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Thesis Committee Member's Report

**for thesis "Mathematical Modeling of the derivatives dynamics"
submitted by Eduard Anatolevich Stepanov to achieve
the degree "Candidate of physics-mathematical sciences",
code 05.13.18 — Mathematical modeling, numerical methods and program complexes.**

(also in Russian)

ОТЗЫВ

члена диссертационного совета на диссертацию Степанова Эдуарда Анатольевича на тему: «Математические модели динамики стоимости деривативов», представленную на соискание ученой степени кандидата физико-математических наук по специальности 05.13.18 — Математическое моделирование, численные методы и комплексы программ.

19 сентября 2019 г.

1 General comments

1.1 Actuality

The thesis is devoted to pricing Asian options by using the Black-Scholes model (see, e.g., Hull, 1997 and references therein). This topic is long time outdated, and was considered in the literature long time ago. Good review of what has been already done on this subject can be found in Mudzimbabwe, 2009. As such, from the financial point of view, the research provided by the author is outdated by at least a decade. Hence, in my opinion, actuality of this research is low. For this reason, perhaps, it doesn't make sense to consider it as a research work on financial mathematics or engineering, because it doesn't solve any problem inherent to financial engineering. This is also demonstrated by some statements in the thesis that are either incorrect or unjustified from the finance standpoint. Accordingly, in what follows I will treat this work as pure mathematical modeling/simulation where finance is just the field where this simulation could be applied for.

1.2 Structure

From this prospective the structure of the thesis is also questionable. Indeed, the first chapter outlines some models used in financial engineering, starting from the Black-Scholes model for European (plain

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vanilla) options and then going through some examples of exotic options up to some stochastic volatility models. There are at least two problems with this survey.

First, the modern finance use many other and more sophisticated models, so the survey is also outdated. Moreover, every complex model has been originally introduced not just for increasing complexity, but rather to be capable catching the behavior of traded financial instruments observed at the market but not replicated by the previous models. Therefore, every step towards the next generation of the models has an explicit reasoning and aim that are not described or presented in Chapter 1. Therefore, the whole chapter looks like a collection of models combined together for no obvious reason.

The second problem is that logically this survey would be helpful, if the author first indicates the existing problems of the older models, and then decides to be concentrated on a more recent model. In such a way he could contribute to both: solving some real financial problem, and, perhaps, also solving some computational problem important for the financial engineering. Instead, the author finally decided to work just with the Black-Scholes model which is the oldest one. Therefore, the survey doesn't serve any goal stated in the thesis, and, thus, can be easily omitted.

This, however, could be ignored, if, as I said, we consider the thesis as a work on mathematical modeling. From this prospective, again this chapter can be fully excluded from the thesis with no any damage for it.

1.3 Style and rigorousness

First of all, I think the title of thesis should be changed. The author doesn't propose any model, instead he numerically solves the equations following from the existing models.

I also found many typos in the text which is bad, but suitable. However, also I see some definitions that are simply wrong. For instance, the definition of the Wiener process (Brownian motion, BM) in (1.1) is wrong (missed the requirement that $W(0) = 0$, and also the scaling property of the BM reads $W_t = \sqrt{t}\epsilon$, see, e.g., Karatzas и Shreve, 1991. Therefore, if this is further propagated to the discretization of the SDE in the Monte-Carlo method, the latter contains an error. The comparison of implied vs historical volatility at page 189 is naive and incorrect. The stock volatility obtained by using the historical time-series is computed under the real (physical) probability measure, while the implied volatility is computed under the risk neutral measure. To compare them, one needs to do a measure change which is well-known in the literature. Also, what the phrase "more accurate results are obtained using the implied volatility [24], [?]"at the bottom of this page means, since the Black-Scholes volatility (implied) can be found only by calibration of the Black-Scholes model to a single market price. As such, this is a direct map of the option price, and it is mostly used as the language prefered by traders. Indeed, the notion of the implied volatility allows comparison of prices for various options, while doing this compariosn in terms of the dollar prices is difficult because of the different scale. This list can be continued (see the more detailed comments below).

The description of the models is not accurate. Many important definitions are missed, equations and the initial and boundary conditions are presented with no references. Therefore, it is hard for readers to follow. Also, the proposed boundary condition for the PDE (2.1) is not a model as it is claimed.

