time is of crucial practical importance. Environmental problems, especially global ones, cannot be solved overnight but over time, which implies that a dynamic approach to tackle them is a must. By following this line of thought and accounting for the stochasticity in the system, Dr. Parilina is making an excellent contribution in this area.

Presentation: In the introduction, the author mentions that the thesis is based on the results of 39 of her publications. The thesis is an impressive work that puts together all these contributions in a unified framework and shows how they are related and how they contribute to the literature on the sustainability of cooperation in a dynamic and stochastic context. The thesis reads very well, and the quality of the language is excellent. All concepts are properly introduced and the flow in presentation is smooth.

Diffusion: The results of this thesis have been presented in many scientific forums, both national and international. As a sign of appreciation of her work, Dr. Parilina was invited at a numerous occasion to participate in international conferences and to give seminars in research centers. The candidate mentions that she published 85 papers, with 39 of them directly related to this thesis. This is a remarkable diffusion for a scholar at this stage in her career. Moreover, many of these papers are published in recognized and excellent academic journals.

Recommendation: I consider that the thesis presented by Dr. Parilina is excellent and, without any doubt, meets the requirements for the degree of Doctor of Physico-mathematical Sciences in Discrete Mathematics and Mathematical Cybernetics.

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