

## **Report on the thesis “GAME-THEORETIC METHODS FOR STABILITY ANALYSIS IN ENVIRONMENTAL POLLUTION CONTROL PROBLEMS”, by Su Shimai**

The thesis is made of an introduction and three chapters. In my report, I first evaluate each of the chapters separately, and next make some general comments on the whole work using few standard metrics. The report ends with a conclusion (verdict).

### **A. Evaluation of the different chapters**

#### **A.1 Assessment of the Introduction**

The introduction includes a series of classical sections typically found in a Ph.D. thesis such as a general description of the topic, the objectives, the specific tasks, the main contributions to the literature and practice, the research methods used, the organization of the thesis, etc. The introduction also integrates a literature review.

I found that the introduction reads well and defines clearly what comes next. I have two comments. First, I think it would have been better to write, here and there, “equilibrium analysis” instead of “stability analysis” as this last term has various meanings in different areas of applied mathematics. Second, I would have preferred to have the review of literature spread over the three chapters rather than being part the introduction. In this manner, the reader could better appreciate the relevance of each reference and the contribution of the specific chapter. Of course, this does not eliminate the need to cite in the introduction some general references to the class of environmental problems studied. Both comments are of editorial nature (and reflect my taste as a reader) and do not change my appreciation of the scientific contribution of the thesis.

#### **A.2 Assessment of Chapter 1**

There is a significant game-theoretic literature dealing with environmental agreements, which can be divided into streams based on the mode of play, that is, noncooperative mode of play or a cooperative one. In the later, the literature has considered a framework where only one coalition that contains some players (possibly the whole set of players) can

form, while the remaining players act individually. In this chapter, the candidate departs from this framework by considering a cooperative game with coalition structure, which mimics much better how the game is actually played in reality. From this perspective, this chapter constitutes an important scientific contribution to the problem of formation of environmental agreements. The reason explaining why such model has not been often implemented in the literature is the technical and mathematical difficulties involved in the computations of the different equilibrium solutions. Indeed, determining equilibrium strategies and outcomes in an  $n$ -player game with coalition structure is challenging due to the large number of configurations that must be considered.

After analyzing all possible configurations and showing that some of them are not stable, the candidate proposes three mechanisms to restore/enforce stability, namely, payment schemes, taxes, and restriction on the set of coalitions. The results obtained in the different sections are nicely reported, the proofs are rigorous and to the best of my checking the chapter is mathematically error-free. This being said, I would like to make three comments:

- Although the model is written and solved for 4 players, the candidate argues that his approach can be extended to any number of players. This is conceptually true, but it would have been nice to discuss further the difficulties involved in extending the model to a large number of players.
- Some conclusions are made on the results, in particular on page 28. They are clear, but a discussion distinguishing what is expected from what it is not, and highlighting the differences with respect to the literature, would have been of interest to scholars and decision makers involved in the design of environmental agreements. One crucial question here is whether or not it is worth it, qualitatively, and quantitatively speaking, to consider coalition structures.
- The model involves two developing countries and two developed countries. The literature has repeatedly attributed the difficulties in reaching an international environmental agreement to free-riding behavior and to the absence of a supra national organization that can enforce such an agreement. Consequently, it is not clear to me to see practically who would be in charge of implementing a taxation system, or excluding some coalitions, in order to reach a stable agreement.

I recommend that these elements be considered if the candidate decides to pursue a research program in this area.

### A.3 Assessment of Chapter 2

This chapter is made of two somehow distinct parts. In the first part, the candidate considers two countries that differ in their vulnerability to environmental damage. A starting point is that one country (developing country) does not suffer at all from pollution stock, whereas the other (developed country) suffers from pollution accumulation. The model is a differential game where each player aims at maximizing its payoff by choosing its emissions, subject to the state dynamics that describe the evolution of the pollution stock. The model is of the linear-quadratic variety, and therefore the equilibria and optimal solution can be fully characterized analytically. In particular, one can easily compute the strategies, payoffs, and the state trajectory and its steady-state value.

The payoff function is made of the two parts. The first one measures the revenues that can be obtained from emissions, here as proxy of production. The functional form is quadratic. The second part is a quadratic damage cost in the pollution stock. For the non-vulnerable player, this cost is zero.

