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REPORT

of the member of Thesis' Council on the thesis **"Multi-loop calculation of critical exponents in models of critical dynamics and statics"** submitted for the degree Doctor of Philosophy in Physics, specialty 01.04.02 Theoretical physics
(кандидат физико-математических наук по специальности:
01.04.02 – Теоретическая физика)
by Ella V. Ivanova at Saint-Petersburg State University

Thesis is devoted to the problem of precise theoretical calculations of some universal quantities, such as the critical exponents, in the theory of the continuous phase transitions and critical phenomena by using the field theoretic renormalization group technique in higher orders of the corresponding perturbation expansions. The multi-loop results for the critical exponents in the static φ^4 model with the cubic symmetry and in the so-called A and E models of the critical dynamics are obtained and analyzed. Namely, the known results in the framework of the O(n)-symmetric φ^4 model up to the six-loop approximation are generalized to the φ^4 model with the cubic symmetry, the five-loop calculations of the critical exponents in the model A of the critical dynamics are performed, and the model E of the critical dynamics is studied up to the two-loop level of approximation. To obtain these results, the dynamical diagram reduction method was developed and applied, the so-called Sector Decomposition technique was generalized to problems of the critical dynamics, as well as the resummation of the obtained five- and six-loop perturbation series was performed using the conformal-Borel method.

The problems studied in the thesis are without doubt up to date and the obtained nontrivial results and developed techniques can further be used for direct precise investigations of other phenomenologically important models of the critical statics as well as dynamics to understand their critical properties at the fundamental theoretical level. This fact also represents great practical significance of the obtained results.

The thesis consists of the Introduction, four chapters, the Conclusion, three Appendices, and the Bibliography with 57 items. In the Introduction, the relevance and the elaboration extent of the studied topic are given; the main goals of the thesis are enumerated; the scientific novelty, the theoretical and practical relevance, as well as the reliability of the obtained results are briefly discussed. Besides, the used research methods, the approbation of the research, the list of published papers of the author

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related to the topic of the thesis, as well as the list of the main thesis statements to be defended are also given in the Introduction.

In the first chapter, the field theoretic description of the models of critical statics is given and the relation between the static and dynamic models is briefly discussed. The renormalization procedure is described in the framework of the minimal subtraction scheme and the so-called Sector Decomposition technique to isolate divergences in integrals over Feynman parameters is introduced and demonstrated.

Further, in the second chapter, the static φ^4 model with the cubic symmetry is investigated using the results obtained earlier in the framework of the O(n)-symmetric φ^4 model up to the six-loop approximation. The six-loop expressions for the coupling constants of the model are found and the regions of stability of all possible scaling regimes are discussed. The explicit expressions for the critical number of components of the field, below which the Heisenberg scaling regime and above which the cubic regime is stable, is obtained. Besides, the corresponding six-loop expressions for critical exponents η, $1/\nu$, γ are determined.

Chapter three is dedicated to the investigation of the models A and E of the critical dynamics. The method of the Sector Decomposition is generalized to the dynamic models and the introduced scheme of diagram reduction is described and used for the simplification of the multi-loop calculations. The dynamic critical exponent z of the model A is calculated up to the five-loop level of approximation.

Finally, in the fourth chapter the conformal-Borel technique is introduced and used for the resummation of the multi-loop asymptotic series obtained in the previous two chapters for various critical exponents. The obtained final results are discussed and compared to the existing values in the literature.

The obtained results are briefly summarized in the Conclusion and some technical material is placed in the appendices.

The novel results obtained in the thesis are present in Chapters 2-4 and their reliability is checked up by comparison to the results obtained earlier by other authors (if it was possible).

However, as for the text of the thesis, I have a few notes, comments, and questions.

1. The text contains some misprints. For example: On page 9, in description of the Chapter 2, there must be indices η, γ, and ν instead of η, z, and ν; The thesis

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contains three appendices but in the end of the Introduction only two of them are mentioned; The significant Appendix C, which contains interesting technical details, is not mentioned here. On page 67, there is a reference to Table 9 but in fact the corresponding results are present in Table 6; References [20] and [28] are not complete (the complete form of Ref. [28] is in the Introduction); The same is valid for Ref. [49].

2. For completeness and readability of the text, it would be better if definitions of all studied critical exponents would be given explicitly. The same is also valid for the normalization factor C in Eq. (3) on page 11. Especially since the same symbol is used on page 17 for completely different quantity. It would be also fine (again, at least, for completeness) to define the explicit form of the $\zeta(n)$-function on page 29.
3. The critical exponents were determined by using the conformal-Borel resummation representation of the asymptotic series. Here the following natural questions immediately arise: What results would one obtain using a different resummation technique? Would the results be the same?
4. In the resummation procedure on page 63, the special form of the asymptotic behavior is assumed, which is given in Eq. (150). Is there any physical evidence that allows one to suppose unambiguously the asymptotic behavior of the series in the form of alternating terms with different signs?

However, the abovementioned shortcomings do not lessen the overall quality of the performed work. In fact, the dissertation thesis represents a masterpiece of theoretical work with a lot of nontrivial results obtained by using sophisticated theoretical methods.

Thus, the dissertation research carried out by Ella V. Ivanova "Multi-loop calculation of critical exponents in models of critical dynamics and statics", presented for the degree of Doctor of Philosophy in Physics meets the qualification requirements for candidate dissertations on specialty 01.04.02 Theoretical physics.

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