1.4 Novelty

New boundary conditions First of all, for the model with constant coefficients considered in the thesis, the PDE (2.1) for the Asian option can be reduced to a one-dimensional parabolic equation. Then it can be solved by either using the Fourier transform, or even analytically, see, e.g., Kumar, Waikos и Chakrabarty, 2011; Elshegmani и др., 2011; Mudzimbabwe, 2009 and references therein. Therefore, the problem with this boundary condition disappears.

For more complex models, e.g., with time-dependent interest rates or local volatility models, which do require usage of numerical methods, this problem would be interesting to investigate, but the thesis doesn't operate with them.

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Second, from the general theory of PDEs, the correct boundary conditions at $x \to 0$ are determined by the speed of the diffusion term as we approach the boundary in a direction normal to the boundary. To illustrate, consider a PDE

$$C_t = a(x)C_{xx} + b(x)C_x + c(x)C, \tag{1}$$

where $C = C(t, x)$ is some function of the time t and the independent variable $x \in [0, \infty)$, $a(x), b(x), c(x) \in C^2$ are some known functions of x. Then, as shown by Oleinik и Radkevich, 1973, no boundary condition is required at $x = 0$ if $\lim_{x \to 0}[b(x) - a_x(x)] \geq 0$. In other words, the convection term at $x = 0$ is flowing upwards and dominates as compared with the diffusion term. A well-known example of such consideration is the Feller condition as applied to the Heston model, see, e.g., Lucic, 2008.

To make it clear, no boundary condition means that instead of the boundary condition at $x \to 0$ the PDE itself should be used at this boundary with coefficients $a(0), b(0), c(0)$. It is easy to check, that this is just the case here, so no boundary condition is required.

Also, the boundary condition in (2.19f) appears from nowhere. There is no any discussion or derivation why this boundary condition has been chosen, and any mathematics or empirics behind that. Therefore, even assuming I am wrong (I am not, I believe), I have no chance to validate this result.

Finally, all numerical examples are presented for the value of volatility $\sigma = 0.01$. Needless to say, that i) 1% volatility is never observed at the market, so is totally unrealistic, and ii) with so low volatility the stochastic term almost doesn't play any role as it is negligible, and the PDE behaves like a hyperbolic one. It is well-know that the option price as a function of strike (or spot) in this case has a form of an ice-hockey stick, i.e., is equal to the option intrinsic value. This is also displayed at the figures in the thesis.

Therefore, overall, I cannot treat this part as a contribution.

Analysis of the developed model at extreme values of volatility As I have already mentioned, there is some confusion here. The trading activity at the market might be halted when the stock volatility is low. This has nothing related to the implied volatility of options. Also, as this is well-know, the Black-Scholes model looses the description of so called "volatility smile"and "skew Hull, 1997, so it is impossible to describe any real market by using it. Perhaps, what the author means is the analysis of his numerical solution for some low values of the implied volatility, but again, this is irrelevant to a trading halt.

Finite-difference and Monte-Carlo algorithms for pricing options All algorithms presented in the thesis are very standard and outdated. For instance, see Itkin, 2017; Glasserman, 2003 and references therein. There are some questions about the Monte-Carlo algorithm:

- It doesn't use any variance reduction technique, so is inefficient.
- Section 2.4.1 discusses random number generators (RNG) used by the author for the Monte-Carlo algorithm. But all the RNG in this section, e.g., `cuRAND`, are the uniform RNG (i.e., they sample from a uniform distribution). Since the increments dW_t are normally distributed, the standard approach is to sample random numbers from the normal distribution by using Box-Muller, or Ahrens-Dieter, or any other corresponding algorithm, Glasserman, 2003. Therefore, I am not sure what actually the author's implementation computes.

Comparison of various numerical methods Also mentioned in above, all methods presented in the thesis are pretty standard and even obsolete. The comparative analysis of them could be found in textbooks, again see, e.g., Itkin, 2017; Glasserman, 2003 and references therein.

Path-integral approach for solving the Asian option PDE This part is good, but also the approach is pretty standard and already described in the literature.