Three scenarios are analyzed: full cooperation, noncooperation, and a cost-benefit sharing mechanism. The solutions in the three scenarios are characterized and compared. One main objective is to check if the cost-benefit-sharing (CBS) mechanism is Pareto improving with respect to the noncooperative equilibrium. To define the payoff functions in the CBS scenario, two sharing parameters are introduced, that is,  $0 < \tau < 1$  for revenues and  $0 < \theta < 1$  for damage cost.

One point that requires a clarification is the revenue sharing. In all the literature to which this chapter refers and intends to contribute, the revenue is defined by  $e_1 - \frac{1}{2}e_1^2$ , so it is not clear to me why the sharing parameter is applied only to the first term. What would be the economic meaning? I think the candidate should have done a better job in justifying his choice of sharing mechanism. The idea of borrowing a framework from supply chain and apply it to a pollution control problem is interesting, but it requires a discussion, especially that the focal point is not the same in the two literature streams. Indeed, such sharing mechanisms are used in supply chain to get rid of inefficiencies in pricing, while here the focal point is pollution control.

The second part of the chapter is devoted to the analysis of a pollution control problem involving three players, with each player is either vulnerable or non-vulnerable to environmental damage. Cooperative, noncooperative, and partial cooperative scenarios are studied. In the partial cooperative scenario, various coalition structures are considered in the spirit of what is done in Chapter 1, modulo the fact that the model here is a differential game.

In this part, the results are well established, and the analysis is nicely done. In particular, the candidate does an excellent work characterizing Nash stability of the different coalition structures in the partial cooperative case. In my opinion, here lies the main contribution of the chapter.

#### **A.4 Assessment of Chapter 3**

Using a differential-game framework, the candidate assesses the impact of lack of information on terminal cost, initial pollution stock, and upper bound on pollution emissions on the results. The game is in finite time and belongs to the linear-state variety. The information structure is in open loop, and so are the Nash equilibria computed in the different sections.

The mathematical developments in this chapter are error-free. Within each context of lack of information, the strategies and outcomes are illustrated numerically and the impact of lack of information is assessed.

A long series of results are provided in this chapter and their derivation is rigorous. The comparison of the results is instructive and allows to judge the value of information. However, the candidate should have better positioned what he is doing and justify better some choices. To illustrate, the chapter includes in the beginning the solution of cooperative game, but in all the rest the mode of play is noncooperative. It is not clear why the cooperative game part is included. The switch from pollution control problem to resource exploitation problem comes as a surprise to the reader and gives the impression of lack of unity in the chapter. On the technical side, it is hard nowadays to justify the use of open-loop strategies in an area that has been developing for 30 years. Also, why the pollution stock is not decaying in this chapter? At least a justification/discussion should have been included.

#### **B. General Comments**

In this section, I make some general comments on the thesis.

**Contribution:** The thesis contributes in a novel way to the area of pollution control in a multi-player context. Although most models used are taken from the literature, the thesis still contributes significantly to several areas, e.g., presence of coalition structure, asymmetric players, and lack of information.

**Literature:** The directly relevant literature is well covered. I did not observe any important omissions. However, as mentioned earlier, the literature could have been covered in the different chapters to highlight the area to which a chapter belongs and aims at

contributing. Most importantly, I would have appreciated that the candidate contrasts, here and there, his results with those obtained in the literature.

**Organization:** Each of the three chapters is well organized. I would have appreciated that the candidate spends some space in the introduction of each chapter to provide a clear road map and to highlight the interest with the question to come.

**Presentation:** The presentation and quality of writing is generally good. Some editing effort is required to publish those results that are not yet published.

**Dissemination of results:** The different chapters seem to have been presented at international conferences, and two papers were published in international academic journals, which is excellent.

### **C. Verdict**

In my opinion, the thesis is ready to be defended publicly.

I have a long series of questions, some mathematical, while others being related to the economic interpretation of setups and results. I will raise them during the defense.

Montreal, April 27, 2024

A handwritten signature in black ink, appearing to read 'G. Zaccour', with a long horizontal stroke extending to the right.

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