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2 Contribution

To me the most interesting part is about a parallel finite-difference solver which runs on CUDA. The author demonstrates that by using this approach he achieves speedup of calculations by factor 65x. Note, that the standard parallel toolbox of Matlab provides speedup of the order of 20-25. Therefore, the results of the author on making the solver parallel looks good.

3 More detailed comments

Page 6, Line Fig.1:
This paragraph talks about derivatives. So why the graph of historical time-series of S&P500 is provided there which is not a derivative instrument?

Page 169, paragraph 8:
At the moment the most widespread tools for the options describing are stochastic differential equations, partial differential equations and the path integral. This is not true. The PDE doesn't exist at all for path-dependent instruments (the Feynman-Kac theorem is not applicable). The path-integral approach is very limited and slow, and also very rarely used in practice. So, the main mathematical tool for modeling financial assets are the SDEs. Nowadays, also various model-free approaches become very popular, e.g., those based on Deep and Reinforced Learning and Artificial Neural Networks.

Page 205, paragraph 2:
It is based on the fact that in most cases of the financial world the emphasis is on the calculation speed, while their accuracy goes to the background. This is the main criterion for selecting the most efficient computational algorithm. How did the author come to this conclusion if he is not a quant/trader and doesn't work in the industry? A simple example could be, that the trading quotes could be specified up to a penny (there is even a special market of penny options, but we don't consider it here). Therefore, the pricer should be accurate, at least up to a a half of cent. This, for instance, for the GOOG stock with the price $1239 requires the relative accuracy $\approx 4 \cdot 10^{-6}$. Therefore, both speed and accuracy are important.

Page 270, paragraph 3:
Thus, this approach is useful in cases when it is required to find the option value for a number of underlying asset values or to find it with high accuracy. However, in practice, as far as the author knows, this situation is extremely rare. Yes, the author doesn't know this since he is not a quant. In reality, this is a very important feature of the method since not only the asset price has to be computed but the values of option Greeks (Delta, Gamma, Theta). The Greeks are used to hedge positions in traded options with other similar instruments on a day-by-day basis. And computing the Greeks requires knowledge of the price on a grid for various values of the underlying, and with high accuracy as well.

I stop here because physically don't have time to mention all of them.

4 Comments on the author conclusions

Some comments of conclusions the author put at the end of the thesis.

- "Проведенные вычисления показали, что в представленном виде многие алгоритмы не применимы на практике в силу их крайней неэффективности."This is not a news. It is known for a long time.

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- "Однако более точные модели описания рынка основаны на уравнениях в частных производных и континуальном интеграле."Why is that? Any reference, or explanation? To me this is simply wrong.

5 Conclusion

A can provide an example of a similar work, Mudzimbabwe, 2009, that was defended in 2009 as the Master thesis. As compared with the thesis of Stepanov, that thesis is much better organized, rigorous in mathematical sense, and uses various modern techniques such as: variance reduction for Monte-Carlo, transformation of the original 2D PDE to the 1D problem, etc. Also it is consistent from the financial point of view.

What is not in Mudzimbabwe, 2009, and what the author of this thesis used to accomplish:

1. implemented a parallel finite-difference solver and ran it on GPU;
2. built a package which contains his implementation of options pricers for some[1] Asian options considered under the Black-Scholes model;
3. implemented a path integral solver for pricing some Asian options under the Black-Scholes model.

I think that these three bullet points are the main contribution of this work.

Since I have been working in the USA almost for last 20 years, I am not sure which requirements are stated by the document "О порядке присуждения ученых степеней в Санкт-Петербургском государственном университете"от 01.09.2016 № 6821/1. By analogy with the US Universities, I can say that the author of the thesis proved some ability to perform research in the area of mathematical modeling, which, however, is not sufficient with the aim of receiving the degree of PhD. In other words, the thesis is more coherent to be a Master thesis, rather than the PhD one (see all my comments and criticism in above). Again, I am not sure to which extent the PhD degree maps to the requested degree of "кандидата физико-математических наук по специальности 05.13.18 — Математическое моделирование, численные методы и комплексы программ". Also I have nothing to say about "Пункт 11 указанного Порядка диссертантом не нарушен."

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[1] Here "some"means that there exist 8 different types of the Asian options, while in the thesis only some of them are discussed.

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